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# ADVANCED ARITHMETIC
FOR
# SCHOOLS & COLLEGES
BY
THOMAS W. PIPER.

# ADVANCED ARITHMETIC

FOR

# SCHOOLS AND COLLEGES.

BY

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# PREFACE.

ARITHMETIC may be studied with two different *objects*, and may be taught by two different *methods.*

The first object in studying Arithmetic is to acquire accuracy and facility in computation; the second is to cultivate the reasoning powers, by exercising them upon the various problems presented by Arithmetic.

It does not necessarily follow that the two different *methods* adopted for teaching Arithmetic should correspond to the two *objects* mentioned above; indeed, it is hoped that the present treatise will be in itself a proof that Arithmetic may be so taught as to secure both these objects at once.

It is much to be feared that in many cases Arithmetic is taught in such a way as to exercise the *memory* alone; its higher object, to exercise and strengthen *the reasoning faculties,* being more or less completely overlooked. Rules are given, illustrated and examined upon, sometimes without any explanation at all, sometimes, however, with an explanation of the Reason of the Process *following* instead

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# ARITHMETIC.

## CHAPTER I.

## DEFINITIONS.

**1. Arithmetic** is the science which treats of Numbers.

**2. Number** is the expression of the relation of any magnitude compared with the Unit.

By Magnitude is meant everything which is capable of being augmented or diminished.

**3. A Unit** is a magnitude taken as a term of comparison, which magnitude is supposed to be one and undivided.

The Unit must always be of the same kind of magnitude as that which it is used to measure. Thus, in estimating *length*, we use the length of a given line as the Unit; in calculating *surface*, we use the surface of a small square, called "a square foot," as the Unit; in determining *weight*, we use a given weight, called a "pound," as the Unit.

The Unit is not necessarily indivisible into other units; in fact, it generally is divided into units of lower denominations, as will hereafter be explained; but we may illustrate our remarks here by an example. Thus, estimating the strength of an army, we may say perhaps that it contains nine *regiments*, where *a regiment* is the Unit; or, we may say that it contains nine thousand *men*, where *a man* is the Unit.

**4. Numbers** are considered as being either **Abstract** or **Concrete.**

ABSTRACT NUMBERS are those of which the nature of the unit is not determined.

CONCRETE NUMBERS are those of which the nature of the unit is determined.

In the expressions *five horses*, *seven tables*, the numbers *five* and *seven* are called Concrete Numbers, because they refer to the particular units *one horse*, *one table;* but when we speak of *five* and *seven* without reference to any particular unit, these numbers are both Abstract Numbers.

**5.** In common arithmetic, all numbers are expressed by means of the significant figures, 1, 2, 3, 4, 5, 6, 7, 8, 9 (named respectively *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*), and the figure 0, which is called *a cypher* or *zero*, and which has no value in itself.

The name *Digits* has been applied to these ten figures, but they are more properly called *the nine digits and zero.*

**6.** The value of every figure depends upon its position.

1st. When any one of the figures mentioned above stands by itself, its value is expressed by its name. Thus, the digit 5 standing by itself signifies *five units;* and this value of the digit is called its **Simple or Intrinsic Value.**

2nd. When any one of the digits is followed on the right by another digit, its value is increased ten-fold. Thus, the digit 6 followed on the right by 4 forms therewith the number 64 (*sixty-four*), where the value of the figure 6, being *sixty*, is ten times its simple value.

This value of the digit is called its **Local Value.**

**7.** The simple value of the figure 1 is called UNITY, and a collection of *ones*, or *unities*, is called a WHOLE NUMBER, or AN INTEGER.

Thus, the digits 2, 3, 4, 5, 6, 7, 8, 9 are *Integers.*

# NOTATION AND NUMERATION.

## The Arabic System.

8. **Notation** is the art of writing down in figures any number expressed in words.

**Numeration** is the art of reading in words any number expressed in figures.

It has been said above that all numbers are expressed in figures by means of the nine digits and zero. We have now to see how this is done.

Let it be remembered that—

1st. All whole numbers less than ten are expressed by means of the nine digits.

2nd. The value of any digit is increased ten-fold by writing a figure on the right of it.

It follows that—

1st. TEN may be expressed by writing 1 and a 0 on its right—thus, 10; for the value of the 1 is increased ten-fold by the zero which follows it.

Similarly,

TWENTY, THIRTY, FORTY, FIFTY, SIXTY, SEVENTY, EIGHTY, and NINETY, may be expressed thus—

20, 30, 40, 50, 60, 70, 80, and 90 respectively.

2nd. A HUNDRED may be expressed by first writing *ten*, 10, and then placing a 0 after it—thus, 100: for the value of the number 10 is increased ten-fold by writing a figure after it.

Similarly,

TWO HUNDRED, THREE HUNDRED, FOUR HUNDRED, FIVE HUNDRED, &c., may be expressed thus—

200, 300, 400, 500, &c.

3rd. NUMBERS BETWEEN TEN AND TWENTY, BETWEEN TWENTY AND THIRTY, &c., AND ALSO BETWEEN ONE HUNDRED AND TWO HUNDRED, &c., may now be expressed by an application of the foregoing statements, thus—

**Ex. 1.** *Let it be required to write the number Thirty-six.*

We first consider that we have the number **Thirty** to express, and, as *thirty* is equal to *ten times three*, we first write down 3, and then make it *thirty* by writing some other figure after it. Now, we might use a cypher for this purpose, as before, but since we have to express **Six** besides *thirty*, we write a 6 after the 3, and then read the figures 36 as **Thirty-six.**

**Ex. 2.** *Let it now be required to write the number Four Hundred and Thirty-six.*

Since **Four Hundred** may be expressed by 400, and **Thirty-six** by 36, we are able to express **Four Hundred and Thirty-six** by 436—*i.e.*, by substituting the figures 3, 6, in place of the two cyphers in 400.

9. It has been stated above that

"*The value of any digit is increased ten-fold by writing a figure on the right of it.*"

This proposition may also be stated as follows :—

*The local value of any digit is increased ten-fold by moving it one place farther to the left whenever this digit forms one in a group of figures expressing one number.*

This will be more easily understood from the following explanations :—

1. Whenever a group of figures expresses one whole number,

*The figure on the extreme right* is said to occupy the place of UNITS;

*The figure occupying the place second from the right* is said to occupy the place of TENS; and

*The figure occupying the place third from the right* is said to occupy the place of HUNDREDS.

2. When the number consists of more than three digits, they are usually divided into *groups of three digits each*, thus—

76,543,218,

where it will be observed that *the grouping proceeds from the right.*

Each of these groups is read as a distinct and independent number—thus, in the above example,

76 is read *Seventy-six;*
543 is read *Five Hundred and Forty-three;*
218 is read *Two Hundred and Eighteen;*
but, whereas the
218 is read *Two Hundred and Eighteen* simply, the
543 is read *Five Hundred and Forty-three* THOUSAND,
and the
76 is read *Seventy-six* MILLION:

The whole eight figures being therefore read thus—*Seventy-six Million, Five Hundred and Forty-three Thousand, Two Hundred and Eighteen.*

**10. The complete system of Notation** will now be evident from the following table:—

| TRILLIONS. | | | BILLIONS. | | | MILLIONS. | | | THOUSANDS. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | HUNDREDS. | TENS. | UNITS. |
| 2 | 7 | 8, | 2 | 1 | 3, | 7 | 0 | 5, | 8 | 6 | 4, | 9 | 1 | 6 |

and so on to quadrillions, &c.

*N.B.*—1. A difficulty presents itself in writing such a number as FIVE HUNDRED AND EIGHT. This may easily be overcome thus—

Let the pupil consider that, if the number to be written had been *Five Hundred and, say, Sixty-eight*, it would be written 568; but, as nothing was said about Sixty, the place of the Six must be supplied by a cypher—thus, 508.

In all cases where the number to be written does not include digits in all the possible places, the following rule should be followed:—*Put a cypher in every place in which a digit might have been placed.*

2. It should also be carefully noticed that the number 2572 is equal to 257 tens and 2 units, also to 25 hundreds 7 tens and 2 units, &c. A knowledge of this will assist the pupil in understanding the reason of the processes of Addition, Subtraction, &c.

## Exercise 1.

1. Write down in figures—

Seventy, one hundred, seven hundred, nine hundred, sixteen, nineteen, eleven, thirteen, twelve, fifty-two, ninety-one, forty-three, thirty-eight, one hundred and sixty-four, two hundred and twenty-two, seven hundred and eighty-nine, three hundred and fifty-five, eight hundred and nineteen.

2. Write down in words—

30, 200, 500, 400, 800, 14, 17, 18, 16, 15, 39, 62, 74, 93, 24, 729, 485, 619, 324, 713, 842.

3. Write down in figures—

Forty-two thousand nine hundred and eighty-one;
Seven hundred and thirty-four thousand six hundred and thirteen;
Four million six hundred and twenty-five thousand three hundred and thirty-seven;
Eight hundred and thirty-nine million one hundred and forty-two thousand three hundred and seventeen.

4. Write down in words—

3645, 7381, 9165, 82743, 694172, 31429, 748362, 8142, 7381416, 2839743, 28614912, 843916485, 721916811, 3141627185, 239111225218, 432651412759888.

5. Write down in figures—

Two hundred and three, six hundred and one, five thousand four hundred, sixteen million nineteen thousand and four, four hundred million thirty thousand and twelve, eighteen million three hundred and forty thousand, two hundred million one thousand and one, two billion and sixteen, thirty billion one hundred million forty thousand and fifty.

6. Write down in words—

404, 850, 603, 7940, 6089, 17408, 209106, 18000003, 400500060, 810010099, 82000416829, 3001020116410.

7. Write down in figures—

Three *tens* six *units*, sixty-nine *tens* four *units*, eighty-six *tens* three *units*, four hundred and thirty-three *tens* two *units*, forty-three *hundreds* three *tens* two *units*, seven thousand four hundred and six *tens* one *unit*, seven hundred and forty *hundreds* six *tens* one *unit*, twenty-three thousand eight hundred and forty-one *tens* nine *units*, two thousand three hundred and eighty-four *hundreds* one *ten* nine *units*.

## The Roman Notation.

**11.** The system of Notation described above is the one in general use at the present time, and is called the "**Arabic Notation,**" because it was introduced into Europe by the Arabs, who had obtained it from the Hindoos. Another and more cumbrous system was in use among the Romans, of which the following is a brief description:—

1st. Instead of *figures* being used to express numbers, the following *letters* were employed—viz., I, V, X, L, C, D, M, of which the simple values were respectively 1, 5, 10, 50, 100, 500, 1000.

*Five hundred* was primarily represented by IƆ, which was afterwards contracted to D. CIƆ was also employed to represent *one thousand.*

2nd. *If two characters of the same value were placed side by side*, or, *if a character was followed by one of less value than itself*, the number denoted by the expression was the *sum* of their simple values—thus, XX = 20; XI = 11.

3rd. *If a character was followed by one of greater value than itself*, the number denoted by the expression was the *difference* of their simple values—thus, IX = 9; XL = 40.

4th. The value of any character was increased a thousand-fold by drawing a line over it—thus, $\overline{\text{V}} = 5000$.

5th. The value of IↃ was increased ten-fold by every Ↄ annexed to it; and the value of CIↃ was increased ten-fold by adding to it a C and a Ↄ—thus, 5000 was indicated by IↃↃ, and 10000 was denoted by CCIↃↃ.

## Exercise 2.

1. Write in words and Arabic figures the numbers expressed by CC, XC, XCIX, LXXV, CCXLIII, $\overline{\text{VI}}\text{V}$, $\overline{\text{M}}$, $\overline{\text{CXLIII}}\text{LI}$.

2. Express in the Roman Notation the numbers 30, 17, 1142, 6018, 24029, 81000.

---

## USE OF SIGNS IN ARITHMETIC.

**12.** In arithmetic we make use of the following signs to indicate the various operations:—

+ (*plus*), placed between two or more numbers, signifies that they are to be added together—thus, $7 + 3 + 4 + 2$, indicates that we are to find one number equal to the sum of these four numbers.

— (*minus*), placed between two numbers, signifies that the number *before* which it stands is to be taken from the number *after* which it stands—thus, $7 - 3$ indicates that we are to find what number remains when *three* has been taken from *seven*.

× (*into*), placed between two or more numbers, signifies that they are to be multiplied together—thus, $6 \times 4 \times 3$ indicates that we are to find one number equal to *six* times *four* times *three*.

Multiplication is also sometimes signified by placing a dot between the number to be multiplied together—thus, $6 \times 4 \times 3$ might be expressed by $6.4.3$.

$\div$ (*by*), placed between two numbers, signifies that the number *after* which it occurs is to be divided by the number *before* which it occurs—thus, $8 \div 2$ indicates that we are to divide *eight* by *two.*

Division is also sometimes expressed by writing the number to be divided *above* a line, and the number by which it is to be divided *below* that line—thus, $8 \div 2$ may be expressed by $\frac{8}{2}$.

$=$ (*equal*), placed between two numbers, signifies that they are equal to each other—thus, $6 + 1 = 3 + 4$.

(The expression "$6+1=3+4$" is called an *equation*.)

$\sim$ placed between two numbers, signifies that the less of them is to be taken from the greater—thus, $3 \sim 8$ indicates that *three* is to be taken from *eight.*

$.\dot{}.$ (*therefore*) is often used in stating a method by which an answer has been obtained.

**Brackets**, which are of several kinds—*e.g.*, ( ), { }, [ ]—are used to denote that all numbers included within any pair of them are to be considered as forming but one number, and are therefore to be equally affected by any number not included within the same pair of brackets, thus—

$$(6 + 3) \times 8 = 9 \times 8 = 72.$$

$$\begin{aligned} \text{Also,}\quad & [8 + 2\,\{9 + 3\,(4 + 3) + 17\} + 21] \times 9 \\ & = [8 + 2\,\{9 + 21 + 17\} + 21] \times 9 \\ & = [8 + 94 + 21] \times 9 \\ & = 123 \times 9 \\ & = 1107. \end{aligned}$$

**A Vinculum** is a sign sometimes used instead of brackets. It consists of *a line* drawn over the numbers to be considered as forming one number—thus, $2 \times \overline{8 + 3} = 2 \times 11 = 22$.

## Exercise 3.*

**Find the value of—**

1. $4 + 3 + 2 + 7$, $6 + 7 + 9 + 10$, $21 + 18 + 13$, $17 + 2 + 40$.
2. $19 - 6$, $28 - 13$, $17 - 2$, $104 - 5$, $77 - 61$, $804 - 54$.
3. $6 \times 2 \times 8$, $7 \times 1 \times 4 \times 2$, $2 \times 3 \times 5$, $8 \times 16 \times 2 \times 4$, $17 \times 5 \times 8$.
4. $18 \div 3$, $20 \div 10$, $64 \div 4$, $81 \div 9$, $74 \div 2$, $136 \div 4$.
5. $86 \sim 14$, $109 \sim 3$, $21 \sim 500$, $701 \sim 683$, $20 \sim 1000$.
6. $3 \times (2 \times 4)$, $8 \times (17 \times 1)$, $(16 - 3) \times 2$, $18 - 14 + 7$, $(61 + 29) \div 3$.
7. $\{(28 \div 7) + 9\} \times 2$, $\{(16 + 18 - 2) \div 8 + 4\} \times 3$, $\{(154 + 44) \div 9\} \times 6$.
8. $[\{(19 + 17) \div 6 + 4\} \times 10 - 91] \div 3$, $[\{27 + 14 + 6(3 + 15 - 13) - 11\} \div 5] \times 7$.
9. $\overline{7 \times 4} \times \overline{6 \times 2}$, $\overline{8 - 3 + 7 + 9} \div \overline{4 + 5 - 2}$, $\overline{(18 + 16 - 2 \times 12)} \times 8$.

# SIMPLE RULES.

## I. Addition.

**13. Addition is of two kinds, viz.:—**

1st. Simple Addition, in which the numbers to be added together are either all abstract numbers or all concrete numbers of the same denomination—as all *pounds*, all *yards*.

2nd. Compound Addition, in which the numbers to be added together are all concrete numbers of the same kind, but not expressed in one denomination—as when one quantity expressed in *yards, feet, and inches*, is to be added to another quantity also expressed in *yards, feet, and inches*.

---

* The pupil had better omit this exercise till he has worked as far as § 30, when he had better return to it.

**14. The object of Addition is to find a number equal to two or more numbers taken together.**

This one resulting number is called the **sum** of the numbers which were added together to produce it—thus, 8 is the *sum* of $5+2+1$.

**15. Simple Addition** may be distinguished into two cases, viz. :—

*Case* 1. Addition of numbers which are each *less than ten.*

*Case* 2. Addition of numbers some, or all, of which are *greater than ten.*

Or, in other words,

*Case* 1. Addition of numbers requiring only one figure to express them.

*Case* 2. Addition of numbers some, or all, of which require more than one figure to express them.

**16. Addition of numbers requiring only one figure to express them** is a purely mental process, involving a knowledge of the Addition Table merely, thus—

**Ex.** *Add together* 6, 8, 4, *and* 3.

One process serves here for the whole operation. We calculate thus—

*Six* and *Eight* make *Fourteen,*
*Fourteen* and *Four* make *Eighteen,*
*Eighteen* and *Three* make *Twenty-one;*
∴ Set down 21, *Answer.*

**17. Addition of numbers some or all of which require more than one figure to express them,** is not so easily performed, nor is the reason of the process by which it is done so easily understood.

**Ex. 1.** *Add together* 23 *and* 64.

*First,* we may calculate thus—

*Three* and *Four* make *Seven,* and
*Twenty* and *Sixty* make *Eighty;*
Therefore, *Twenty-three* and *Sixty-four* make *Eighty-Seven.*
Set down 87, *Answer.*

Or, *Secondly*, we may proceed thus—

*Twenty-three* = 2 tens + 3 units,
*Sixty-four* = 6 tens + 4 units;
∴ *Twenty-three* + *Sixty-four* = (6 + 2) tens + (4 + 3) units
= 8 tens + 7 units
= 87, *Answer.*

By inspecting this process we see that **the answer in this case is a number which has the sum of the units of the given number in its units' place, and the sum of the tens in its tens' place.** Let us take another example, and see whether this is so in every case.

**Ex. 2.** *Add together* 69 *and* 27.

*Sixty-nine* = 6 tens + 9 units,
*Twenty-seven* = 2 tens + 7 units;
∴ *Sixty-nine* + *Twenty-seven* = (6 + 2) tens + (7 + 9) units
= 8 tens + 16 units
= 8 tens + (1 ten + 6 units)
= 9 tens + 6 units
= 96, *Answer.*

This example has shown us that, **when the sum of the units requires two figures to express it, the right-hand figure of the sum is put in the units' place of the answer, while the left-hand figure is carried to the sum of the tens, and § 10, N.B. 2 teaches us that the same thing will obtain for the sum of the tens, the hundreds, &c.,** as in the following example:—

**Ex. 3.** *Add together* 439, 62, 8341, *and* 796.

The sum of the *units* = 9 + 2 + 1 + 6 = 18.
As above, set down 8 in the units' place, and carry 1 to the tens.
The sum of the *tens* = 3 + 6 + 4 + 9 + 1 carried = 23.
Set down 3 in the tens' place, and carry 2 to the hundreds.
The sum of the *hundreds* = 4 + 3 + 7 + 2 carried = 16.
Set down 6 in the hundreds' place, and carry 1 to the thousands.
The sum of the *thousands* = 8 + 1 carried = 9.
Set down 9 in the thousands' place.

*Answer*, 9638.

By the help of the three preceding examples, the pupil will be enabled to understand the following Rule:—

**RULE.—Having expressed in figures the numbers to be added together, proceed as follows:—**

**1. Arrange them one beneath the other in such a manner that their units' figures may be in an upright row, and their remaining figures in rows parallel to the row of units, thus—**

**2. Add up the row of units, express their sum in figures, and put the units' figure of this sum beneath the row of units.**

$$\begin{array}{r} 439 \\ 62 \\ 8341 \\ 796 \\ \hline 9638 \end{array}$$

**3. Take the remaining figure (or figures) of this sum, and add it to the sum of the figures in the row of tens.**

**4. Take the units' figure of this new sum, and write it beneath the row of tens.**

**5. Take the remaining figure (or figures) of this sum, and add it to the sum of the figures in the row of hundreds, and proceed in a manner similar to that followed with the sum of the tens.**

**6. Repeat this process continually till the row of figures on the extreme left is reached, when this difference must be observed in dealing with the sum of the figures in this row, that, *instead of setting down the units' figure only, set down all the figures which express this sum.***

The Rule may be illustrated by Example 3, thus :—

**The sum of its units** is 18, which is equal to 1 *ten* + 8 *units*.
Set down 8 under the row of *units*.

**The sum of its tens** is 22, which, with the 1 *ten* carried from the row of *units*, makes 23 *tens*, which is equal to 2 *hundreds* + 3 *tens*.
Set down 3 under the row of *tens*.

**The sum of its hundreds** is 14, which, with the 2 *hundreds* carried from the row of *tens*, makes 16 *hundreds*, which is equal to 1 *thousand* + 6 *hundreds*.
Set down 6 under the row of *hundreds*.

**The sum of its thousands** is 8, which, with the 1 *thousand* carried from the row of *hundreds*, makes 9 *thousands*.
Set down 9 under the row of *thousands*.

*Answer*, 9638.

# Exercise 4.

## Exercises in Simple Addition.

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| 728 | 8291 | 914682 | 208410 |
| 362 | 7463 | 289371 | 692783 |
| 429 | 2839 | 432968 | 400667 |
| 731 | 6274 | 244312 | 862009 |

| (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|
| 7246 | 44812 | 2293148 | 2200639 |
| 839 | 9687 | 768820 | 100080 |
| 682 | 82391 | 300 | 99635 |
| 413 | 864 | 1946 | 874310006 |
| 28 | 42 | 8752 | 43 |

9. *Add together* 401, 920, 6004, and 13; also, 70050, 191006, 74, 803, 3916, and 71; also, 413172, 200060, 1924300, 81, 706, and 90020.

10. *Find the sum of* 19749, 2839614852, 439700080, 61010492, 800639721050, 3829, 64280, 3007004, and 261.

11. *Add together* eight thousand six hundred and seventeen, nine hundred and five, twenty thousand eight hundred and seventy, ninety-two, and four hundred and sixteen.

12. *Add together* thirty-eight thousand four hundred, one hundred and forty-nine thousand six hundred and ninety-two, eight hundred and five, seventy thousand eight hundred and forty-one, eight hundred and twenty-nine thousand and sixty-three.

13. *Find the sum of* seven million ten thousand eight hundred and twenty-nine, eighty thousand nine hundred and ninety-nine, sixty-three million four hundred thousand and eighty, nine hundred and seventeen, nine hundred and fourteen million four hundred and eight thousand and three, and three hundred and sixty-nine thousand and eighty-one.

14. *Find the sum of* twenty million ten thousand and sixteen, eighty-three thousand and ninety-one, five hundred and twenty-six million and twenty, eight million, eight hundred and forty-one thousand and thirty-eight, forty-three million six hundred and seventy-one thousand two hundred, five thousand and nineteen, three hundred and eighty-two thousand six hundred, and one hundred and eighty-two million and four.

## II. Subtraction.

18. **The object of Subtraction** is to find a number which shall express the difference between two given numbers; in other words, we use this part of Arithmetic to find what number will remain when a less number has been taken from a greater.

The number taken away is called the **Subtrahend**, the number from which it is taken is called the **Minuend**, and the number which remains when the former has been taken from the latter is called the **Difference**.

Like Addition, Subtraction is of two kinds, **Simple** and **Compound**, and the same distinction obtains as in Addition.

19. We find by Simple Addition that the sum of 26 and 52 is 78; therefore, if 26 be taken from 78, the remainder must be 52.

Let us now see how we can find this remainder, and then proceed to discover a rule by which the remainder may be found when any number is subtracted from a greater number.

Let the numbers be arranged one under the other as in Addition, and having the minuend above the subtrahend, thus—

78 (Minuend.)
26 (Subtrahend.)
—
52 (Difference.)

We already know that the difference is 52, so we may put it down in its proper place beneath the subtrahend, and we then observe that it may be found by subtracting—

1st. The **units** in the subtrahend from the **units** in the minuend (*i.e.*, 6 from 8), and putting down the difference (*i.e.*, 2);

2nd. The **tens** in the subtrahend from the **tens** in the minuend (*i.e.*, 2 from 7), and putting down the difference (*i.e.*, 5).

Let us now proceed to a more difficult example, and see whether an application of the same process will produce the answer, thus—

*We know by Simple Addition that* 69331 + 9273 = 78604;
*therefore,* 78604 − 9273 = 69331.

∴ Let the question stand thus—

*What is the difference between* 78604 *and* 9273?

$$\begin{array}{r} 78604 \\ 9273 \\ \hline 69331 \end{array}$$ *Answer.*

*In the units' column* we take 3 from 4, and put down the remainder 1 beneath the 3.

*In the tens' column* we find a difficulty; we cannot subtract 7 *tens* from *no tens.* We therefore take a *hundred, i.e.,* 10 *tens,* subtract the 7 *tens* from it, and, since 3 *tens* remain, place a 3 beneath the 7.

By the help of the preceding examples, the pupil will be enabled to understand the following Rule:—

**RULE FOR SIMPLE SUBTRACTION.**—Having expressed the minuend and subtrahend in figures, and arranged the latter underneath the former in such a manner that *units* are under *units*, *tens* under *tens*, &c., proceed as follows:—

1. Take the units in the subtrahend from the units in the minuend, and place the figure which expresses the difference in the units' place of the answer.

2. Proceed in the same way with *tens*, *hundreds*, &c., placing the remainders in the tens', hundreds', &c., places of the answer.

3. If a figure in the subtrahend exceeds the figure in the corresponding place of the minuend, take a 1 and place it on the left of the figure in the minuend. From the number thus formed subtract the number expressed by the figure in the subtrahend, and, having found the *difference*, express it by a figure in the corresponding place of the answer, *and carry* 1.

Add this 1 to the next figure on the left in the subtrahend, and take the sum from the corresponding figure in the minuend.

The Rule may be illustrated by an example, thus:—

*From* 75632 *take* 9481.

*In the units' column:* 1 from 2 leaves 1; set down 1.

*In the tens' column:* 8 from 13 leaves 5; set down 5, and carry 1.

*In the hundreds' column:* $(4+1)$ from $6 = 5$ from 6, which leaves 1; set down 1.

*In the thousands' column:* 9 from 15 leaves 6; set down 6, and carry 1.

*In the ten thousands' column:* the 1 carried being taken from 7 leaves 6; set down 6.

$$\begin{array}{r}75632\\9481\\\hline 66151\end{array}$$

$\therefore$ the Answer $= 66151$.

The correctness of the working of sums in Subtraction may be proved by adding together the Subtrahend and the Difference, the sum of which ought to reproduce the Minuend.

## Exercise 5.

### Exercises in Simple Subtraction.

| | | | | |
|---|---|---|---|---|
| (1.) | (2.) | (3.) | (4.) | (5.) |
| 89 | 48 | 796 | 949 | 8467 |
| 45 | 31 | 482 | 327 | 2336 |

| | | | |
|---|---|---|---|
| (6.) | (7.) | (8.) | (9.) |
| 14827 | 968354 | 7423 | 83061 |
| 3604 | 250123 | 2612 | 27038 |

| | | | |
|---|---|---|---|
| (10.) | (11.) | (12.) | (13.) |
| 508613 | 426381 | 526381 | 70006014 |
| 7081 | 30907 | 6409 | 209318 |

| | | | |
|---|---|---|---|
| (14.) | (15.) | (16.) | (17.) |
| 19106382 | 72946325 | 142857310 | 10101160330 |
| 8570991 | 869999 | 141963882 | 801209337 |

18. What number remains when one million four hundred thousand six hundred and nineteen has been taken from one hundred millions and two?

19. What is the difference between sixteen thousand and eighteen, and one million?

20. If eighty-nine be added to fourteen thousand six hundred and forty-three, how many must be added to the sum to make it ten millions?

21. Add together 493 and 18741, from the *sum* take away 28, and then say by how many this remainder exceeds the sum of 801 and 1931.

22. How much greater is 314920638 than 8996673?

23. By how many is 698390997 less than 4590132000?

24. How many must be added to the sum of 99, 416305, and 2180, that it may exceed the difference between 19104 and 605 by 1143200?

25.* *Add together the following numbers:*—Nine millions seventy thousand four hundred, three hundred and thirty-three thousand and six, four hundred and seven thousand and ninety, fifty-six thousand seven hundred; *subtract from the sum*, seven hundred and eighty-four thousand and six, and write out the answer in words.

26. *Add together* four millions seventy-eight thousand and ninety, fifty thousand six hundred and seventy-four, three hundred and five, ten millions four hundred and seven thousand and ninety-eight, seventy thousand six hundred and three; *from the sum take away* six hundred and six thousand seven hundred and seventy, and write out the answer in words.

27. *Add together* five thousand and thirteen, eighty thousand and forty-seven, twenty-nine, five hundred and sixty-eight thousand and seven. *Subtract* the result from seven hundred and three thousand seven hundred and three, and write the remainder in words.

28. *Add together* two millions seventy thousand and fifty-nine, one hundred and twenty-seven, fifty thousand and sixteen, ninety thousand and thirty-two, four hundred and seven thousand one hundred and seven. *Take away* nine hundred and seven thousand nine hundred and five, and then say how much the remainder differs from five hundred thousand, and write out the answer in words.

## III. Multiplication.

**20. The object of Multiplication** is to find the sum of a given number when repeated a certain number of times.

The number which is to be repeated, or, in other words, which is to be *multiplied*, is called the **Multiplicand**; the number which indicate how often it is to be repeated—*i.e.*, the number by which it is to be multiplied—is called the **Multiplier**; and the number which is found by multiplying the former by the latter is called the **Product**.

If more than two numbers be multiplied together, the result is called the **Continued Product**.

Multiplication is either **Simple** or **Compound**, and the same distinction is made as in Addition and Subtraction.

**21.** Multiplication is simply a shortened method of Addition—thus, if we want to know the value of 9 + 9 + 9, we find by Addition that it is 27, but by **the Multiplication Table** we find at once that *three nines make twenty-seven.*

## The Multiplication Table.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **2** | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **2** | 4 | **6** | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | **6** | 9 | **12** | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | **12** | 16 | **20** | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | **20** | 25 | **30** | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | **30** | 36 | **42** | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | **42** | 49 | **56** | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | **56** | 64 | **72** | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | **72** | 81 | **90** | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | **90** | 100 | **110** | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | **110** | 121 | **132** |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | **132** | 144 |

The manner of using this Table will be best shown by an example, thus—

*To find the product of* 7 × 8, take the horizontal column which has 7 in its left-hand square, and find what number stands in the square at which this column intersects the vertical column which has 8 in its top square. The square at the intersection contains the product—viz., 56.

*N.B.*—1. This Table should be known by heart; and it should be noticed that the product of any two numbers is the same whichever of them is taken as the multiplier—*e.g.*, 7 times 8 = 8 times 7; 19 times 141

= 141 times 19. This is illustrated in the above Table by a double row of dark figures forming the diagonal of the square.

2. No Table has been given for Addition or Subtraction. The pupil will in most cases be able to add and subtract numbers without the assistance of a Table.

**22. Simple Multiplication** is generally said to be of two kinds—viz., SHORT MULTIPLICATION and LONG MULTIPLICATION.

I. **Short Multiplication** is the name given to the process when the multiplier is 12 or less than 12. It is so called because in this case we write down the answer figure by figure at once.

**Ex. 1.** *Find the sum of* 7 *times* 6438—

BY SIMPLE ADDITION—

Having written the multiplicand *seven* times we find that

*The sum of the units is* 56: put down 6 and carry 5.

*The sum of the tens is* 21, which, with the 5 carried from the units' column, will make 26: put down 6 and carry 2.

*The sum of the hundreds is* 28, which, with the 2 carried from the tens' column, will make 30: put down 0 and carry 3.

*The sum of the thousands is* 42, which, with the 3 carried from the hundreds' column, will make 45: put down 45.

6438
6438
6438
6438
6438
6438
6438
**45066**

*Answer*, **45066.**

BY SIMPLE MULTIPLICATION—

Writing down the multiplicand once, and placing the multiplier beneath the units' figure of the multiplicand, we proceed as follows:—

8 × 7 = 56: put down 6 and carry 5.

3 × 7 = 21, which, with the 5 carried, will make 26: put down 6 and carry 2.

4 × 7 = 28, which, with the 2 carried, will make 30: put down 0 and carry 3.

6 × 7 = 42, which, with the 3 carried, will make 45: put down 45.

6438
7
**45066**

*Answer*, **45066.**

By comparing these two processes we find that in *Short*

*Multiplication the same results are obtained as in Simple Addition.*

II. **Long Multiplication** is the name given to the process generally employed when the multiplier exceeds 12.

The number at which Short Multiplication ceases to be employed and the use of Long Multiplication commences is purely optional, and depends merely on the number to which the Multiplication Table has been learnt; but 12 is the number usually adopted in schools.

**Ex.** 2. *Multiply* 5792 *by* 6438.

We must first understand that, *if any multiplicand be separated into parts, and each part be multiplied by a given number, the sum of the products thus found will be equal to the undivided multiplicand* × *that given number*, thus—

$$17 = 9 + 8$$
$$\therefore\ 17 \times 6 = (9 \times 6) + (8 \times 6).$$

We know (by § 21, N.B. 1) that 5792 × 6438 = 6438 × 5792. Also, we know that—

Since $6438 = 6000 + 400 + 30 + 8$,

$\therefore\ (5792 \times 6438) = (5792 \times 6000) + (5792 \times 400) + (5792 \times 30) + (5792 \times 8)$.

Let us apply this to find the **Rule for Long Multiplication.**

Having placed the multiplier under the multiplicand, units under units, tens under tens, &c., proceed thus:—

By Simple Multiplication and the use of § 10, N.B. 2, we find that—

$$\begin{array}{cccc} \text{THOUSANDS.} & \text{HUNDREDS.} & \text{TENS.} & \text{UNITS.} \\ 5 & 7 & 9 & 2 \\ 6 & 4 & 3 & 8 \end{array}$$

(1.) 5792 × 8 = 46336

(2.) 5792 × 3 = 17376.

∴ 5792 × 3 *tens* = 17376 *tens* = 17376

(3.) 5792 × 4 = 23168.

∴ 5792 × 4 *hundreds* = 23168 *hundreds* = 23168

(4.) 5792 × 6 = 34752.

∴ 5792 × 6 *thousands* = 34752 *thousands* = 34752

Now, by Simple Addition, the sum of these products = 37288896

∴ 5792 × 6438 = 37288896, *Answer.*

The reason of the following Rule will now be evident:—

**RULE—**

**1. Beginning with 8,** ***the units' figure of the multiplier,*** **multiply the whole multiplicand thereby, and write the product, 46336, beneath the line, as before.**

**2. Proceeding to 3,** ***the tens' figure of the multiplier,*** **multiply the whole multiplicand thereby,** ***but 6, the units' figure of the present product, must be placed exactly under 3, the present multiplier, i.e., under the tens' figure of the last product.***

**3. Advancing to 4,** ***the hundreds' figure of the multiplier,*** **multiply the whole multiplicand thereby, and write the product under the last product, in this case also placing 8, the units' figure of the present product, immediately under 4, the present multiplier.**

**4. Coming now to 6,** ***the thousands' figure of the multiplier,*** **multiply the whole multiplicand thereby, and write the product under the last product, observing the same rule in placing 2, the units' figure of the present product, exactly under 6, the present multiplier.**

**The whole column is then added up as before, and the sum is the answer.**

*N.B.*—It is not essential that we should multiply by the units' figure *first:* an inspection of the following methods of working an example will show that we can use any digit of the multiplier as the initial multiplier, provided only that in every case the units' figure of the product be placed exactly under the figure of the multiplier by which that product was obtained, thus—

**Ex.** *Multiply* 64109 *by* 70638.

(1.) Beginning with the *hundreds'* figure—

$$\begin{array}{r}
64109\\
70638\\
\hline
384654\phantom{00}\\
448763\phantom{0000}\\
192327\phantom{0}\\
512872\\
\hline
\mathbf{4528531542}
\end{array}$$

(2.) Beginning with the *ten thousands'* figure—

$$\begin{array}{r}
64109\\
70638\\
\hline
448763\phantom{0000}\\
512872\\
384654\phantom{00}\\
192327\phantom{0}\\
\hline
\mathbf{4528531542}
\end{array}$$

# Exercise 6.

## Exercises in Simple Multiplication.

| (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|
| 7243 | 9164 | 82413 | 66413 | 814372 |
| 3 | 5 | 7 | 6 | 4 |

| (6.) | (7.) | (8.) | (9.) |
|---|---|---|---|
| 284175 | 2355667 | 91643027 | 6240328 |
| 8 | 9 | 11 | 12 |

| (10.) | (11.) | (12.) | (13.) |
|---|---|---|---|
| 42874 | 6129408 | 22638409 | 91428364 |
| 13 | 23 | 57 | 85 |

| (14.) | (15.) | (16.) | (17.) |
|---|---|---|---|
| 6284 | 238074 | 72351 | 8436209 |
| 916 | 9615 | 6849 | 703 |
| | | | 25308627 |
| | | | 59053463 |
| | | | 5930654927 |

| (18.) | (19.) |
|---|---|
| 2883050503 | 42691470008 |
| 7006 | 61003 |

| (20.) | (21.) | (22.) | (23.) |
|---|---|---|---|
| 387159 | 96413 | 623100804 | 61792 |
| 60834 | 20074 | 900683 | 31846 |

| (24.) | (25.) | (26.) |
|---|---|---|
| 240639 | 27816304 | 324139 |
| 260083 | 2090604 | 620065 |

27. Find the product of 14 times the difference between 6183 and 200.

28. Find the product of the sum and difference of 2010 and 161438.

29. Find the continued product of forty-two, one hundred and six, and eighteen thousand and nineteen.

30. Multiply 620149293 by 81792648, add the sum of 4962 and 41382714 to the product, and say how much the result exceeds the continued product of 18162, 409, and 73.

31. Find the sum of the continued products of 86, 21, and 714, 26, 308, and 704, 213, 4005, and 6072; multiply the sum by itself, and say how many must be added to, or subtracted from, the product to make it equal to 6 times the continued product of 64, 32, 7, 108, and 26.

## IV. Division.

**23. The object of Division** is to find how many times a given number contains another given number.

The number to be divided is called the **Dividend**, the number by which it is to be divided is called the **Divisor**, and the number which expresses the number of times which the former contains the latter is called the **Quotient**.

The same distinction holds between **Simple** and **Compound Division** as between *Simple* and *Compound Addition*.

**24.** Division bears the same relation to Subtraction that Multiplication does to Addition, for Division may be described as—

"*The method by which we find how many times one number must be taken to make up another.*"

Or, "*The method by which we find how many times one number may be taken from another.*"

**Ex.** Let it be required *to find how many times* **three** *may be taken from* **twelve.**

We find that, taking *twelve* counters ... ... ... ... we can divide them into **four** groups of *three*, so that *three* may be taken **four** times from *twelve;* in other words, if *twelve* be divided by *three*, the Quotient will be *four*, which is thus expressed, $12 \div 3 = 4$.

**25.** The object of Division is exactly the opposite of that of Multiplication, being in fact the undoing of what may have been done by Multiplication, thus—

$8 \times 3 = 24$; this is an example of Multiplication, our object being to find what number is equal to the sum of *eight threes*. But in $24 \div 3 = 8$ we have an example of Division, our object now being to find how many times the *three* was taken to make *twenty-four*.

**The Multiplication Table** also serves as a **Division Table,** thus—

To find how many times 8 is contained in 72, find the horizontal

**column which has 8 in its left-hand square, then seek 72 in that column, and, having found it, pass the hand to the top of the vertical column in which the 72 was found. The figure 9 standing there expresses the number of times that 8 is contained in 72.**

**26.** In Division, as in Multiplication, there are two different names given to the two methods which are respectively the converse of *Short Multiplication* and the converse of *Long Multiplication.*

The method by which Division by any number less than 13 is performed is called **Short Division**, and that employed for dividing by a number greater than 12 is called **Long Division.**

**27.** *If a number be divided into several parts and each of these parts be divided by a given divisor, the sum of their quotients will be equal to the single quotient found by dividing the whole number by the same divisor,* thus—

$$72 = 30 + 18 + 24$$

$$\text{and } 72 \div 3 = (30 \div 3) + (18 \div 3) + (24 \div 3)$$

$$\textit{i.e.}, 24 = 10 + 6 + 8.$$

This proposition will be immediately found useful.

**28. Short Division** being the converse of *Short Multiplication,* it will evidently be the most satisfactory way of finding the Rule by which Short Division may be performed, to take a number, multiply it by another number, and, using the *Product* for a *Dividend* and the *Multiplier* for a *Divisor,* find some method by which the original number may be found as a *Quotient,* thus—

**Ex. 1.** $213 \times 3 = 639. \therefore 639 \div 3 = 213.$

Therefore, let the question be—

"*How many times is* 3 *contained in* 639?"

We already know that the answer is 213, but our present object is to find this answer, supposing ourselves to be totally ignorant of it.

1st. Since $639 = 600 + 30 + 9$

$$\begin{aligned} \text{it follows that } 639 \div 3 &= (600 \div 3) + (30 \div 3) + (9 \div 3) \\ &= 200 + 10 + 3 \\ &= \mathbf{213}, \textit{ Answer.} \end{aligned}$$

2nd. But it is evident that the figures of this answer are the same as would be found by dividing 639 by 3 in the manner following:—

$$3 \,|\, \underline{639}$$
$$\mathbf{213}$$

Having arranged the dividend and the divisor as above, say—

*Threes* in *Six?* (*i. e.*, How many *threes* are contained in *six?*) Two: put down **2** under the 6.

*Threes* in *Three?* One: put down **1** under the 3.

*Threes* in *Nine?* Three: put down **3** under the 9.

*Answer*, **213.**

*The Reason of this Process* is apparent from the consideration which preceded it, and may be thus expressed—

*Three* is contained twice in *six;* therefore it is contained *two hundred* times in *six hundred*, and the 2 being put in the place of hundreds indicates this *two hundred.*

*Three* is contained once in itself; therefore it is contained one *ten* times in *three tens;* and the 1 being put in the place of tens indicates this *ten.*

*Three* is contained three times in *nine*, and the 3 being put in the place of units indicates this *three.*

And therefore
the three figures **213** express the answer.

**Ex. 2.** $487 \times 3 = 1461. \quad \therefore 1461 \div 3 = 487.$

Therefore, let the question be—

"*How many times is* 3 *contained in* 1461?"

$$3 \,|\, \underline{1461}$$
$$\mathbf{487}$$

In attempting to apply the above process to this example we encounter a difficulty at the very outset, for, the thousands' figure of the dividend being 1, we cannot divide it by 3. It is evident therefore that *the answer* *must be less than a thousand*, and, this being the case, it is also evident that *the answer must be expressed by less than four figures*, because any whole number of four figures must be at least a thousand. We therefore conclude that, *when the divisor is greater than the first figure on the left of the dividend, the first figure on the left of the answer should be placed under the second (or third) figure of the dividend, counting from the left.*

The difficulty is now easily overcome by reducing the thousands to hundreds, thus—

One thousand four hundreds = fourteen hundreds;
and, since 14 ÷ 3 = 4 and 2 over,
14 hundreds ÷ 3 = 4 hundreds and 2 hundreds over;
therefore put 4 under the 4 in the dividend (*i.e.*, put 4 in the place of hundreds), to express the *four hundreds.*

Or, which comes to the same thing, proceed by the following Rule:—

**RULE.—When the first figure on the left of the dividend is less than the divisor, take the first two (or three, if necessary) figures of the dividend, find how many times the number expressed by these figures contains the divisor, put beneath the second (or third, if three be taken) figure from the left of the dividend a figure to express the times the divisor is contained in the number, and carry the figure which is over.**

The remainder of the answer may now be found by a process which depends upon the foregoing explanations; and to put the whole process clearly, the example will now be worked completely through, thus—

$$\begin{array}{r|l} 3 & \underline{1461} \\ & \phantom{1}487 \end{array}$$

*Threes in one?* there are *none;* leave the space beneath the 1 vacant.

*Threes in fourteen?* there are *four* and *two* over; put a 4 under the 4 and carry 2.

Bringing down the 6, the next figure in the dividend, and placing it beside the 2 carried, we have a number 26; therefore,

*Threes in twenty-six?* there are *eight* and *two* over; put an 8 under the 6 and carry 2.

Bringing down 1, the next figure in the dividend, and placing it beside the 2 carried, we have a number 21; therefore,

*Threes in twenty-one?* there are *seven;* put down a 7 under the 1.

*Answer*, 487.

**29. Long Division.** It will have been observed that Long Multiplication is not a process founded on any principle which differs from that of Short Multiplication, but that it is in fact nothing more than an abbreviated form for performing Short Multiplication, with high numbers for multipliers. The same may be said of *Long Division* as compared with *Short Division*—the former is nothing but a varied and accommodated form of the latter, as we shall now proceed to show—

**Ex.** $2906 \times 42 = 122052. \quad \therefore \ 122052 \div 42 = 2906.$

Therefore, let the question be—

"*How many times is* 42 *contained in* 122052?"

We will find the answer in two ways, viz.—

**I. By Short Division—**

It is evident that, if we knew the *forty-two times* Table, we could do the work by Short Division; let us therefore make out the Table as below.

Then say—

*Forty-twos in one?* there are none; therefore leave the space beneath the 1 vacant.

*Forty-twos in twelve?* there are none; therefore leave the space beneath the 12 vacant.

$$\begin{array}{r|l} 42 & \underline{122052} \\ & \phantom{12}2906 \end{array}$$

| | | 42 |
|---|---|---|
| 1 | are | 42 |
| 2 | ,, | 84 |
| 3 | ,, | 126 |
| 4 | ,, | 168 |
| 5 | ,, | 210 |
| 6 | ,, | 252 |
| 7 | ,, | 294 |
| 8 | ,, | 336 |
| 9 | ,, | 378 |

*Forty-twos in*[*] *one hundred and twenty-two?* there are *two* and *thirty-eight* over; therefore put down a 2 beneath the 2 in the thousands' place of the dividend, and carry 38.

Bringing down 0, the hundreds' figure of the dividend, and placing it beside the 38 carried, we have 380; therefore,

*Forty-twos in three hundred and eighty?* there are *nine* and *two* over; put down 9 beneath the 0 and carry 2.

---

[*] Consider thus:—Twice 42 = 84, and three times 42 = 126. Therefore 122 contains more than two forty-twos and less than three forty-twos—*i.e.*, it contains two forty-twos and a remainder—*i.e.*, it contains two forty-twos and 122 − 84—*i.e.*, it contains two forty-twos and thirty-eight remainder.

Bringing down 5, the tens' figure of the dividend, and placing it beside the 2 carried, we have 25; therefore,

*Forty-twos in twenty-five?* there are none; put a 0 under the 5, and bringing down 2, the units' figure of the dividend, place it beside the 25, thus making 252, and say,

*Forty-twos in two hundred and fifty-two?* there are *six;* put down 6 under the 2.

*Answer*, **2906.**

## II. By Long Division—

It is evident that we may place the figures 2906 (*i.e.*, the quotient) in any place on our paper, provided only that we put the figures 2, 9, 0, 6, in their proper order.

In Long Division we find it convenient to place them on the right hand of the dividend, thus—

```
42) 122052 (2906
    84
    ---
    380
    378
    ----
      252
      252
      ---
```

The method by which these figures are found will now be explained, thus—

*Forty-twos in one?* there are none.

*Forty-twos in twelve?* there are none.

*Forty-twos in one hundred and twenty-two?* When we were doing the sum by Short Division, we consulted the Table at this point, and found that the dividend contained the divisor twice, but not three times. To find the remainder, we took 84 (*i.e.*, twice 42) from 122, and the difference, 38, was the remainder. But now, instead of putting the 2 *beneath* the dividend, we place it *on the right* of the dividend, and we show how the whole calculation has been accomplished, for we put 84 (*i.e.*, twice 42) beneath the 122, and then, by subtracting, we find the remainder, 38.

Also, we bring down the 0, the hundreds' figure of the dividend, and place it beside the 38 which remained from the last operation. In this way we get the new number 380, and say—

*Forty-twos in three hundred and eighty?* The Table tells us it will "*go*" nine times; we therefore place a 9 on the right of the 2 in the quotient, multiply 42 by 9, place the product 378 beneath the 380, subtract the former from the latter, and find the remainder to be 2.

Next, we bring down 5, the tens' figure of the dividend; and having thus obtained the new number 25, we say—

*Forty-twos in twenty-five?* there are none; therefore put a **0** in the quotient, and bring down 2, the units' figure of the dividend; and having thus obtained the new number 252, we say—

*Forty-twos in two hundred and fifty-two?* The Table tells us it will *go* exactly six times; therefore put a **6** in the quotient; multiply 42 by this 6; put down the product, 252, beneath the 252; subtract, and nothing remains, neither is there any other figure in the dividend.

Therefore the ***Answer is*** **2906.**

*N.B.*—It is not usual to make out a table before commencing to work sums in Long Division. The general method is to try such a figure in the quotient as seems most likely to give a proper number for subtracting from the number produced by the carrying figure and the figure brought down from the dividend. The following is the rule generally used to test whether the trial figure is the right one:—

**RULE.—If the number produced by multiplying the divisor by the trial figure in the quotient *exceed* the number from which the product is to be subtracted, the trial figure is too great, and a less one must be tried; but if the product *be less* than the number from which it is to be subtracted, proceed to find the difference. If this difference *exceed* the divisor or be equal to it, the trial number is too small, and a greater one must be substituted for it; but if the difference *be less* than the divisor, the number tried is the correct one. Also, if the product be *equal* to the number from which it is to be subtracted, the trial figure is the correct one.**

**30.** It now remains to show that the process by which Long Division is performed is the exact converse of the method by which Long Multiplication is performed.

**Ex.** $427 \times 6358 = 2714866. \therefore 2714866 \div 6358 = 427.$

By inspecting the process by which 427 is multiplied by 6358, we find that 2714866 = 2543200 + 127160 + 44506.

$$\begin{array}{r} 6358 \\ 427 \\ \hline 44506 \\ 127160 \\ 2543200 \\ \hline 2714866 \end{array}$$

Therefore,

$$(2714866 \div 6358) = (2543200 \div 6358) + (127160 \div 6358) + (44506 \div 6358);$$

And 6358)2543200 + 127160 + 44506(400 + 20 + 7 = 427, *Answer.*

```
         2543200
         -------
                  127160
                  127160
                  ------
                           44506
                           44506
                           -----
```

## Exercise 7.

### Exercises in Simple Division.

1. 684÷2.
2. 7824÷2.
3. 90684032÷2.
4. 7192018÷2.
5. 691722÷3.
6. 8269172÷3.
7. 928480÷4.
8. 91628030÷5.
9. 7726164÷6.
10. 4270147÷7.
11. 36412838÷7.
12. 21963824÷7.
13. 81602496÷8.
14. 3276419328÷8.
15. 712830915÷8.
16. 18450639÷9.
17. 241387641÷9.
18. 8613927813÷9.
19. 44667788÷11.
20. 31462915÷11.
21. 316990837÷11.
22. 814369248÷12.
23. 719842683÷12.
24. 416827019÷12.

25. Find how many times 9 is contained in the sum of 36 times 418, 103 times 99142, and 1742 times 10649218.

26. How many times may 7 be taken away from the difference between 3081 times 364196, and 38507 times the sum of 16418, 71430, and 139164218?

27. How many twelves are required to make up the product of 44839 times 16836?

28. Divide 4719585 by 15; 121259976 by 19; 144073854 by 18; 163370127 by 23; 81789279 by 13.

29. Divide 7628409 by 19; 700600448800 by 67; 8139621 by 29; the continued product of 64, 83, 705, 6281, and 1004 by 13.

30. 2831265÷45.
31. 200914630÷79.
32. 84268044÷93.
33. 84000000000÷101101.
34. 2186315712÷604.
35. 620397182÷443.
36. 2659329034÷638.
37. 2008006009977÷47.
38. 491024326÷587.
39. 361209103÷6172.
40. 4957133022380÷60401.
41. 8463920684÷8147.
42. 1303272875÷60875.
43. 2426729184÷62288.

44. 573880765072596 ÷ 714928364
45. 71264193610 ÷ 412936.
46. Divide 30 millions by 101.
47. Multiply 6481391 by 18, and divide the product by 613004.
48. How many times may 634005008 be taken from 574601274770432?
49. How many times must 613492 be taken to make 4930021712?
50. Divide the continued product of 14, 13, 17043 and 618279 by 191.

**31.** We shall now point out some abbreviated methods* of working exercises in the Simple Rules.

**Ex.** 1. *Multiply* 6492 *by* 42.

Since 42 = 7 × 6, the answer is found as below:—

$$\begin{array}{rr} & 6492 \\ & \underline{7} \\ \text{7 times the multiplicand} = & 45444 \\ & \underline{6} \end{array}$$

∴ (6 times 7 times =) 42 times the multiplicand = 272664, *Answer.*

**Ex.** 2. *Divide* 272664 *by* 42; *and* 9143826 *by* 63.

$$42\left\{\begin{array}{r|r} 6 & 272664 \\ \hline 7 & 45444 \\ \hline & 6492 \end{array}\right. \text{, } \textit{Answer.} \qquad 63\left\{\begin{array}{r|l} 9 & 9143836 \\ \hline 7 & 1015981+7 \\ \hline & 145140+1 \end{array}\right\} 16$$

The 16 over is found by multiplying the first divisor, 9, by the last remainder, 1, and adding the product to the first remainder, 7.

**Ex** 3. *Multiply* 9643287 *by* 378427.

$$\begin{array}{lr} & 9643287 \\ & \underline{(378)(42)(7)} \\ \text{7 times the multiplicand} = & 67503009 \\ \left.\begin{array}{l}\text{42 times the multiplicand} = \text{6 times 7 times the multi-} \\ \quad\text{plicand} = \text{6 times 67503009}\end{array}\right\} = & 405018054 \\ \left.\begin{array}{l}\text{378 times the multiplicand} = \text{9 times 42 times the} \\ \quad\text{multiplicand} = \text{9 times 405018054}\end{array}\right\} = & \underline{3645162486} \\ & \textit{Answer}\text{, } 3649280169549 \end{array}$$

---

* The student is referred to the author's "*Mental Arithmetic for Pupil Teachers and Students in Training Colleges*," for further information on this subject.

**Ex. 4.** *Multiply* 2384 *by* 19.

2384
19
45296

(4×9)=36; set down 6 and carry 3.

(8×9)+3 carried+4, the units' figure of the multiplicand = 79; set down 9 and carry 7.

(3×9)+7 carried+8 the tens' figure of the multiplicand = 42; set down 2 and carry 4.

(2×9)+4 carried+3 the hundreds' figure of the multiplicand = 25; set down 5 and carry 2.

2 carried+2 the thousands' figure of the multiplicand = 4; set down 4.

*Answer*, 45296.

**Ex. 5.** *Multiply* 843719 *by* 80.

Since 80 = 10 × 8, we may use the same process here that we did in Ex. 1; but, multiplying 843719 by 10, we get 8437190, where it will be noticed that *the same result would be obtained by simply annexing a cypher to the right of the multiplicand.*

The operation is therefore usually performed as below :—

843719
80
67497520, *Answer.*

1st. Bring down the cypher.

2nd. Multiply the multiplicand by 8.

**Ex. 6.** *Multiply* 43926 *by* 64000.

This will be done in a manner similar to that by which Ex. 5 was done, thus—

43926
64000
175704000
263556
2811264000, *Answer.*

**Ex. 7.** *Divide* 271830 *by* 30.

3,0 | 27183,0
9061, *Answer.*

1st. Cut off a cypher from the divisor, and also one from the dividend.

2nd. Divide 27183, the remainder of the dividend, by 3, the remainder of the divisor: the quotient 9061 is the answer.

**Ex. 8.** *Divide* 49386 *by* 630.

1st. Cut off a figure from the divisor and one also from the right of the dividend.

2nd. Divide the remainder of the dividend by the remainder of the divisor: the quotient is *the whole number* of the answer.

The *remainder* is found by placing the 6 which was cut off from the dividend, beside the 24 which remained after subtracting 504 from 528.

$$\begin{array}{r} 63{,}0)4938{,}6(78 \\ 441\phantom{,6(78} \\ \hline 528\phantom{,6(78} \\ 504\phantom{,6(78} \\ \hline 24\phantom{,6(78} \end{array}$$

*Answer*, 78 *and* 246 *over*, *i.e.* $78\frac{246}{630}$.

**Ex. 9.** *Multiply* 86423 *by* 999.

$$\text{By Subtraction}\left\{\begin{array}{rl} 1000 \text{ times } 86423 = & 86423000 \\ 1 \text{ time } 86423 = & 86423 \\ \hline 999 \text{ times } 86423 = & 86336577, \textit{Answer}. \end{array}\right.$$

# Exercise 8.

## Miscellaneous Exercises in the Simple Rules.

1. Multiply 31482 by 974; divide the product by 583. What must be taken from the quotient to leave exactly 100?

2. Explain, as you would to children, how to subtract 1607 from 10340.

3. *Divide* nine millions nine hundred and ninety-eight thousand five hundred and fifty, *by* seven thousand eight hundred and forty-two, and write out the answer in words.

4. Explain the difference between the methods of Long and Short Division.

5. *Divide* one million four hundred and seventy thousand three hundred and twelve, *by* two hundred and sixteen, and explain each step of the process.

6. Find the value of $\frac{(37004 \times 8607 + 157)}{5} - \frac{5048 \times 206}{8} + \frac{371700.}{59}$

7. What number subtracted 28 times from 479632 will leave 20 as a remainder?

8. Divide 3672972 by 2 × 3 × 6 (short division). Explain every step of the process.

9. *Divide* nine millions and forty-four thousand four hundred and nineteen, *by* eight thousand and eleven

10. *Divide* six millions seven hundred and twenty-three thousand eight hundred and sixty-four, *by* forty thousand and twenty-three.

11. Find the value of (503407 × 82) − (3792 × 48) + (94185 ÷ 273).

12. In a question of Division, if any three of the four terms, divisor, dividend, quotient, remainder, be given, the remaining term may be found. Prove this by examples.

13. What is the difference between the thousandth part of a million and the 25th part of 20000?

14. *Multiply* four thousand and ninety-six *by* six hundred and twenty-five; and *divide* eight millions *by* one hundred and twenty-five; using, in each case, the quickest and easiest method that you know.

15. In a Multiplication sum the multiplicand was 72038, and the product was 332023142; what was the multiplier?

16. In a Division sum the quotient was 9008, and the dividend was 3675291024; what was the divisor?

17. The divisor of a Long Division sum was 81200, and the quotient was 84106000; what was the dividend?

18. In dividing £27770000 amongst a body of men it was found that after each man had received £324 there yet remained £5792; how many men was the money divided amongst?

19. If the quotient of a Division sum be $\frac{1}{16}$ of the divisor, and the divisor be 83 times greater than the remainder, find the dividend when the remainder is 212.

20. Multiply 200436972018 by 138064672968 in the shortest manner you know.

21. What number is that to which if 38 and 5 times 38 be added, and the sum so found be further increased by 7 times itself, the total sum is 2400?

22. What number is that which being increased by 23, and then multiplied by 302, gives a product which is greater by 183946 than the product found by multiplying 500 by one-half of itself?

23. If 80800 be multiplied by one-tenth of itself, and the product be reduced by the *sum* of 1401 taken fifteen thousand times, what number can be taken exactly one hundred times from the *difference*?

24. What number is that which being increased by 60390, 70109, and 609300, and the *sum* being multiplied by 38, gives a *product* of 54784562?

## *CHAPTER II.*

### The Greatest Common Measure and Least Common Multiple.

### DEFINITIONS.

**32.** 1. When one number will divide another number without remainder, the first is said to be a MEASURE of the second, and the second is said to be a MULTIPLE of the first.

Thus, 6 and 7 will both divide 42 without remainder; therefore 6 and 7 are each said to be *measures* of 42, and 42 is said to be a *multiple* of 6 and also of 7.

2. When one number is a measure of another, the first is said TO MEASURE the second.

Thus, in the above example 6 is said to measure 42.

3. When one number will divide each of two or more numbers without remainder, it is said to be a COMMON MEASURE of these numbers.

Thus, 4 will measure 24, 32, and 112; therefore 4 is a *Common Measure* of 24, 32, and 112.

4. The greatest number which will divide each of two or more numbers an exact number of times (*i.e.*, without remainder), is called the GREATEST COMMON MEASURE of these numbers.

Thus, in the above example, 8 is the greatest number which will measure 24, 32, and 112, therefore 8 is said to be the *Greatest Common Measure* (G.C.M.) of 24, 32, and 112.

5. When one number can be exactly divided by two or

more numbers, it is said to be a COMMON MULTIPLE of these numbers.

Thus, since 72 can be exactly divided by 8 and also by 12, it is said to be a *Common Multiple* of 8 and 12.

6. The least number which can be divided by each of two or more numbers is called the LEAST COMMON MULTIPLE (L.C.M.) of these numbers.

Thus, 24 is the *Least Common Multiple* of 8 and 12, because it is the least number which can be divided by each of these without remainder.

7. A COMPOSITE NUMBER is a number which is the product of two or more other numbers—*i.e.*, a number which is exactly divisible by other numbers besides itself and unity.

Thus, 63 is a composite number, because it is the product of 7 × 3 × 3. Hence, every *multiple* of two or more numbers is a *composite number*.

8. THE FACTORS OF ANY NUMBER are those numbers which when multiplied together will produce that number.

Thus, 8 and 3 are the *factors* of 24.
Hence, every *measure* of a number is a *factor* of that number.

9. A PRIME NUMBER is one which cannot be resolved into factors—*i.e.*, a number which is not exactly divisible by any number besides itself and unity.

Thus, 7, 3, 41, 113 are *prime numbers*, neither of them being reducible to factors.

Every measure of a number is not necessarily *a prime number*, but every number may be reduced into *prime factors*—*i.e.*, into factors which are prime numbers. Thus, 24 may be measured by 6 and by 4, but neither 6 nor 4 is a prime number, the first being the product of 3 × 2, and the second of 2 × 2.

N.B.—In speaking of the factors of a number it is usual to exclude Unity, so that a number divisible only by itself and Unity is considered as having no factors nor measures.

10. Numbers which have no common measure but unity are said to be PRIME TO EACH OTHER.

Thus, 14 and 55 are prime to each other, because neither 7 nor 2, the measures of 14, is a measure of 55.

## HOW TO FIND THE G.C.M. and the L.C.M., of two or more numbers.

**33.** Two problems are suggested by the consideration of this part of our subject, viz. :—

1st. How to find the G.C.M. of two or more numbers :

2nd. How to find the L.C.M. of two or more numbers.

In investigating the methods by which each of these may be found, we shall be materially assisted by the following proposition :—

"*A Common Measure of any two numbers is also a measure of their sum, of their difference, and of any multiple of either of them.*"

Thus, 7 is a Common Measure of 28 and 91;
7 is also a measure of their *sum*, *i.e.* of 28+91, *i.e.* of 119.
7 is also a measure of their *difference*, of 91−28, *i.e.* of 63.
Also, 7 is a measure of 5 times 91, and of any other *multiple of* 91.
And, 7 is a measure of 8 times 28, and of any other *multiple of* 28.

### I. To find the G.C.M. of two numbers.

**RULE.—Divide the greater number by the less: (if there be no remainder the less number is the G.C.M., but, if there be a remainder) divide the first divisor by this remainder; if there be still a remainder, divide the first remainder by this second remainder. Continue this process till a remainder is found which divides the last remainder exactly. This last divisor is the G.C.M.**

**Ex.** *Find the G.C.M. of* 1633 *and* 923.

```
923)1633(1
     923
     ---
     710)923(1
         710
         ---
         213)710(3        ∴ 71 is the G.C.M. of 923 and 1633.
             639
             ---
              71)213(3
                 213
                 ---
```

*The Reason of the Process* depends upon the preceding proposition, § 33, thus—

Any number which is a measure of 923 and 1633 is a measure also of their *difference*, *i.e.* of 1633 − 923, *i.e.* of 710,

And is therefore a measure of the *difference* of 923 and 710, *i.e.* of 213,

And therefore of any *multiple* of 213, *e.g.* of 639,

And therefore of 710 and 639,

And therefore of their *difference*, *i.e.* of 710 − 639, *i.e.* of 71,

And therefore of 213 and 71,

And therefore, since 71 is *a measure* of itself and 213, it is also a measure of the two given numbers 923 and 1633.

Also, 71 is the *Greatest Common Measure* of the given numbers, for it has been shown that any number which is *a measure* of 923 and 1633 is also *a measure* of 71, and, since 71 is the greatest measure of itself, it is the *Greatest Common Measure* of 923 and 1633.

*The Truth of the Result* may be thus demonstrated:

71 is a measure of itself and 213,

$\therefore$ 71 is a measure of itself and three times 213, *i.e.* of 639, and

$\therefore$ 71 is a measure of itself and of 639 + 71, *i.e.* of 710,

$\therefore$ 71 is a measure of itself and 213 + 710, *i.e.* of 923,

$\therefore$ 71 is a measure of itself and 923 + 710, *i.e.* of 1633,

$\therefore$ 71 is a measure of *itself*, 923, and 1633,

$\therefore$ as above, 71 is the G.C.M. of 923 and 1633.

## II. To find the G.C.M. of three (or more) numbers.

**RULE.—Find the G.C.M. of any two of the numbers; find also the G.C.M. of this G.C.M. and the remaining number. The G.C.M. last found is the G.C.M. of the three given numbers.**

**To find the G.C.M. of *four* numbers, take the G.C.M. of the first three numbers, and find the G.C.M. of this G.C.M. and the remaining number;**

*Or*, find the G.C.M. of the first two, and the G.C.M. of the last two, numbers: the G.C.M. of these two G.C.M.'s will be the G.C.M. of the four given numbers.

**Ex. 1.** *Find the G.C.M. of* 27, 63, *and* 42.

The G.C.M. of 27 and 63 is 9.

The G.C.M, of 9 and 42 is 3.

$\therefore$ The G.C.M. of 27, 63, and 42 is 3, *Answer.*

**Ex.** 2. *Find the G.C.M. of* 39, 52, 336, *and* 210.

The G.C.M. of 39 and 52 is 13.
The G.C.M. of 336 and 210 is 42.
∴ The G.C.M. of 39, 52, 336, and 210 = the G.C.M. of 13 and 42
= 13, *Answer.*

**Ex.** 3. *Find the G.C.M. of* 138, 299, 759, 2415, *and* 2254.

The G.C.M. of 138, 299, and 759 is 23.
The G.C.M. of 2415 and 2254 is 161.
∴ The G.C.M. of 138, 299, 759, 2415 and 2254 = the G.C.M. of 23 and 161
= 23, *Answer.*

We shall require the following proposition in finding the *Least Common Multiple* of two or more numbers :—

### To reduce a number to its prime factors.

**RULE.—Divide the given number by any prime factor of that number; do the same if possible to the quotient, and repeat the process till a quotient is found which is itself a prime number.**

**The prime factors of the given number will consist of the divisors and the last quotient.**

**Ex.** *Reduce* 6120 *to its prime factors.*

The prime factors of 6120 are
5, 2, 2, 2, 3, 3, 17,
or, in other words,
$6120 = 5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 17.$

$$\begin{array}{r|l} 5 & \underline{6120} \\ 2 & \underline{1224} \\ 2 & \underline{612} \\ 2 & \underline{306} \\ 3 & \underline{153} \\ 3 & \underline{51} \\ & 17 \end{array}$$

## 34. III. To find the L.C.M. of two or more numbers.

(1.) *A Common Multiple of any two or more numbers can always be found by multiplying them together.*

Thus, a common multiple of 6 and 7 is 6 × 7, *i.e.* 42; a common multiple of 5, 3, 7, 13, is $5 \times 3 \times 7 \times 13$, *i.e.* 1365.

But, since *any composite number* × *another number* = *the product of the prime factors of these numbers,*

We may express the same proposition thus—

*A common multiple of two or more numbers can be found by multiplying together all their prime factors*, thus—

A common multiple of 9, 6, and 28=(3×3)×(3×2)×(7×2×2)=1512.

(2.) *Now it is evident that any number which contains* 3×3 *is a multiple of* 9;

*Also, that any number which contains* 3×2 *is a multiple of* 6;

*Lastly, that any number which contains* 7×2×2 *is a multiple of* 28.

*From which it clearly follows that—*

(3×3) is a multiple of 9 because it contains 3×3: also that

(3×3×2) is a multiple of 6 because it contains 3×2: it is also a multiple of 9 because it contains 3×3: therefore it is a common multiple of 9 and 6. Further,

(3×3×2×2×7) is a multiple of 28 because it contains 7×2×2: it is also a common multiple of 9 and 6 because it contains 3×3×2: therefore it is a common multiple of 9, 6, and 28.

*i.e.*, 252 is a Common Multiple of 9, 6, and 28; and this number, being the least number which will meet the requirements of the several given numbers, is the *Least Common Multiple* of these numbers.

(3.) It is also evident that *if one number contains another number an exact number of times, the number which contains the other is a multiple of that other,*

From which it follows that—

*In finding the L.C.M. of any numbers we may leave out of consideration all those given numbers which are contained in other given numbers.*

Thus, in finding the L.C.M. of 2, 8, 9, and 27, we may strike out the 2 because it is contained in the 8, and the 9 because it is contained in the 27, so that the L.C.M. of 2, 8, 9, and 27 may be found by finding the L.C.M. of 8 and 27.

(4.) Putting together all these considerations we find a method for finding the L.C.M. of two or more numbers, which will perhaps be best explained by an example :—

**Ex.** *Find the L.C.M. of* 2, 7, 9, 36, 25, 55, *and* 350.

*First :* Strike out 2 and 9, because they are each contained in 36, and 7 and 25, because they are each contained in 350. It now remains to find the L.C.M. of 36, 55, and 350..

*Second :* Reduce 36, 55, and 350 to their prime factors : thus

$36=3\times3\times2\times2.$
$55=11\times5.$
$350=7\times5\times5\times2.$

Then, the Least Multiple of 36 $=(3\times3\times2\times2).$
$\therefore$ the L.C.M. of 36 and 55 $=(3\times3\times2\times2)\times11\times5.$
And, $\therefore$ the L.C.M. of 36, 55 and 350 $=(3\times3\times2\times2\times11\times5)\times5\times7.$
$=$ 69300, *Answer.*

(5.) A more usual method is as follows :—

*First :* Strike out the 2, the 9, the 7, and the 25, as above.

*Second :* Since two of the remaining numbers have a common measure, 5, divide them by 5, thus—

*Third :* Since two of the remaining numbers have a common measure, 2, divide them by 2, thus—

$$\begin{array}{r|l} 5 & \cancel{2},\ \cancel{7},\ \cancel{9},\ 36,\ \cancel{25},\ 55,\ 350. \\ \hline 2 & 36,\quad 11,\quad 70. \\ \hline & 18,\quad 11,\quad 35. \end{array}$$

*Fourth :* Multiply together the remaining numbers and the numbers by which we have divided, thus—

$35\times11\times18\times2\times5=$ 69300, *Answer.*

It will be seen that this last process is in effect the same as the one preceding it, for,

A *Common Measure* of 36, 55, and 350 will be $36\times55\times350,$
or, $(3\times3\times2\times2)\times(11\times5)\times(7\times5\times5\times2).$

The L.C.M. may soon be found by—

1st. Reducing the 5, the prime factor of 55, to unity, because it is contained in the prime factors of 350.

2nd. Reducing the 2, the prime factor of 350, to unity, because it is contained in the prime factors of 36.

But this is the same thing in the end as dividing first by 5 and then by 2, as in the last method.

## "Casting out the Nines."

**35.** We shall now insert that proof of the truth of an operation in Multiplication which is familiarly known as *Casting out the Nines.*

*The Reason of the Process* is involved in the truth of the two following propositions, viz.—

1. *Any number divided by* 9 *leaves the same remainder as the sum of its digits divided by* 9, thus—

463 ÷ 9 leaves 4; also (4+6+3) ÷ 9 = 13 ÷ 9, leaves 4.

The truth of the general statement of this proposition will appear from considering that

Since 10 = 9+1
Any multiple of 10 is an exact number of nines + 1. *e.g.*
10 = 9+1 ∴ 80 = an exact number of nines + 8.
Also 100 = 99+1 ∴ 700 = an exact number of nines + 7.
1000 = 999+1 ∴ 6000 = an exact number of nines + 6.
Therefore 6000+700+80 = an exact number of nines + 6+7+8.
*i.e.* 6780 ÷ 9 leaves the same remainder as (6+7+8) ÷ 9.

∴ *By adding together the digits composing a number and dividing their sum by* 9 *we find the same remainder that would be found by dividing the number itself by* 9.

N.B.—It is not necessary to add the nines in finding the sum of the digits of a number, because these being exactly divisible by 9 will not affect the number left as a remainder.

2. *If two numbers be each divided by* 9, *the product of their remainders being divided by* 9 *will leave a remainder similar to that which would be left if the product of the two numbers were divided by* 9, thus—

6271 ÷ 9 leaves 7; and 1034 ÷ 9 leaves 8.
Then (7 × 8) ÷ 9 leaves 2.
And (6271 × 1034) ÷ 9 leaves 2.

The truth of the general statement of this proposition will appear from considering that

**$6271 \times 1034 = (696 \text{ nines} + 7) \times (114 \text{ nines} + 8)$.**
**$= (696 \text{ nines} \times 114 \text{ nines}) + (7 \times 114 \text{ nines}) + (696 \text{ nines} \times 8) + (7 \times 8)$.**
**$=$ an exact number of nines $+ (7 \times 8)$.**
**$=$ an exact number of nines $+ 56$.**

From which it follows that

**The remainder found by *dividing* the product of the two numbers 6271 and 1034 *by* nine** $\left.\right\} = \left\{\right.$ **The remainder found by first *dividing* these numbers each *by* nine, and then *dividing* the product of their remainder *by* nine.**

By the help of these two propositions we arrive at the process generally used to prove the truth of an operation in Multiplication, which process may perhaps be best explained by an example, thus—

**Ex.** $9643287 \times 378427 = 3649280169549$.

To prove the truth of this result, proceed as follows :—

**1st. Add together all the digits of the *multiplicand* except the nines, thus—**

**$6+4+3+2+8+7 = 30$.**

**Divide the sum by 9, and set down the remainder, thus—**

**$30 \div 9 = 3$ and 3 over : set down 3.**

**2nd. Proceed similarly with the *multiplier*, thus—**

**$3+7+8+4+2+7 = 31$.**

**$31 \div 9 = 3$ and 4 over : set down 4.**

**3rd. Treat the *product* in the same way, thus—**

**$3+6+4+2+8+1+6+5+4 = 39$.**

**$39 \div 9 = 4$ and 3 over : set down 3.**

**4th. *Multiply* the 3 which remained from the multiplicand *by* the 4 which remained from the multiplier, and divide the product by 9, thus—**

**$(4 \times 3) \div 9 = 12 \div 9 = 1$ and 3 over.**

**Since now the 3 which here remains is the same as the number, 3, which remains from the product, the product is *probably* correct.**

**N.B.—If, however, an error of *nine* or *any multiple of nine* has been made, it will not be detected by this method.**

## Example 9.

1. Find the G.C.M. of the following numbers :—

87 and 145; 1878 and 3443; 55328 and 22755; 61037 and 5628; 3304, 2065, and 5369; 56112, 72144, and 200400; 1484, 689, and 4723; 7845, 44455, and 28765; 658, 752, 1222, 1410, and 2444; 19497, 13065, 21507, 7839, 12462, 50451, and 163614.

2. Find the L.C.M. of the following numbers :—

4 and 17; 5 and 6; 81 and 4; 14, 13, and 5; 61, 17, and 3; 8 and 104; 91, 16, and 28; 56, 21, 104, and 39; 72, 64, 1108, 28, and 63; 105, 403, 248, 221, 119, and 12648.

3.* Find the L.C.M. of 12, 16, 20, and 30; of 357, 24, and 29; of 891, 972, and 924.

4. What multiple of 5 results from multiplying 314725 by 9?

5. What number is the same multiple of 5 that 148995 is of 9?

6. Of all the numbers, besides Unity, which may be subtracted from 7905 an exact number of times, which is that which may be taken the *greatest*, and which the *least* number of times?

7. What is the least number from which 34, 510, 91, and 16 may each be taken an exact number of times?

8. What is the least number by which 17595 must be multiplied, that 36 may be taken from the product an exact number of times?

9. A school was found to contain such a number of boys that whether arranged in sixes, sevens, nines, or elevens, there were always five over. How many children, *at least*, did that school contain?

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* The following exercises are to be found in the Government Examinations for Certificates for Male Students of the First Year.

# *CHAPTER III.*

## Weights and Measures.

**36.** All the preceding articles have had reference to operations with Abstract Numbers. We shall now proceed to the consideration of Concrete Numbers, and first of all we shall inquire into the principles upon which the different systems by which quantities are measured with regard to their Weight, Extent, Cubical Capacity, or Value, have been founded—*i.e.*, we shall investigate the meaning and use of the TABLES OF WEIGHTS AND MEASURES.

A moment's consideration is sufficient to show that every perfect system of Weights and Measures must provide measures for estimating (1) SPACE, (2) WEIGHT, (3) MONEY, and (4) TIME, and that it is essential for the convenience of merchants that these measures should be of one uniform standard, at least in each country, and if possible throughout the world. But in England and in France, as well as other countries, so many different values were formerly attached to measures bearing the same name, but used in different parts of these respective countries, that laws have been passed by their respective Governments for remedying the evil by establishing certain specified weights and measures as the only lawful ones.

In England a system was established by Act of Parliament in 1824, and came into operation on January 1st, 1826, and the System of Weights and Measures thereby rendered authoritative is known as the "*Imperial Weights and Measures.*"

In France the new system was established by law in 1801, and is called the "*Metric System of Weights and Measures.*"

We shall now proceed to describe these systems in detail, the English because it is the system established by law in this country, and the Metric because of its completeness and simplicity, and also because attempts have been made and are still being made to supersede our present English system by the Metric or some other similar system.

## I. The English System of Weights and Measures.

**Def. 1.** *A Measure* is the instrument by which we estimate the extent, the weight, or the value of anything.

**Def. 2.** *To measure* is to find how many times any one quantity contains another quantity which is fixed and invariable, and which is known as the Unit of Measure.

The Act of 1824 settled that in measuring—

### I. Space :—

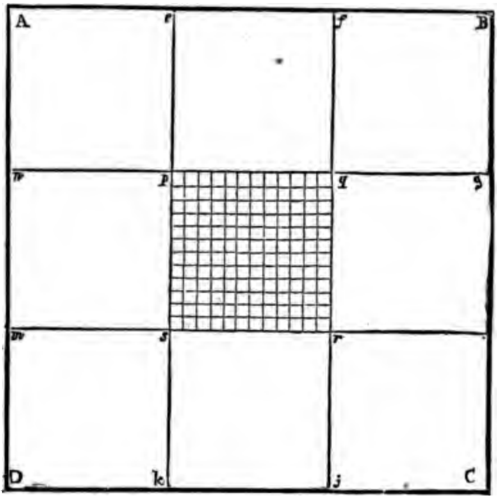
a. *The Unit of* LENGTH should be *the Imperial Yard.*

This was a yard measure made by order of Parliament in 1760, and the exact dimensions of which might at any time be recovered (if injured or lost), by finding the length of a pendulum which oscillates seconds in vacuo at the level of the sea in the latitude of London. The length of such a pendulum is 39·13929 inches, and 36 of these inches make the Imperial Yard.

b. *The Unit of* SURFACE would consequently be a square having a yard for its side, such a square being called *a Square Yard.*

**Def.** *A number squared* is the product found by multiplying that number by itself once, thus—*four squared* (which is written thus—$4^2$) $= 4 \times 4 = 16$.

By consulting the Table on page 54 it will be seen that the Imperial Yard contains 3 *feet*, and each foot contains 12 *inches.* Now let AB represent *a yard* divided into feet; then the square ABCD will represent *a square yard.* Let BC also be divided into three equal parts, Bg, gh, hC, and through the points e, f, g, h, draw straight lines parallel to the side of the square :—



Then the whole figure ABCD is divided into nine small squares, each of which represents *a square foot;* so that evidently

1 *square yard* = 9 *square feet* (*i.e.*, *three squared* square feet).

If the sides of the interior figure p, q, r, s, be divided each into twelve equal parts, each of these parts will represent *an inch*, and each of the small squares formed by joining the opposite points will represent *a square inch*, and the figure p, q, r, s, will be divided into 144 such *square inches;* or, in other words—

1 *square foot* = 144 *square inches* (*i.e.*, *twelve squared* square inches).

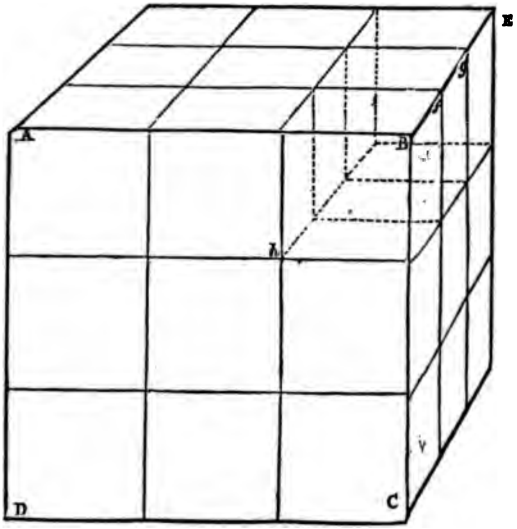
In the same way we find the relative values of all the terms of Square or Superficial Measure. (See Table 3, page 55.)

(c.) *The Unit of Solidity* would therefore be a cube having a square yard for each of its faces, such a cube being called *a* CUBIC YARD.

**Def.** *A number cubed* is the product found by multiplying that number by itself twice, thus: *four cubed* = 4 × 4 × 4 which is written thus $4^3$.

Let the face ABCD of the cube DE represent *a square yard*, as before, divided into 9 *square feet*. Then it is evident that if BE also represent a yard, the line Bf will represent a foot, and the cube of which hf is the diagonal will represent *a cubic foot*. It is also clear that the cube DE will contain 3 × 3 × 3 such cubes as hf, *i.e.*—

*a cubic yard* = *27 cubic feet* (*i.e.*, *three cubed* cubic feet).



In this way we find the relative values of all the terms of Cubic Measure. (See Table 4, page 55.)

The reason of the following comparisons will now be apparent:

I. *In Long Measure* 1 yard = 3 feet, but
*In Square Measure* 1 yard = $3^2$ feet = 9 feet, and
*In Cubic Measure* 1 yard = $3^3$ feet = 27 feet.

II. *In Long Measure* 1 foot = 12 inches, but
*In Square Measure* 1 foot = $12^2$ inches = 144 inches, and
*In Cubic Measure* 1 foot = $12^3$ inches = 1728 inches.

N.B.—In Square Measure the yards are generally called "*Square yards*" and the feet are termed "*Square feet.*" Also in Cubic Measure the yards and feet are called "*Cubic yards*" and "*Cubic feet*" respectively.

(d.) *The Unit of Capacity* should be the IMPERIAL GALLON.

This was selected as being very nearly the same as the gallon then in use. It is the space occupied by 10 lb. Av. of distilled water, weighed in air, at a temperature of 62° F., and with the barometer standing at 30 inches. It was also defined as containing 277·274 cubic inches.

2. **Weight.**—*The Unit of Weight* should be the IMPERIAL POUND TROY.

This was half the weight of a double-pound Troy, also made in 1760. The weight of this pound was 5760 grains, while a pound Avoirdupois contained 7000 such grains.

The value of this Unit can at any time be recovered by finding the weight of a cubic inch of distilled water weighed at the temperature of 62° F., the barometer at the same time standing at 30 inches. The weight of this water being 252·458 grains, that of the Imperial Pound Troy may easily be obtained therefrom.

In measuring *Time* and *Money* we do not depend upon the above-named Act altogether, although we must refer to it to define *ounce*, *&c.*, in the latter case.

3. **Time.**—The standard *Unit of Time* is a MEAN SOLAR DAY, which is the mean, or average, of all the *apparent solar days* in the year.

*An apparent solar day* is the interval between two successive transits of the sun over the meridian of any place.

N.B.—The Common or *Civil Year* contains 365 of these days, but the *Solar Year*, which is the interval between the sun's departure from and return to a certain fixed point in the Ecliptic, has been accurately determined by astronomers to contain 365·242218 mean solar days, *i.e.* 365 days 5 hours 48 min. 47½ secs. nearly. From this it follows that the Civil Year is nearly 6 hours shorter than the Solar Year, and, if some precautions were not taken to remedy this error, we should be thrown into great confusion as regards the time of the return of the Seasons;

December would sometimes be the time of frost and snow and sometimes the time of greatest heat.

In the time of *Julius Cæsar* the length of a Solar Year was supposed to be exactly 365·25 days,* *i.e.*, exactly 365 days 6 hours; he therefore ordered that every fourth year should contain 366 days instead of 365. Such a year was called *Leap Year* or *Bissextile Year*, and the additional day—which was added to the February month, thereby making it to have 29 instead of 28 days—was called *the Intercalary day.*

It is evident, however, that this correction is too great, for

A Julian Year = 365·25 days.
A Solar Year = 365·242218 days.
∴ The Julian Year is too great by ·007782 days.

This error was soon discovered and steps were taken to remedy it. At the first Council of Nice, A.D. 325, the vernal equinox fell on March 21st. Now the second Council of Nice was held A.D. 1582, *i.e.* 1257 years afterwards, and it was found that the vernal equinox that year happened on March 11th, *i.e.* ten days earlier than the Julian Calendar would have made it occur. This difference is explained by what we said above: thus—

*In one year by the Julian Calendar the end of the year is* ·007782 *days behind.*

*Therefore, in* 1257 *years it becomes* ·007782 *days* × 1257, 9·781974 *days behind,* i.e. *nearly* 10 *days behind.*

It was in consequence of this that Pope Gregory ordered that ten days should be omitted in that year (A.D. 1582), by making October 15th follow October 4th, thus causing the vernal equinox to fall in the next year on March 21st; and he also took steps to prevent the recurrence of this error by ordering that for the future in every circle of 400 years *three* of the leap years should contain only 365 days; in other words, the additional day should be omitted whenever the year completed a century, the number of *hundreds* of which was not exactly divisible by 4, thus—1600, 2000, &c., are leap years because 16 and 20 are each exactly divisible by 4; but 1300, 1700, 1900 are *not* leap years, because 13, 17, 19 are not exactly divisible by 4.

The calculation made by Julius Cæsar was called the *Old Style*, and is still used in Russia. The Gregorian style is called the *New Style*, which was of course almost immediately adopted by Roman Catholic countries, but was not introduced into England before 1752, when the error amounted to eleven days.

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* This was called *a Julian Year.*

4. **Money.**—The *Unit of Money* is the POUND, and the coin which represents this value is called a *Sovereign.*

THE STANDARD GOLD, from which the Sovereign and other gold coins of this realm are made, consists of 22 parts *pure gold* and 2 parts *copper* melted together. Coins are made and stamped, and thereby rendered legal money, at an office in London called *the Mint*, and at this office 1 ℔ Troy of Standard gold is coined into 46$\frac{29}{40}$ sovereigns; in other words, the Mint price per ℔ Troy of Standard gold is £46$\frac{29}{40}$, *i.e.* £46, 14s. 6d. Therefore, the Mint price of an oz. Troy of Standard gold is $\frac{1}{12}$ of £46, 14s. 6d., *i.e.* £3, 17s. 10½d.

THE STANDARD SILVER, from which the silver coins are made, is a compound of which 37 parts in every 40 are *pure silver* and 3 parts *copper*. A ℔ Troy of this silver is coined into 66 shillings, so that the Mint price per oz. of Standard silver is 5s. 6d.

A ℔ Av. of COPPER is coined into 24 pennies, but no more than twelve of these are a legal tender, nor is more than 40s. in the silver coinage a legal tender according to the present law.

37. The following Tables should be committed to memory:—

## I. MONEY or VALUE.

2 Farthings (*fg.*) = 1 Halfpenny.
4 Farthings or 2 Halfpence = 1 Penny (*d.*)
12 Pence = 1 Shilling (*s.*)
2 Shillings = 1 Florin (*fl.*)
20 Shillings = 1 Pound (£.)

N.B.—1. £, s, d, q, are abbreviated forms of the Latin words *libra*, *solidus*, *denarius*, *quadrans*, the names of certain Roman coins.

N.B.—2. Farthings are indicated as parts of a penny, thus—

¼d. (i.e. *one-fourth*) signifies *a farthing*, or one-fourth of a penny.
½d. (i.e. *one-half*) signifies *a halfpenny*, or one-half of a penny.
¾d. (i.e. *three-fourths*) signifies *three farthings*, or three-fourths of a penny.

N.B.—3. The following coins have been used in England in former times, and their values should be remembered:—

**Silver Coins.**
A Groat = 4 pence.
A Tester = 6 pence.

These two are still used under different names.

**Gold Coins.**
A Noble = 6s. 8d.
An Angel = 10s. 0d.
A Half-Guinea = 10s. 6d.
A Guinea = £1 1s. 0d.
A Mark or Mek = 13s. 4d.
A Carolus = £1 3s. 0d.
A Jacobus = £1 5s. 0d.
A Moidore = £1 7s. 0d.

## II. SPACE or EXTENT.

### 1. Lineal Measure.

12 Lines = 1 Inch.
3 Barleycorns = 1 Inch (1 *in.*)
3 Inches = 1 Palm.
4 Inches = 1 Hand.
9 Inches = 1 Span.
18 Inches = 1 Cubit.
12 Inches = 1 Foot (1 *ft.*)
5 Feet = 1 Pace.
6 Feet = 1 Fathom.
3 Feet = 1 Yard (1 *yd.*)
22 Yards = 1 Chain (used in measuring Land).
5½ Yards = 1 Rod, Pole, or Perch (1 *p.*)
40 Poles (220 yards) = 1 Furlong (1 *fur.*)
8 Furlongs (1760 yards) = 1 Mile (1 *m.*)
3 Miles = 1 League (1 *lea.*)
69½ Miles, or 60 Geographical Miles = 1 Degree (1 *deg.*)
1 mile 295 yards 6 inches = 1 Knot.

### 2. Cloth Measure.

2¼ Inches = 1 Nail.
4 Nails = 1 Quarter (1 *qr.*)
4 Quarters = 1 Yard (1 *yd.*)
5 Quarters = 1 English Ell.
3 Quarters = 1 Flemish Ell.

### 3. Square or Superficial Measure.

144 Square Inches (*sq. in.*) = 1 Square Foot (1 *sq. ft.*)
9 Square Feet = 1 Square Yard (1 *sq. yd.*)
30¼ Square Yards = 1 Square Pole (1 *sq. p.*)
40 Poles = 1 Rood (1 *r.*) = 25000 Sq. Links.
4 Roods (4840 *sq. yds.*) = 1 Acre (1 *a.*) = 10 Sq. Chains.

### 4. Cubic Measure.

1728 Cubic Inches (*cub. in.*) = 1 Cubic Foot (1 *cub. ft.*)
27 Cubic Feet = 1 Cubic Yard (1 *cub. yd.*)

### 5. Measure of Capacity.

4 Gills or Noggins = 1 Pint (1 *pt.*)
2 Pints = 1 Quart (1 *qt.*)
4 Quarts, or 2 Pottles = 1 Gallon (1 *gal.*)
2 Gallons = 1 Peck (1 *pk.*)
4 Pecks = 1 Bushel (1 *bush.*)
8 Bushels = 1 Quarter (1 *qr.*)
5 Quarters = 1 Load.

In Coal Measure.
3 Bushels = 1 Sack.
12 Sacks = 1 Chaldron.

In Ale and Beer Measure.
36 Gallons = 1 Barrel (1 *bar.*)
54 Gallons = 1 Hogshead (1 *hhd.*)

In Wine Measure.
63 Gallons = 1 Hogshead.
2 Hogsheads = 1 Pipe.
2 Pipes = 1 Tun.

## III. WEIGHT.

### 1. Table of Troy Weight.

24 Grains (*grs.*) = 1 Pennyweight (1 *dwt.*)
20 Pennyweights = 1 Ounce (1 *oz.*)
12 Ounces = 1 Pound (1 *lb.*)

### 2. Table of Avoirdupois Weight.

16 Drams (*drs.*) = 1 Ounce (1 *oz.*)
16 Ounces = 1 Pound (1 *lb.*)
28 Pounds = 1 Quarter (1 *qr.*)
4 Quarters (112 *lb.*) = 1 Hundredweight (1 *cwt.*)
20 Hundredweights = 1 Ton.

N.B.—1. *A Stone* generally means 14 lb., but in buying butcher's meat or fish, a stone is 8 lb.

N.B.—2. A lb. Troy = 5760 Grains, but a lb. Av. = 7000 Grains.

## IV. TIME.

60 Seconds (written thus, 60") = 1 Minute (1').
60 Minutes (written thus, 60') = 1 Hour.
24 Hours = 1 Day.
7 Days = 1 Week.
4 Weeks = 1 Lunar Month.
13 Lunar Months, or
12 Calendar Months, or } = 1 Year.
365 Days
100 Years = 1 Century.

N.B.—The twelve Calendar months into which the year is divided do not all contain the same number of days, but the number in each may easily be remembered by means of the following lines :—

Thirty days hath September,
April, June, and November ;
February has twenty-eight alone,
While all the rest have thirty-one ;
But leap-year coming once in four,
February then has one day more.

## 38. II. The French, or Metric System of Weights and Measures.[*]

In establishing a new system of Weights and Measures, it was the design of the French Government to base the Unit of Lineal Measure upon some principle of nature so that it might be perfectly invariable; and, having once determined this Unit, upon it to found a system which should be marked by great simplicity in its arrangement, and admit of very easy computations.

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[*] The student is advised to leave the consideration of the Metric System till he has made himself acquainted with Decimals (see page 116).

For a fuller description of this System the student is referred to Philips' "*Diagrams and Synoptical Tables of the Metric System of Weights and Measures, with Hand-Book,*" by W. RICKARD, LL.D. ; published by G. Philip & Son, London and Liverpool.

The Unit of Lineal Measure finally adopted was *the ten-millionth part of the quarter of the terrestrial meridian.* This Unit is called a **Metre**, and is equal to about 39·3708 English inches.

**39.** In describing this system it will be necessary to explain—1st, The manner of forming the Tables; and, 2nd, The connection between the different Units in this system.

### (1.) The manner of forming the Tables.

(a.) In every Table the name of the Unit is the principal name used to signify the different quantities, prefixes being used to signify Multiples and Submultiples of the Unit.

Thus, in Lineal Measure the name chiefly used is "*Metre*," which an inspection of the Table will show to appear in every name used in that Table.

(b.) To signify **Multiples** of the Unit, certain *Greek* prefixes are used, viz.:—

*Deca*, or Deka, which signifies 10.
*Hecto*, which signifies 100.
*Kilo*, which signifies 1000.
*Myria*, which signifies 10000.

(c.) To signify **Sub-multiples** of the Unit, certain *Latin* prefixes are used, viz.:—

*Deci*, which signifies $\frac{1}{10}$ (*one-tenth*).
*Centi*, which signifies $\frac{1}{100}$ (*one one-hundredth*).
*Milli*, which signifies $\frac{1}{1000}$ (*one one-thousandth*).

### (2.) The Connection between the different Units.

(a.) *The Unit of Lineal Measure* is, as we have said, the METRE.

(b.) *The Unit of Superficial Measure* is the ARE or *Square Dekametre—i.e.*, a Square having ten metres for its side.

**N.B.—Measurements of Surface are not always expressed in the Are and its Multiples or Sub-multiples, but are sometimes expressed in Square Metres. Hence the value of the following Table :—**

**1 Square Metre = (10 × 10 *i.e.*) 100 Square Decimetres.**
**= (100 × 100 *i.e.*) 10000 Square Centimetres.**
**= (1000 × 1000 *i.e.*) 1000000 Square Millimetres.**
**= ($\frac{1}{10} \times \frac{1}{10}$ *i.e.*) $\frac{1}{100}$ of a Square Dekametre.**
**= ($\frac{1}{100} \times \frac{1}{100}$ *i.e.*) $\frac{1}{10000}$ of a Square Hectometre.**
**= ($\frac{1}{1000} \times \frac{1}{1000}$ *i.e.*) $\frac{1}{1000000}$ of a Square Kilometre.**
**= ($\frac{1}{10000} \times \frac{1}{10000}$ *i.e.*) $\frac{1}{100000000}$ of a Square Myriametre.**

(c.) *The Unit of Cubic Measure, or Measure of Solidity,* is the STERE, or *Cubic Metre—i.e.*, a perfect cube having each of its faces a square metre.

**N.B.—Measurements of content being sometimes expressed in Cubic Metres, the following Tables will be found useful.**

**1 Cubic *Metre* = 1 Stere.**
***Decimetre* = ($\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ *i.e.*) $\frac{1}{1000}$ Stere.**
***Centimetre* = ($\frac{1}{100} \times \frac{1}{100} \times \frac{1}{100}$ *i.e.*) $\frac{1}{1000000}$ Stere.**
***Millimetre* = ($\frac{1}{1000} \times \frac{1}{1000} \times \frac{1}{1000}$ *i.e.*) $\frac{1}{1000000000}$ Stere.**

(d.) *The Unit of Capacity* is the LITRE, or *Cubic Decimetre—i.e.*, the content of a perfect cube, having a square decimetre for its face.

(e.) *The Unit of Weight* is the GRAM or GRAMME. This is the weight of a *Cubic Centimetre* of distilled water, weighed in a vacuum at the temperature of its greatest density.

(f.) *The Unit of Value* is the FRANC. This is a silver coin, the weight of which is 5 Grammes, and the diameter of which is 23 Millimetres: it is worth about $9\frac{3}{4}$d. English money.

**40.** In the following Tables the English equivalents are put side by side with the French or Metric quantities, not that they may be committed to memory, but simply for comparison and reference.

## 1. Lineal Measure.

| | | | | Eng. In. |
|---|---|---|---|---|
| 10 *Milli*metres (10 Millim.) | = | 1 Centimetre (1 Centim.) | = | ·393708 |
| 10 *Centi*metres (10 Centim.) | = | 1 Decimetre (1 Decim.) | = | 3·93708 |
| 10 *Deci*metres (10 Decim.) | = | 1 METRE (1 m.) | = | 39·3708 |
| 10 *Metres* | = | 1 Dekametre (1 Dekam.) | = | 393·708 |
| 10 *Deka*metres (10 Dekam.) | = | 1 Hectometre (1 Hectom.) | = | 3937·08 |
| 10 *Hecto*metres 10 Hectom.) | = | 1 Kilometre (1 Kilom.) | = | 39370·8 |
| 10 *Kilo*metres (10 Kilom.) | = | 1 Myriametre (1 *Myriam.*) | = | 333708 |

(8 Kilom. = approximately, 5 English miles.)

## 2. Square Measure.

| | | | | Sq. Yds. |
|---|---|---|---|---|
| 10 *Centi*ares (10 Centiar.) | = | 1 *Deci*are | = | 11·96033 |
| 10 *Deci*ares (10 Deciar.) | = | 1 ARE = 1 sq. dekametre | = | 119·6033 |
| 10 Ares | = | 1 *Dek*are | = | 1196·033 |
| 10 *Dek*ares (10 Dekar.) | = | 1 *Hect*are | = | 11960·33 |

## 3. Cubic Measure.

| | | | | Cubic Ft. |
|---|---|---|---|---|
| 10 *Deci*steres (10 Decist.) | = | 1 STERE (unit) | = | 35·317 |
| 10 Steres (10 St.) | = | 1 *Deka*stere | = | 353·17 |

## 4. Measure of Capacity.

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 *Centi*litres (10 Centil.) | = | 1 *Deci*litre | = | ·1760773 pints | = | 10 cub. centim. |
| 10 *Deci*litres (10 Decil.) | = | 1 LITRE | = | 1·760773 pints | = | 1 cub. decim. |
| 10 Litres (10 Lit.) | = | 1 *Deka*litre | = | 17·60773 pints | = | 1 centistere |
| 10 *Deka*litres (10 Dekal.) | = | 1 *Hecto*litre | = | 176·0773 pints | = | 1 decistere |
| 10 *Hecto*litre (10 Hectol.) | = | 1 *Kilo*litre | = | 1760·773 pints | = | 1 cub. metre,<br>or 1 stere |

## 5. Measure of Weight.

| | | | | Eng. Grains. |
|---|---|---|---|---|
| 10 *Milli*grams (10 Millig.) | = | 1 *Centi*gram | = | 0·154323 |
| 10 *Centi*grams (10 Centig.) | = | 1 *Deci*gram | = | 1·543234 |
| 10 *Deci*grams (10 Decig.) | = | 1 GRAM (unit) | = | 15·432348 |
| 10 Grams (10 Gr.) | = | 1 *Deka*gram | = | 154·32348 |
| 10 *Deka*grams (10 Dekag.) | = | 1 *Hecto*gram | = | 1543·2348 |
| 10 *Hecto*grams (10 Hectog.) | = | 1 *Kilo*gram | = | 15432·348 |
| 10 *Kilo*grams (10 Kilog.) | = | 1 *Myria*gram | = | 154323·48 |

It is convenient to remember that—

1 Kilog. = 2·68 lb. troy = 2⅔ lb. troy nearly = 2⅕ lb. avoir. nearly.
1 Quintal = 100 Kilog. = 2 English cwts. nearly.
1 Millier, or Ton = 1000 Kilog. = 20 English cwts. nearly.

### 6. Measure of Value.

10 Centimes (10 c.) = 1 Decimes (1 d.)
10 Decimes (10 d.) = 1 FRANC.

The value of the following coins should be remembered:

A *Sou* = 5 Centimes (5 c.)
A *Decime* = 2 Sous = 10 Centimes (10 c.)
A *Napoleon* = 20 Francs (20 fr.)

**41. The Advantages of the Metric System** may be thus stated:—

1. All the operations known at present as *Compound Arithmetic* would be reduced to operations in the *Simple Rules*.

2. *Reduction* would involve nothing more than shifting the decimal place to the right or left, as the case may be, and *Practice* would be entirely abolished.

3. *Proportion*, *Interest*, *Stocks*, *&c.*, would be rendered much more easy to work.

N.B.—The advantages of this system are so great and manifest that several systems of Decimal Coinage have been based thereupon, and proposed to supersede our present English system, but up to the present time no one of them has been adopted, though it is not unlikely that a system of this kind may eventually be established by law for use in Great Britain.

## *CHAPTER IV.*

## ELEMENTARY RULES.

### I. Reduction.

**42. Reduction** is the name given to the process by which a quantity expressed in one denomination is converted into another, thus—

40d. = 3s. 4d. : Where we say that forty *pence* has been reduced to *shillings* and *pence*.

100s. = £5 : Where one hundred *shillings* have been reduced to *pounds*.

It must be carefully noticed that the *values* of these quantities are not at all altered by the Reduction : the *values* remain the same, the *names* only are changed.

The term *Reduce* will be found to be of very wide application, thus—when we wish to ask "*What numbers multiplied together will give* 77?" We express the question by saying "*Reduce* 77 *to its prime factors*."

**43.** Reduction is of two kinds, viz :—

1. To terms of a lower denomination : *e.g.*, *Reduce £5 to shillings.*

2. To terms of a higher denomination : *e.g.*, *Reduce* 106 *shillings to pounds.*

**1. To reduce a quantity to a lower denomination.**

**Ex. 1.** *Reduce* £82 *to pence.*

Since £1 = 20s., £82 will be equal to 82 times 20s., *i.e.* (20s. × 82). Also, since 1s. = 12d., (20s. × 82) will be equal to (82 × 20) times 12 pence = (82 × 20 × 12) pence = 19680 pence, *Answer*.

**Ex.** 2. *Reduce £15, 17s. 4½d. to farthings.*

Since £1 = 20s., £15 will be equal to 15 times 20s., *i.e.* (15×20)s.

∴ £15, 17s. = (15×20)s. + 17s.

And, since 1s. = 12d., $\{(15\times20)\text{s.}+17\text{s.}\}$ will be equal to $\{(15\times20)+17\}$ times 12 pence = $[\{(15\times20)+17\}\times12]$ pence.

∴ £15, 17s. 4d. = $[\{(15\times20)+17\}\times12\text{d.}]+4\text{d.}$; *i.e.* the number of pence in £15, 17s. 4d. = $[\{(15\times20)+17\}\times12]+4$.

Also, since 1d. = 4 farthings, $\overline{[\{(15\times20)+17\}\times12]+4}$ pence will be equal to $\overline{[\{(15\times20)+17\}\times12]+4}$ times 4 farthings; *i.e.* the number of farthings contained in £15, 17s. 4d. = $\overline{[\{(15\times20)+17\}\times12]+4}\times4$.

∴ £15, 17s. 4½d. = $\overline{\overline{[\{(15\times20)+17\}\times12]+4}\times4}+2$ farthings,
= 15234 farthings, *Answer.*

The preceding examples enable us to lay down the following rule for reducing quantities to a lower denomination.

**RULE.—Multiply the term in the highest denomination by the number which expresses how many times the unit of this denomination contains the unit of the next lower denomination, and add to the product such units of the lower denomination as may be included in the given sum.**

**Repeat this operation till the given quantity is expressed in the required denomination.**

**Ex.** 2. (*By Rule.*) *Reduce £15, 17s. 4½d. to farthings.*

$$\begin{array}{l}
\text{£}15\ 17\text{s. } 4\tfrac{1}{2}\text{d.}\\
\underline{\ \ 20\ \ }\\
\overline{300}+17 = 317\text{s.}\\
\qquad\qquad\ \ \underline{\ 12\ }\\
\qquad\quad \overline{3804}+4 = 3808\text{d.}\\
\qquad\qquad\qquad\qquad\quad \underline{\ \ 4\ }\\
\qquad\qquad\qquad\quad \overline{15232}+2 = 15234\text{ f.}
\end{array}$$

1st. Multiply 15 by 20, and add 17 to the product.

2nd. Multiply 317 by 12, and add 4 to the product.

3rd. Multiply 3808 by 4, and add 2 to the product.

The more compendious form of stating this would be—

$$\begin{array}{l}
\text{£}15\ 17\text{s. } 4\tfrac{1}{2}\text{d.}\\
\ \ \underline{\ 20\ }\\
\ \ \overline{317}\text{s.}\\
\ \ \underline{\ 12\ }\\
\ \ \overline{3808}\text{d.}\\
\ \ \ \ \underline{\ 4\ }\\
\ \overline{15234}
\end{array}$$

In this case the 17s. is first added as the multiplication proceeds; afterwards the 4d. and the 2 farthings are added as the reduction proceeds.

*Answer*, 15234 farthings.

### 2. To reduce a quantity to a higher denomination.

**Ex. 1.** *Reduce* 420 *shillings to pounds.*

Since 20s. = £1, 420s. will be equal to $\frac{1}{20}$ of £420, *i.e.* £420 ÷ 20
*i.e.* £21, *Answer.*

**Ex. 2.** *Reduce* 15234 *farthings to £. s. d.*

Since 4 farthings = 1d, 15234 farthings will be equal to $\frac{1}{4}$ of 15234 pence, *i.e.* 15234 pence ÷ 4, *i.e.* 3808 pence 2 farthings.

And, since 12 pence = 1s., 3808 pence will be equal to $\frac{1}{12}$ of 3808 shillings, *i.e.* 3808 shillings ÷ 12, *i.e.* 317 shillings, 4 pence.

∴ 3808 pence 2 farthings = 317 shillings, 4 pence, 2 farthings.

Also, since 20 shillings = £1, 317 shillings will be equal to $\frac{1}{20}$ of £317, *i.e.* £317 ÷ 20, *i.e.* £15, 17 shillings.

∴ 317 shillings, 4 pence, 2 farthings, = £15, 17s. 4½d., *Answer.*

The preceding examples enable us to lay down the following rule for reducing quantities to a higher denomination.

**RULE.—Divide the given quantity by the number which expresses how many units of the given denomination make a unit of the next higher denomination.**

**Repeat this operation till the given quantity is expressed in the required denomination.**

**In each case the remainders (if any) will be of the same denomination as the dividend from which they arise, and will be so considered in the answer.**

**Ex. 2.** (*By Rule.*) *Reduce* 15234 *farthings to £. s. d.*

4 | 15234f.
12 | 3808d. 2f.
2,0 | 31,7s. 4d.
£15 17s.

1st. Divide by 4; the remainder, 2, is 2 farthings, or ½d.

2nd. Divide by 12; the remainder, 4, is 4 pence, or 4d.

3rd. Divide by 20; the remainder, 17, is 17 shillings, or 17s.

*Answer*, £15, 17s. 4½d.

**44.** The following solution of Example in Reduction should be carefully noticed.

**Ex. 1.** *Reduce* 17 *tons* 16 *cwts.* 1 *qr.* 19 *lb. to oz.*

tons. cwts. qrs. lb.
17 16 1 19
20
340+16=356 cwt.
4
1424+1 qr.=1425 qrs.
28
11400
2850
39900+19 lb.=39919 lb.
16
239514
39919
*Answer*, **638704** oz.

*Or*, by the more compendious form:—

17 tons 16 cwt. 1 qr. 19 lb.
20
356 cwt.
4
1425 qrs.
28
11419
2850
39919 lb.
16
239514
39919
**638704** oz., *Answer*.

**Ex. 2.** *Reduce* 27 *fur.* 3 *yds. to inches.*

27 fur. 0 poles 3 yds.
40
1080 poles
5½
5400=5 times 1080
540= ½ of 1080
yds. 5940=5½ times 1080
3
feet 17820
12
inches **213840**, *Answer*.

**Ex. 3.** *Reduce* 14 *acres* 2 *roods* 17 *poles to square feet.*

14 acres 2 roods 17 poles<br>
4<br>
56+2 = 58 roods<br>
40<br>
2320+17 = 2337 poles<br>
30¼<br>
70110 = 30 times 2337<br>
584¼ = ¼ of 2337<br>
∴ sq. yds. 70694¼ = 30¼ times 2337<br>
9<br>
sq. ft. 636248¼, *Answer.*

**Ex. 4.** *Reduce* 3192783 *sq. feet to acres, roods, &c. &c.*

9 | 3192783 sq. feet<br>
354753 sq. yds. 6 sq. feet

We want now to reduce 354753 sq. yds. to acres, roods, &c. The first thing to be done is to divide 354753 by 30¼, to bring the quantity to perches.

Now, *if the dividend and the divisor be both multiplied by the same term, the quotient will not be thereby affected* (§ 59), therefore we multiply 30¼ by 4, thus making 121, and 354753 by 4, thus making 1419012, and dividing 1419012 by 121 the quotient is 11727, *i.e.* 11727 perches, and 45 quarters of a sq. yard remains, *i.e.* 11¼ sq. yards remain.

30¼) 354753<br>
4 4<br>
121)1419012(11727<br>
121<br>
209<br>
121<br>
880<br>
847<br>
331<br>
242<br>
892<br>
847<br>
45

It now remains to reduce 11727 perches to roods, &c.

4,0 | 1172.7 perches<br>
4 | 293 roods 7 perches<br>
73 acres 1 rood 7 perches

Therefore, the whole

*Answer is* 73 *acres* 1 *rood* 7 *perches* 11¼ *sq. yds.* 6 *sq. ft.*

**The whole operation may be compendiously performed, thus—**

**9 | 3192783 sq. feet**
**354753 sq. yds. 6 sq. feet**
**4**
**121 { 11 | 1419012**
**11 | 129001 -1 } 45 quarters of a sq. yd. = 11¼ sq. yds.**
**4,0 | 1172,7-4 }**
**4 | 293 roods 7 poles**
**73 acres 1 rood 7 poles 11¼ sq. yds. 6 sq. feet**

**= 73 acres 1 rood 7 poles 11 sq. yds. + 2 sq. ft. + 36 sq. inches + 6 sq. feet.**

**= 73 *acres* 1 *rood* 7 *poles* 11 *sq. yds.* 8 *sq. feet* 36 *sq. inches, Answer.***

**Ex. 5.** *Reduce* 1782593 *inches to miles, furlongs, &c.*

**12 | 1782593 inches**
**3 | 148549 feet 5 inches**
**5½ | 49516 yds. 1 ft. 5 inches**
**2**
**11 | 99032**
**4,0 | 900,2 perches ($\frac{11}{2}$ yds. *i.e.*) 5 yds. 1 ft. 5 in.**
**8 | 225 fur. 2 perches 5 yds. 1 ft. 5 in.**
**28 *miles* 1 *fur.* 2 *perches* 5 *yds.* 1 *ft.* 5 *in., Answer.***

**Ex. 6.** *How many threepenny pieces are worth as much as* 20649 *fourpenny pieces?*

**20649 fourpenny pieces**
**4**
**3 | 82596 pence**
**27532 *threepenny pieces, Answer.***

**Ex. 7.** *How many half-guineas are worth as much as* 4938255 *crowns?*

**4938255 crowns**
**5**
**24691275 shillings**
**2**
**21 { 7 | 49382550 sixpences**
**3 | 7054650**
**2351550 *half-guineas, Answer.***

**1 half-guinea = 10s. 6d. = 21 sixpences,**
**∴ Divide 49382550 by 21.**

**Ex. 8.** *Reduce 43 degrees 7 miles 3 fur. to poles, and prove the result.*

43 degrees 7 miles 3 fur.

$\underline{69\frac{1}{2}}$

$\overline{387}$

258

$\underline{21\frac{1}{2}}$

$2988\frac{1}{2}+7=2995\frac{1}{2}$ miles $=2995$ miles 4 fur.

$\underline{8}$

$23960+4+3=23967$ fur.

$\underline{40}$

958680 *poles, Answer.*

**PROOF.**

1 degree $=69\frac{1}{2}$ miles $=69$ miles 4 fur. $=22240$ poles

$\underline{8}$

$552+4=556$ fur.

$\underline{40}$

22240 poles

∴ The number of degrees in 958680 poles $=$

$958680\div22240=43$ degrees 2360 poles $=$ *43 degrees 7 miles 3 fur.*

**Ex. 9.** *How many pounds Av. are equal to 2275 pounds Troy? Prove the Answer.*

2275 lb. Troy $=(2275\times5760)$ grains $=13104000$ grains.

1 lb. Avoirdupois $=7000$ grains.

∴ 2275 lb. Troy $=(13104000\div7000)$ lb. Av. $=$ **1872** *lb. Av., Answer.*

**PROOF.**

1872 lb. Av. $=(1872\times7000)$ grains $=13104000$ grains.

1 lb. Troy $=5760$ grains.

∴ 1872 lb. Av. $=(13104000\div5760)$ lb. Troy $=$ **2275** *lb. Troy.*

## Exercise 10.

### Examples in Reduction.

1. Reduce (proving each result), £91, 16s. 3½d. to farthings; 281 half-crowns to farthings; 7013 half-guineas to sixpences; £804, 19s. 0d. to half-pence; £13, 10s. 6d. to half-pence; 963 crowns to fourpenny pieces; £71, 18s. 3d. to threepences; 18s. 8d. to fourpences; £19, 10s. 9d. to threepenny pieces; 180565 guineas to half-crowns; and 365 moidores to crowns.

2. Reduce (proving each result), 71834 pence to £. s. d.; 9463 farthings to £. s. d.; 712920 pence to half-crowns; 28395 fourpences to half-crowns; 18139264 threepences to fourpences.

3. Find the number of fourpenny pieces in £39, 17s. 8d., and the number of half-crowns in £55, 17s. 6d.

4. Find the value of 7¾ millions of farthings; and the value of 1000 fourpenny pieces; and reduce 30724689 farthings to £. s. d.

5. Taking a year as 365 days 6 hours, how many years are there in ten million minutes?

6. Find how many seconds there are in 365 days 5 hours 48 minutes 48 seconds.

7. Reduce (proving each result), 41 miles 6 fur. to poles; 213 leagues 6 fur. 5 in. to inches; 200 miles 1 fur. 9 poles 2½ yds. to feet and inches; 10 million inches to miles, fur., &c.; 713 million feet to furlongs; 41 acres 3 roods 14 sq. inches to sq. inches; 720 acres 100 sq. inches to sq. inches; 8134926 sq. feet to acres, roods, &c.; 20 million cubic inches to cubic yards; and 16 billion inches to degrees.

8. Reduce (proving each result), 83 lb. 10 oz. 3 dr. Av. to drs.; 17 tons 4 cwts. 2 qrs. 19 lb. 13 oz. to oz. Av.; 823 cwts. 2 lb. 12 oz. to drs. Av.; 9641827 oz. Av. to tons, cwts., &c.; 249382 grains to lb. Troy, and also to lb. Av.; 17 oz. 14 dwts. 12 grains to grains Troy; 1000 million oz. to tons, &c., Av.; 685440 grains to stones Av.

9. Reduce (proving each result), 2 bush. 3 pks. 2 quarts to pints; 1 load to pecks; 141 galls. 1 pint to pints; 4000000 pints to hhds.; 1916382 pecks to qrs.; 81927368 gills to galls., &c.; 71 galls. 3 qts. to half-pints; also, 31 yards 2 quarters 3 nails 2 inches to inches; and 2378 inches to terms of higher denominations.

10. In the year 1872, how many days were there between January 5th and October 18th?

11. How many silver spoons, each weighing 2 oz. 16 dwts., could be made out of a silver tankard the weight of which is 50 oz. 8 dwts.?

12. A gentleman delivered 128 oz. 16 dwts. 16 grs. of silver to his goldsmith, and required him to make it into tankards of 17 oz. 15 dwts. 10 grs. each; spoons of 21 oz. 11 dwts. 13 grs. per dozen; salts of 3 oz. 10 dwts. each; and forks of 21 oz. 11 dwts. 13 grs. per dozen; and for every tankard, to have one salt, a dozen of spoons, and a dozen of forks. What is the number of tankards he will get?

13. In 1872 the Income Tax was fourpence in the pound; upon how many pounds of income must this tax be levied to raise the sum awarded by the Geneva Arbitrators as the liability of Great Britain to America, the award being £2500000.

**14.** If the wheel of a railway carriage measures 3 yards 1 foot 7 inches, how many revolutions will it make in travelling nine miles and a half?

## II. Compound Addition.

**45.** By Compound Addition we find the sum of several quantities of the same kind, but containing different denominations of that kind.

**RULE.—Having arranged the quantities, so that in each case the terms of the same denomination form a vertical column, find the sum of the numbers in the right-hand column, reduce this sum to terms of its own and the next higher denomination, set down the units of the present denomination beneath its own vertical column, and carry the units of the higher denomination to the next column.**

**Proceed in the same way with all the columns.**

**Ex. 1.** *Add together £7, 19s. $6\frac{1}{2}$d., £13, 6s. $2\frac{1}{2}$d., £104, 0s. $10\frac{3}{4}$d., 19s. $0\frac{1}{2}$d.*

Here we have

(2+2+3+2) *farthings*=9 farthings=$2\frac{1}{4}$d.
∴ set down $\frac{1}{4}$ and carry 2.
(6+2+10+2 carried) *pence*=20 pence=1s. 8d.
∴ set down 8, and carry 1.
(19+6+19+1 carried) *shillings*=45 shillings=£2, 5s.
∴ set down 5, and carry 2.
(7+13+104+2 carried) *pounds*=126 pounds.
∴ set down 126.

| | £ | s. | d. |
|---|---|---|---|
| | 7 | 19 | $6\frac{1}{2}$ |
| | 13 | 6 | $2\frac{1}{2}$ |
| | 104 | 0 | $10\frac{3}{4}$ |
| | | 19 | $0\frac{1}{2}$ |
| *Answer,* | £126 | 5 | $8\frac{1}{4}$ |

**Ex. 2.** *Find the sum of* 7 *tons* 18 *cwts.* 3 *qrs.* 27 *lb.* 11 *oz.* 13 *drs.*, 9 *cwts.* 2 *qrs.* 15 *lb.* 8 *oz.* 9 *drs.*, 3 *tons* 1 *qr.* 13 *lb.* 12 *oz.* 4 *drs.*, 69 *tons* 12 *cwts.* 24 *lb.* 7 *oz.* 11 *drs.*, 11 *cwts.* 2 *qrs.* 6 *lb.* 13 *oz.* 1 *dr.*

| | tons. | cwts. | qrs. | lb. | oz. | drs. |
|---|---|---|---|---|---|---|
| | 7 | 18 | 3 | 27 | 11 | 13 |
| | | 9 | 2 | 15 | 8 | 9 |
| | 3 | 0 | 1 | 13 | 12 | 4 |
| | 69 | 12 | 0 | 24 | 7 | 11 |
| | | 11 | 2 | 6 | 13 | 1 |
| *Answer,* | 81 | 12 | 3 | 4 | 5 | 6 |

# Exercise 11.

## Examples in Compound Addition.

(1.)

| s. | d. |
|---|---|
| 16 | 2 |
| 13 | 10 |
| 4 | 9 |
| 5 | 6 |

(2.)

| s. | d. |
|---|---|
| 19 | 3 |
| 18 | 7 |
| 16 | 4 |
| 2 | 11 |

(3.)

| £ | s. | d. |
|---|---|---|
| 8 | 6 | 3 |
| 21 | 19 | 6 |
| 7 | 18 | 4 |
| 16 | 13 | 10 |

(4.)

| £ | s. | d. |
|---|---|---|
| 4001 | 19 | 6 |
| 879 | 0 | 10 |
| 38016 | 18 | 3 |
| 48617 | 17 | 11 |

(5.)

| £ | s. | d. |
|---|---|---|
| 8 | 3 | 2½ |
| 16 | 17 | 9½ |
| 4 | 16 | 7¾ |
| 61 | 4 | 11¼ |
| 8 | 9 | 6¾ |

(6.)

| £ | s. | d. |
|---|---|---|
| 42 | 10 | 6½ |
| 6018 | 19 | 8 |
| 79 | 17 | 3¾ |
| 20635 | 18 | 2¼ |
| 8 | 13 | 11¾ |

(7.)

| £ | s. | d. |
|---|---|---|
| 50001 | 3 | 6½ |
| 8916 | 17 | 0 |
| 5869 | 18 | 10¾ |
| 8173482 | 19 | 6 |
| 88 | 10 | 4¼ |

(8.)

| £ | s. | d. |
|---|---|---|
| 172149 | 0 | 4 |
| 846 | 18 | 3½ |
| 9 | 17 | 10¼ |
| 26849 | 10 | 8¾ |
| 5000 | 0 | 0½ |

(9.)

| tons | cwts. | qrs. |
|---|---|---|
| 650 | 13 | 1 |
| 82 | 19 | 3 |
| 6080 | 7 | 2 |
| 960 | 14 | 2 |
| 38 | 3 | 0 |

(10.)

| tons | cwts. | lb. |
|---|---|---|
| 7618 | 14 | 96 |
| 83 | 17 | 29 |
| 24388 | 10 | 100 |
| 9618 | 19 | 111 |
| 74 | 2 | 4 |

(11.)

| cwts. | lb. | oz. |
|---|---|---|
| 1614 | 90 | 13 |
| 82 | 48 | 12 |
| 50000 | 108 | 15 |
| 683 | 97 | 14 |
| 555 | 5 | 5 |

(12.)

| tons | cwts. | qrs. | lb. | oz. | drs. |
|---|---|---|---|---|---|
| 4 | 3 | 1 | 0 | 0 | 3 |
| 28 | 19 | 2 | 17 | 13 | 13 |
| 10 | 16 | 2 | 0 | 14 | 15 |
| 908 | 3 | 0 | 26 | 0 | 10 |
| 48 | 19 | 3 | 27 | 15 | 15 |

(13.)

| oz. | dwts. | grs. |
|---|---|---|
| 83 | 13 | 21 |
| 8 | 16 | 16 |
| 306 | 17 | 19 |
| 52 | 3 | 4 |
| 108 | 0 | 23 |
| 2 | 19 | 19 |

(14.)

| oz. | dwts. | grs. |
|---|---|---|
| 428 | 13 | 2 |
| 1627 | 14 | 20 |
| 9 | 6 | 19 |
| 782 | 14 | 5 |
| 38 | 2 | 22 |
| 6 | 8 | 1 |

(15.)

| lb. | oz. | dwts. | grs. |
|---|---|---|---|
| 3124 | 11 | 19 | 2 |
|  | 8 | 0 | 19 |
| 64 | 10 | 18 | 0 |
|  |  | 19 | 23 |
| 614 | 1 | 0 | 7 |
| 8962 | 10 | 3 | 9 |

(16.)

| lb. | oz. | dwts. | grs |
|---|---|---|---|
| 15 | 10 | 18 | 15 |
| 35 | 11 | 8 | 16 |
| 17 | 8 | 14 | 20 |
| 43 | 10 | 3 | 23 |
| 18 | 3 | 12 | 10 |
| 26 | 4 | 6 | 9 |

(17.)

| yds. | ft. | in. |
|---|---|---|
| 600 | 2 | 9 |
| 81063 | 2 | 11 |
| 82 | 1 | 10 |
| 3 | 0 | 11 |
| 513 | 2 | 1 |
| 6 | 0 | 3 |

(18.)

| miles. | fur. | poles. | yds. | ft. | in. |
|---|---|---|---|---|---|
| 13 | 0 | 17 | 2 | 0 | 1 |
| 60 | 3 | 33 | 4 | 1 | 9 |
|  |  | 39 | 5 | 2 | 3 |
| 8 | 7 | 0 | 2 | 0 | 11 |
|  | 6 | 19 | 4 | 1 | 10 |
| 148 | 0 | 2 | 0 | 0 | 9 |

(19.)

| lea. | miles. | fur. | yds. | ft. | in. |
|---|---|---|---|---|---|
| 1600 | 2 | 7 | 140 | 1 | 9 |
| 80 | 1 | 3 | 186 | 2 | 6 |
|  |  | 6 | 219 | 1 | 10 |
| 8247 | 0 | 0 | 3 | 0 | 9 |
| 85 | 1 | 7 | 200 | 2 | 11 |
| 619 | 2 | 0 | 213 | 2 | 7 |

20. Add together 30 ells 4 qrs. 2½ nails, 26 ells 2 qrs. 2¼ nails, 19 ells 3 qrs. 3 nails, 26 ells 4 qrs. 1½ nails, 28 ells 2 qrs. 3½ nails, and 19 ells 3 qrs. 8 nails.

21. Add together 325 sq. yds. 6 ft. 170 in., 10900 sq. yds. 7 ft. 140 in., 8 sq. yds. 3 ft. 139 in., 108 sq. yds. 5 ft. 2 in., 17649 sq. yds. 5 ft., 800 sq. yds. 101 in.

22. Find the sum of 40 acres 2 roods 31 poles, 1640 acres 3 roods 34 poles, 210 acres 26 poles 19 yds. 101 in., 1000 acres 1 rood 6 ft. 10 in.

23. Add together 17 c. yds. 25 ft. 1324 in., 1097 c. yds. 3 ft. 1018 in., 17 c. yds. 14 ft. 201 in., 14 c. yds. 1017 in., 16 c. yds. 26 ft. 1723 in., 81 c. yds. 19 ft., 17 c. ft. 100 in.

24. Add together 189 hhds. 51 gals. 2 qts., 75 hhds. 35 gals. 2 qts., 76 hhds. 18 gals. 1 qt., 15 hhds. 50 gals. 2 qts., 215 hhds. 7 gals. 2 qts. 1 pt., 45 hhds. 15 gals. 3 qts., 179 hhds. 49 gals. 3 qts., 60 hhds. 45 gals. 2 qts., 815 hhds. 7 gals. 2 qts. 1 pt., 95 hhds. 16 gals. 3 qts., 26 hhds. 60 gals. 1 qt. 1 pt., 39 hhds. 24 gals. 1 pt.

25. Find the sum of 377 yrs. 30 wks. 4 days, 5 wks. 6 days 5 hrs. 23 min. 39 sec., 22 days 2 hrs. 28 min. 59 sec., 5 mo. 1 wk. 1 day 14 hrs., 20 days 21 hrs. 49 min. 48 sec., 32 yrs. 21 hrs. 3 min., 94 yrs. 6 days 11 hrs., 714 yrs. 3 days 9 hrs.

26. The floor of a room contains 17 sq. yds. 3 ft. 18 in.; of another, 21 sq. yds. 1 ft. 126 in.; of a third, 33 sq. yds. 4 ft.; and a fourth is half as large again as the sum of the other three. What is their united area?

27. The first of five trucks of coal contains 9 tons 16 cwts. 39 lb.; in the second there is 17 cwts. 18 lb. more than in the first, while the third contains 2 tons 109 lb. more than the first; in the fourth there is as much as in the second and third together, while the fifth contains 18 tons less than the sum of the other four. How much coal do the five trucks contain?

## III. Compound Subtraction.

**46.** By Compound Subtraction we find the difference between two quantities of the same kind, but containing different denominations of that kind.

**RULE.—Having placed the less number below the greater, in such a manner that numbers in the same denomination may be in the same vertical column, subtract, if possible, the subtrahend in the right-hand column from the minuend, and set the remainder beneath the subtrahend. If, however, the subtrahend exceed the minuend, add to this last as many units as form one unit of the next higher denomination, from the sum take the subtrahend, and set down the remainder, carrying one to the next subtrahend. Repeat the process with the next vertical column.**

**Ex. 1.** *From £148, 16s. 10½d. take £73, 11s. 2¼d.*

| £ | s. | d. | |
|---|---|---|---|
| 148 | 16 | 10½ | |
| 73 | 11 | 2¼ | |
| 75 | 5 | 8¼, | *Answer.* |

The reason of the process in this example is quite obvious from what we have said concerning Simple Subtraction.

**Ex. 2.** *From £14683, 7s. 4¾d. take £98, 14s. 10¼d.*

| £ | s. | d. | |
|---|---|---|---|
| 14683 | 7 | 4¾ | |
| 98 | 14 | 10¼ | |
| 14584 | 12 | 6½, | *Answer.* |

The various steps in the *Rule* may be somewhat varied, thus:—

(1.) *One* farthing from *three* farthings? Two; put down ½d.

(2.) *Ten* pence from *four* pence? Borrow a shilling; ten pence from a shilling? Two pence; therefore, two pence and four pence = six pence; put down 6 under the 10.

(3.) *One and fourteen* = fifteen; *fifteen* shillings from *seven* shillings? Borrow a pound; fifteen shillings from a pound? Five shillings; five shillings and seven shillings = twelve shillings; set down 12 under the 14.

(4.) One and ninety-eight = ninety-nine; ninety-nine from fourteen thousand six hundred and eight = (by Simple Subtraction) fourteen thousand five hundred and eighty-four; set down 14584.

∴ *Answer* = £14584, 12s. 6½d.

**Ex. 3.** *Take* 14 *cwts.* 3 *qrs.* 19 *lb. from* 7 *tons* 6 *cwts.* 14 *lb.; and from* 31 *miles* 2 *fur.* 1 *ft. take* 16 *miles* 21 *poles* 5 *yds.* 2 *ft.*

| | tons. | cwts. | qrs. | lb. |
|---|---|---|---|---|
| | 7 | 6 | 0 | 14 |
| | | 14 | 3 | 19 |
| *Answer,* | 6 | 11 | 0 | 23 |

| miles. | fur. | poles. | yds. | ft. |
|---|---|---|---|---|
| 31 | 2 | 0 | 0 | 1 |
| 16 | 0 | 21 | 5 | 2 |
| 15 | 1 | 17 | 5 | 2 |

In the second sum we have to take 5+1, *i.e.* 6, yds. from 0 yds., and by borrowing 1 pole we have to take 6 yds. from 5½ yds., which we cannot do, so that in this case we must borrow 2 poles instead of 1; we then have to take 6 yds. from 11 yds.; 5 will remain, therefore set down 5 and carry 2.

N.B.—Examples in Subtraction may be *proved* by adding together the subtrahend and the difference, which ought, of course, to reproduce the minuend.

# Exercise 12.

## Examples in Compound Subtraction.

(1.)

| £ | s. | d. |
|---|---|---|
| 84 | 19 | 2 |
| 31 | 16 | 1 |

(2.)

| £ | s. | d. |
|---|---|---|
| 748 | 0 | 3 |
| 9 | 10 | 6 |

(3.)

| £ | s. | d. |
|---|---|---|
| 1018 | 14 | 2 |
| 60 | 18 | 9 |

(4.)

| £ | s. | d. |
|---|---|---|
| 3010 | 0 | 4 |
| 809 | 19 | 6 |

(5.)

| £ | s. | d. |
|---|---|---|
| 4081 | 2 | 1¼ |
| 99 | 16 | 8 |

(6.)

| £ | s. | d. |
|---|---|---|
| 74431 | 1 | 2½ |
| 869 | 13 | 0¾ |

(7.)

| £ | s. | d. |
|---|---|---|
| 400001 | 6 | 0¼ |
|  | 18 | 6¾ |

(8.)

| £ | s. | d. |
|---|---|---|
| 1206 | 0 | 0 |
| 48 | 0 | 0¼ |

(9.)

| tons. | cwts. | qrs. | lb. |
|---|---|---|---|
| 9008 | 2 | 1 | 0 |
| 908 | 17 | 0 | 15 |

(10.)

| tons. | cwts. | lb. | ozs. |
|---|---|---|---|
| 6082 | 14 | 90 | 0 |
| 789 | 19 | 110 | 6 |

(11.)

| qrs. | lb. | oz. | drs. |
|---|---|---|---|
| 28 | 3 | 0 | 3 |
| 9 | 17 | 14 | 1 |

(12.)

| lb. | oz. | dwts. | grs. |
|---|---|---|---|
| 94 | 3 | 0 | 0 |
| 28 | 10 | 13 | 21 |

(13.)

| qr. | bush. | pks. | gals. |
|---|---|---|---|
| 179 | 4 | 2 | 0 |
| 6 | 5 | 3 | 1 |

(14.)

| days. | hrs. | min. | secs. |
|---|---|---|---|
| 301 | 12 | 0 | 5 |
| 8 | 7 | 6 | 19 |

15. From 4080 lea. 2 miles 30 poles 2 inches take 71 lea. 2 miles 39 poles 1 ft.

16. From 1900 lb. 1 oz. 14 dwts. 23 grs. take 708 lb. 10 oz. 19 dwts., and express the answer in dwts.

17. By how much does 30 ells 4 qrs. 2½ nls. of cloth exceed 26 ells 2 qrs. 2¼ nls.?

18. By how many cubic inches does 13 c. yds. 19 c. ft. 1001 c. in. exceed 10 c. yds. 17 c. ft. 1019 c. in.?

19. How much must be taken away from the sum of 94 lb. 3 oz., 28 lb. 10 oz. 13 dwts. 21 grs., and 1601 lb. 19 dwts. 17 grs., to make it equal to the sum of 14 lb. 15 dwts., 89 lb. 17 dwts. 19 grs., 3 lb. 10 oz. 20 grs., and 18 lb. 11 oz. 19 dwts. 13 grs.?

20. Take 179 qrs. 6 bush. 3 pks. from 326 qrs. 2 bush. 1 pk.

21. Find the difference between 126 days 7 hrs. 15 min. 39 secs., and 231 days 14 hrs. 2 min. 7 secs.

22. Find the sum and difference of 121 gals. 2 qts. 1 pt., and 69 gals. 2 qts., and to the sum of the two results add 84 gals. 1 pt.

23. How many sq. feet must be added to 1 acre 2 roods 14 poles 6 sq. feet, to make it 2 acres 1 rood?

24. The difference between the areas of two fields is 13 acres 29 sq. yds.; the larger field being 101 acres 3 roods 17 po. 2 sq. yds.; what is that of the smaller, and what is the combined area of the two?

25. A horse and carriage were sold by auction, when it was found that the horse, which was sold for 142 guineas, fetched £98, 12s. 6d. more than the carriage. What was the price of the two together?

26. Find the difference between the sum of 31 tons 6 cwts. 110 lb., 48 tons 19 cwts. 76 lb. 8 oz., 3 cwts. 29 lb. 14 oz., 90 tons 101 lb. 13 oz., and the sum of 1601 tons 14 cwts. 87 lb., 346 tons 16 cwts. 10 lb. 8 oz., 728 tons 19 cwts.

27. There are six pieces of cloth; the first two pieces each contain 15 yds. 2 nails 1 inch less than either of the last 2 pieces, which each contain 37 yds. 1 qr. 3 nails more than either of the remaining pieces, which each contain 68 yards 3 qrs. 2 nails. What is the amount of cloth in the six pieces?

## IV. Compound Multiplication.

**47.** By Compound Multiplication we find what quantity is equal to the result obtained by repeating a Compound quantity (*i.e.*, a quantity composed of denominations which, though differing among themselves, are all of the same kind), any given number of times.

**RULE.—Having placed the multiplier under the lowest denomination of the multiplicand, multiply the term in this denomination by the multiplier, and having reduced the product as in Compound Addition, put down the units of this denomination which remain, and carry the units of the higher denomination to the next product. Repeat the process with each denomination.**

**Ex. 1.** *Multiply* £83, 16*s.* 3½*d. by* 7.

| | £ | s. | d. |
|---|---|---|---|
| | 83 | 16 | 3½ |
| | | | 7 |
| *Answer,* | £586 | 14 | 0½ |

Here we have

(1.) (2 farthings × 7) = 14 farthings = 3½d.;
set down ½d. and carry 3.

(2.) (3d. × 7) + 3d. carried = 24d. = 2s. 0d.;
set down 0 and carry 2.

(3.) (16s. × 7) + 2s. carried = 114s. = £5, 14s.
set down 14 and carry 5.

(4.) (£83 × 7) + £5 carried = £586;
set down 586.

**Ex. 2.** *Multiply £19, 10s. 3¾d. by 63.*

| | £ | s. | d. |
|---|---|---|---|
| *cf.* Ex. 1, § 31, page 33, and proceed as follows: | 19 | 10 | 3¾ |
| | | | 9 |
| 9 times the Multiplicand = | 175 | 12 | 9¾ |
| | | | 7 |
| (7 times 9 times, *i.e.*) 63 times the Multiplicand = | 1229 | 9 | 8¼, *Answer.* |

**Ex. 3.** *Multiply £163, 14s. 10¾d. by 29.*

| | £ | s. | d. |
|---|---|---|---|
| | 163 | 14 | 10¾ |
| | | | 7 |
| 7 times the Multiplicand = | 1146 | 4 | 3¼ |
| | | | 4 |
| (7 times 4 times, *i.e.*) 28 times the Multiplicand = | 4584 | 17 | 1 |
| 1 time the Multiplicand = | 163 | 14 | 10¾ |
| ∴ By Addition, 29 times the Multiplicand = | 4748 | 11 | 11¾, *Answer.* |

**Ex. 4.** *Multiply £83, 16s. 3¼d. by 6428.*

| | £ | s. | d. | |
|---|---|---|---|---|
| | 83 | 16 | 3¼ | (A) |
| | | | 10 | |
| 10 times the Multiplicand = | 838 | 2 | 8½ | (B) |
| | | | 10 | |
| ∴ 100 times the Multiplicand = | 8381 | 7 | 1 | (C) |
| | | | 10 | |
| ∴ 1000 times the Multiplicand = | 83813 | 10 | 10 | |
| | | | 6 | |
| ∴ 6000 times the Multiplicand = | 502881 | 5 | 0 | |
| And, 400 times the Multiplicand = 4 times the line (C) = | 33525 | 8 | 4 | |
| 20 times the Multiplicand = 2 times the line (B) = | 1676 | 5 | 5 | |
| 8 times the Multiplicand = 8 times the line (A) = | 670 | 10 | 2 | |
| ∴ By Addition 6428 times the Multiplicand = *Answer,* | 538753 | 8 | 11 | |

N.B.—This sum might have been done as below :—

£83, 16s. 3¼d. = 80461 farthings,
∴ 6428 times £83, 16s. 3¼d. = 6428 times 80461 farthings,
= 517203308 farthings,
= £538753, 8s. 11d., *Answer.*

## Exercise 13.

### Examples in Compound Multiplication.

1. Multiply £16, 8s. 7d. by 3, 9, 4, 7, 6, 8, 10, 12, and 11.
2. Multiply £201, 17s. 8¾d. by 3, 9, 5, 4, 7, 6, 8, 10, 11, and 12.
3. Multiply £1608, 13s. 11½d. by 42, 36, 81, 84, 63, 35, and 27.
4. Multiply £9700, 18s. 3¼d. by 23, 41, 26, 34, and 107.
5. Multiply £48419, 0s. 1½d. by 9427, 6284, 19729, and 341695.
6. Multiply £317, 4s. 4¾d. by 505, 6240, 7900, 8401, 2004, and 11401.
7. Multiply 14 tons 7 cwts. 63 lb. by 8, 17, 42, 605, 19, and 191.
8. Multiply 72 tons 14 cwts. 2 qrs. 19 lb. 7 oz. 4 drs. by 63, 147, 85, and 640.
9. Multiply 63 lb. 10 oz. 14 dwts. 10 grs. by 6, 9, 17, 83, 403, and 717.
10. Multiply 39 yds. 3 qrs. 2 nails by 4, 3, 7, 19, 63, 48, and 13.
11. Multiply 71 acres 2 roods 19 poles 13 sq. yds. by 65, 81, and 1107
12. Multiply 6 m. 3 f. 5 yds. 2 ft. 7 in. by 9, 17, 71, 306, and 133.
13. Multiply 18 gals. 3 qts. 1 pt. by 16, 17, 19, 401, and 1500.
14. Multiply 605 qrs. 4 bush. 2 pks. by 13, 18, 51, and 96.
15. Multiply 31 weeks 6 days 11 hours 14 min. 19 secs. by 43, 91, and 168.
16. Multiply 4 tons 17 cwts. 2 qrs. 16 lb. by 103, reduce the product by as many lb. as there are complete cwts. therein, and having multiplied the remainder by the number which expresses the complete tons yet remaining, reduce the result to ounces.

## V. Compound Division.

**48.** By Compound Division we find how many times one Compound quantity contains another Compound quantity of the same kind, or else what Compound quantity results from dividing a given Compound quantity into a given number of equal parts.

In Compound Division, as in Simple Division, there are two processes, called, respectively, LONG DIVISION and SHORT DIVISION; but the working of both is essentially the same, being simply adaptations of the Simple Rules, and being both included in the following Rule:—

**RULE.—Arrange the divisor and dividend as in Simple Division; then—**

**(1.)** *If the divisor be a Concrete quantity,* **reduce both it and the dividend to the lowest denomination which is contained in either of them, and then divide as in Simple Division, but—**

**(2.)** *If the divisor be an Abstract number,* **find how many times the number in the highest denomination of the dividend contains the divisor, set the result in the quotient, and reduce the remainder and the number in the next denomination lower, to units of this lower denomination, and perform the same process as with the last denomination, and proceed similarly with the remaining terms of the dividend.**

The two cases for which this rule provides are the obvious result of the following axiom:—*The divisor* × *the quotient* = *the dividend;* from which it follows that, if any two of these be given, the other can be found; thus—

If £20 be multiplied by 8, the product will be £160:

Therefore, (1.) If £160 be divided by 8, the quotient will be £20.
(2.) If £160 be divided by £20, or rather, if 160 be divided by 20, the quotient will be 8.
(3.) If £20 be multiplied by 8, the product will be £160.

In the first case, the *quotient* is found from the dividend and the divisor; in the second case, the *divisor* is found (in the form of a quotient) from the dividend and the quotient (used as a divisor); in the last case, the *dividend* is found (in the form of a product) from the quotient and the divisor (used as a multiplicand and a multiplier respectively).

**Ex. 1.** *Divide* £113, 6*s.* 3*d. by* 7, *and prove the result.*

Here, we have
(1.) £113 ÷ 7 = £16 and £1 over; set down **16.**
(2.) £1 and 6s. = 26s.; ∴ 26s. ÷ 7 = 3s. and 5s. over; set down **3.**
(3.) 5s. and 3d. = 63d.; ∴ 63d. ÷ 7 = 9d.; set down **9.**

|  | £ | s. | d. |
|---|---|---|---|
| 7 \| | 113 | 6 | 3 |
|  | **£16** | **3** | **9**, *Answer.* |

**PROOF.**

(1.) By multiplying the quotient by the divisor, we obtain the original dividend; thus—

| £ | s. | d. |
|---|---|---|
| 16 | 3 | 9 |
|  |  | 7 |
| £113 | 6 | 3 |

(2.) By using the original quotient as a divisor, we obtain the original divisor as a quotient; thus—

```
 £  s.  d.     £   s.  d.
16  3   9 ) 113   6   3
20             20
----         -----
323s.        2266s.
 12            12
-----        ------
3885d.  )    27195d. ( 7
             27195
```

**Ex. 2.** *Find how many times £8, 19s. 7¾d. is contained in £3709, 13s. 8¾d., and prove the result.*

```
 £  s.  d.       £    s.  d.
 8 19  7¾    ) 3709  13  8¾
 20              20
----           ------
179s.          74193s.
 12               12
-----          --------
2155d.         890324d.
  4                4
------        ---------
8623f.        ) 3561299 ( 413 times, Answer.
               34492
              -------
                11209
                 8623
                -----
                25869
                25869
```

**PROOF.**

(1.) By multiplying the divisor by the quotient, we have £8, 19s. 7¾d. × 413 = £3709, 13s. 8¾d., by Compound Multiplication.

(2.) By using the original quotient as a divisor, we obtain the original divisor as a quotient; thus—

```
         £   s.  d.   £  s.  d.
413 ) 3709  13  8¾ ( 8  19  7¾
      3304
      ----
       405
        20
      ----
      8113
       413
      ----
      3983
      3717
      ----
       266
        12
      ----
      3200
      2891
      ----
       309
         4
      ----
      1239
      1239
```

**Ex. 3.** *Divide £7669, 6s. 7½d. by 63.*

*cf.* Ex. 2, § 31, page 33, and proceed as below :—

$$
\begin{array}{r|rrr}
 & \pounds & s. & d. \\
63\left\{\begin{array}{r} 9 \\ 7 \end{array}\right. & \begin{array}{r} 7669 \\ \hline 852 \\ \hline \pounds 121 \end{array} & \begin{array}{r} 6 \\ \hline 2 \\ \hline 14 \end{array} & \begin{array}{r} 7\tfrac{1}{2} \\ \hline 11\tfrac{1}{2} \\ \hline 8\tfrac{1}{2} \end{array}
\end{array}
\text{, } \textit{Answer.}
$$

## Exercise 14.

### Examples in Compound Division.

1. £96, 18s. 4d. ÷ 2.
2. £1018, 17s. 6d. ÷ 2.
3. £141, 16s. 9d. ÷ 3.
4. £7146, 13s. 3d. ÷ 3.
5. £9011, 13s. 8d. ÷ 4.
6. £5093, 6s. 4¼d. ÷ 5.
7. £6203, 17s. 1d. ÷ 5.
8. £821, 14s. 4¾d. ÷ 7.
9. £2752, 17s. 3¾d. ÷ 9.
10. £4562, 16s. 8¼d. ÷ 11.
11. £54754, 0s. 3d. ÷ 12.
12. £22022, 19s. 0d. ÷ 8.
13. £673, 9s. 7½d. ÷ 13.
14. £4066, 9s. 4¾d. ÷ 113.
15. £14435, 12s. 1½d. ÷ 18.
16. £3765, 16s. 9¾d. ÷ 131.
17. £131637, 11s. 9½d. ÷ 65.
18. £1635, 13s. 8¾d. ÷ 41.
19. £139418, 11s. 0d. ÷ 72.
20. £5984, 18s. 2¾d. ÷ 47.
21. £5315295, 18s. 9¾d. ÷ 121.
22. £3625, 18s. 11¾d. ÷ 49.
23. £371824784, 6s. 11d. ÷ 5826.
24. £11771, 5s. 0d. ÷ 144.
25. £314919, 18s. 3¼d. ÷ 7403.
26. £1096, 9s. 2½d. ÷ 1234.
27. £12280, 14s. 7d. ÷ 3650.
28. £601012702, 7s. 8½d. ÷ 3005.
29. £2408, 4s. 9¼d. ÷ 43.
30. £2940937220, 15s. 3¾d. ÷ 9803.
31. £47, 16s. 1¾d. ÷ 67.
32. £89, 11s. 10½d. ÷ 470.
33. £70, 0s. 7½d. ÷ 81.
34. £19895, 16s. 8d. ÷ 50000.
35. 3251 tons 14 cwts. ÷ 72.
36. 2302 cwts. 3 qrs. 15 lb. ÷ 97.
37. 1194 lb. 13 oz. 4 drs. ÷ 47.
38. 3119 lb. 5 oz. 12 dwts. 12 grs. ÷ 36.
39. 3570 yds. 3 qrs. 2 nls. ÷ 53.
40. 4571 acres 1 r. 24 po. ÷ 12.
41. 577 gals. 2 qts. ÷ 10.
42. 96 loads 1 qr. 2 bush. ÷ 100.
43. 3612 gals. ÷ 96.
44. 353862 fur. 31 poles 2 yds. 1 ft. ÷ 2017.
45. 38337 Eng. ells 3 qrs. 2 nls. 1 in. ÷ 487.
46. 373113 yds. 0 qrs. 0 nls. ÷ 216.
47. 82071 hhds. 55 gals. 3 qts. ÷ 235.
48. 126748 ac. 2 po. 29¼ yds. ÷ 367.

49. 21484 hrs. 48 min. 56 secs. ÷ 904.

50. How many times is 17 hrs. 57 min. 45 secs. contained in 15717 hrs. 11 min. 15 secs.?

51. How many times can 8½d. be taken from £1, 13s. 3½d.?

52. How many cwts. of sugar at £12, 13s. 2½d. per cwt. may be bought for £405, 2s. 8d.?

53. How many times must £2, 9s. 6½d. be paid to clear off a debt of £2318, 11s.?

54. How many pieces of lead each weighing 25 lb. 6 oz. 12 drs. can be made from one large piece weighing 1194 lb. 13 oz. 12 drs.?

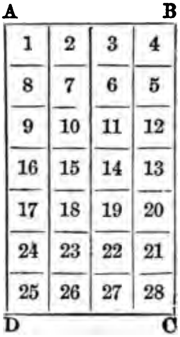
55. How many times is 345 ac. 1 rood 17 sq. po. 30 sq. yds. contained in 126748 ac. 2 p. 29¼ sq. yds.

## VI. Mensuration of Surfaces and Solids.

**49.** We have already explained (page 49) the manner in which the number of square inches in a square foot, and the number of cubic inches in a cubic foot, &c., &c., have been respectively determined: we shall now proceed to the consideration of other questions of a similar character.

**Def.** *A Rectangle* is a four-sided figure of which all the angles are right angles.

**Ex. 1.** *How many square inches are contained in a surface which is 7 inches long, and 4 inches broad?*



| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 8 | 7 | 6 | 5 |
| 9 | 10 | 11 | 12 |
| 16 | 15 | 14 | 13 |
| 17 | 18 | 19 | 20 |
| 24 | 23 | 22 | 21 |
| 25 | 26 | 27 | 28 |

By constructing the diagram ABCD, we see that such a surface would contain 7 × 4 sq. inches, *i.e.* 28 sq. inches, and the reason of the following rule is also apparent by help of this diagram and the remarks on page 49.

**RULE.—To find the area of any regular rectangular surface, express its length and breadth in units of one denomination of length, and then multiply the units in the length by the units in the breadth: the product will be the area expressed in corresponding units of square measure.**

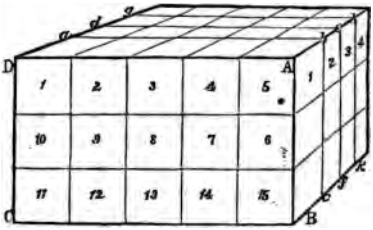
N.B.—Since the AREA = the LENGTH × the BREADTH, it follows that if any two of these be given the third may be found, thus—

(1.) The AREA may be found by multiplying the *Length* by the *Breadth.*

(2.) The LENGTH may be found by dividing the *Area* by the *Breadth.*

(3.) The BREADTH may be found by dividing the *Area* by the *Length.*

**Ex. 2.** *How many cubic feet are contained in a block of stone which is 5 feet long, 4 feet broad, and 3 feet high?*



In the accompanying diagram the area of the front face of the block is 5 × 3, *i.e.* 15 sq. feet. If, now, the block were cut into four slices of equal thickness, through the lines, *abc*, *def*, *ghk*, it is clear that each slice might be cut up into 15 little cubes, each being a *cubic foot;* and, therefore, the four slices would altogether produce 4 times 15 cubic feet, so that there are 60 cubic feet in the block.

The reason of the following rule is now obvious—

**RULE.—To find the content of any regular rectangular solid, express its length, breadth, and height in terms of the same unit of length, and then find the continued product of the number of units in each term: the result will be the content of the solid expressed in terms of the corresponding unit of cubic measure.**

N.B.—Since the **Cubic Content = Length × Breadth × Height,** it follows that if any three of these be given the fourth may be found, thus—

(1.) **The Cubic Content** may be found by multiplying together the *Length, Breadth, and Height.*

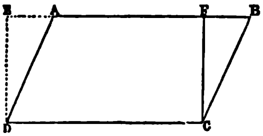
(2.) **The Length** may be found by dividing the *Cubic Content* by the product of the *Breadth* and *Height.*

(3.) **The Breadth** may be found by dividing the *Cubic Content* by the product of the *Length* and *Height.*

(4.) **The Height** may be found by dividing the *Cubic Content* by the product of the *Length* and *Breadth.*

**Ex. 3.** *Find the area of a parallelogram the length of which is* 7 *feet and the perpendicular height* 5 *feet.*

**Def.** *A parallelogram* is a four-sided figure of which the opposite sides are parallel.



Let ABCD be the parallelogram, and let CD represent 7 feet, and CF 5 feet: we want to know the area of ABCD.

It is evident that by producing BA to E, and drawing DE perpendicular to CD, we get a figure EAD, equal to BCF; so that if we take BCF from the parallelogram, and add EAD to the remainder, we shall have the figure EDCF equal to the parallelogram ABCD. But the area of EDCF is 7 × 5, *i.e.* 35 square feet (as in Ex. 2), therefore the area of ABCD is also 35 sq. feet.

Hence the reason of the following rule—

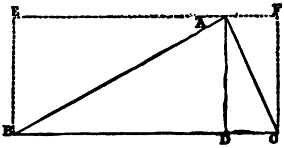
**RULE.—To find the area of a parallelogram, express its length and height in terms of the same unit, and multiply* the two results together: the product is the area expressed in terms of the corresponding unit of square measure. (Euc. I. 35.)**

**Ex. 4.** *Find the area of a triangle which is* 10 *feet long and* 6 *feet high.*

---

* The phraseology here has been retained for the sake of brevity. Strictly speaking we can neither multiply nor divide the length, breadth, &c., by each other, but the operations indicated are really performed upon the *Abstract* number of units in these quantities.

Let the triangle be represented by ABC, and its height by AD. Draw the line EAF parallel to BC, EB parallel to AD, and FC also parallel to AD; then the figure BEFC is a parallelogram, and its area = BC × AD.



*i.e.* EADB + ADCF = BC × AD;
but ABD is evidently equal to $\frac{1}{2}$ of EADB,
and ACD is equal to $\frac{1}{2}$ of ADCF,
∴ ABD + ACD = $\frac{1}{2}$ of EADB + $\frac{1}{2}$ of ADCF,
= $\frac{1}{2}$ of (EADB + ADCF) = $\frac{1}{2}$ of EBCF = $\frac{1}{2}$ of BC × AD.

Hence the reason of the following rule—

**RULE.—The area of a triangle being one-half that of the parallelogram which has the same base and the same height; to find the area of a triangle reduce its base and height to units of the same denomination of length, multiply the results together, and divide the product by 2: the quotient will be the area of the triangle expressed in corresponding units of square measure. (Euc. I. 41.)**

Thus, in the given example, the area of ABC is $\frac{1}{2}$ of 10 times 6 sq. feet, *i.e.* $\frac{1}{2}$ of 60 sq. feet, *i.e.* 30 square feet.

## 50. Solution of Exercises in Mensuration.

**Ex. 1.** *Find the content of a block of stone* 17 *ft.* 9 *in. long,* 14 *ft.* 3 *in. broad, and* 5 *ft.* 6 *in. thick.*

The cubic content = (17 ft. 9 in.) × (14 ft. 3 in.) × (5 ft. 6 in.)
= 213 in. × 171 in. × 66 in.
= 2403918 cub. in. = **1391 cub. ft. 270 cub. in.**, *Ans.*

**Ex. 2.** *How many yards of carpet* 30 *in. wide will be required for a room* 28 *feet long by* 18$\frac{1}{2}$ *feet wide?*

The area of the room = 28 feet × 18$\frac{1}{2}$ feet.
= 336 in. × 222 in. = 74592 sq. in.

We want now to know how long a piece of carpet must be to cover this area, supposing the carpet to be 30 inches wide; in other words, we have the *area* given—viz., 74592 sq. inches—and the *width* given—viz., 30 inches—and from these we find the *length*, for

$$\text{The length} = \frac{\text{area}}{\text{width}} = \frac{74592 \text{ sq. inches}}{30 \text{ inches}} = 2486 \text{ inches} + 12 \text{ inches over.}$$

$$207 \text{ ft. } 2 \text{ in.} = 69 \text{ yards } 2 + \text{ inches, } \textit{Answer.}$$

**Ex. 3.** *If a room be* 24 *feet* 6 *inches in length,* 18 *feet* 3 *inches in width, and* 11 *feet* 9 *inches in height, how much paper* 27 *inches wide would be required to cover its walls?*

(1.) There are two walls, each 24 feet 6 inches long, and two other walls, each 18 feet 3 inches long, the combined length of which will therefore be

$$2\,(24 \text{ ft. } 6 \text{ in.}) + 2\,(18 \text{ ft. } 3 \text{ in.}) = 49 \text{ ft.} + 36 \text{ ft. } 6 \text{ in.} = 85 \text{ ft. } 6 \text{ in.}$$

(2.) The height is 11 ft. 9 in.,

$$\therefore \text{The area} = (85 \text{ ft. } 6 \text{ in.}) \times (11 \text{ ft. } 9 \text{ in.}) = 1026 \text{ in.} \times 141 \text{ i}$$
$$= 144666 \text{ sq. in.}$$

(3.) As in Ex. 2, we have

$$\text{The length} = \frac{\text{area}}{\text{width}} = \frac{144666 \text{ sq. in.}}{27 \text{ in.}} = 5358 \text{ in.} = 446 \text{ ft. } 6 \text{ in.}$$

$$= 148 \text{ yds. } 2 \text{ ft. } 6 \text{ in., } \textit{Answer.}$$

## Exercise 15.

### Examples in Mensuration of Surfaces and Solids.

1. Find the area of a rectangular surface which is 81 ft. long and 17 ft. broad.
2. How many square feet are contained in a room which is 93 ft. long and 75 ft. broad?
3. What is the area of a square room, the side of which is 87 ft.?
4. What is the area of a rectangular playground which is 101 yds. 2 ft. long and 73 yds. broad?
5. How many yards of carpet will be required to cover a room which is 84 ft. long and 75 ft. wide, if the carpet be 2 ft. 1 in. in width?

**6.** How many cubic inches of stone are contained in a block which is 49 inches square at the end and 38 feet long?

**7.** How many cubic feet of air are contained in a room which is 15 ft. high, 20 ft. broad, and 61 ft. long?

**8.** Find the cubic content of 181 beams of timber, each of which is 23 ft. 7 in. long, and the end of each of which is a square, containing 32 sq. ft. 101 sq. in.

**9.** How many children can be accommodated with 8 sq. feet of space in a school, the principal room of which is 638 inches long and 239 inches wide, and possessing also two class-rooms, of which one is 239 inches by 204 inches, and the other 204 inches by 221 inches; and how many square inches will remain unutilised if only as many children are admitted as can be supplied with the given amount of surface? And how many children can be accommodated with 80 cubic feet in the principal room alone, supposing it to be 15 feet high, and how much space will remain unused?

**10.** What is the length of a room which is 31 ft. 7 in. wide, and contains 2010 sq. ft. 116 sq. in.?

**11.** How deep was a trench which contained 119 c. yds. 14 c. ft. 672 c. in., and was 9 ft. 7 in. wide?

**12.** How much longer must a trench be made so that it may contain 380 c. yds. 10 c. ft. 1440 c. in., if it be 7 ft. 3 in. wide, 8 ft. 4 in. deep, and 136 ft. 8 in. long?

**13.** What is the area of a parallelogram whose base is 4 ft. 7 in., and perpendicular height 2 ft. 9 in.

**14.** What is the perpendicular height of a triangle which is 9 ft. 7 in. long, and the area of which is 72 sq. ft. 97 sq. in.?

**15.** How many books, each of which is 1 ft. 1 in. long, 8 in. broad, and 1 in. thick, can be packed in a box which is 54 ft. 2 in. long, 12 ft. 8 in. wide, and 21 ft. 7 in. deep?

**16.** What is the united area of the four walls of a room which is 16 ft. 3 in. in height, 14 ft. 7 in. in width, and 40 ft. 2 in. long, and how much paper 25 inches wide will be required to cover them?

## Exercise 16.

### Miscellaneous Exercises in the Elementary Rules.

**51.** The following Exercises have been mostly collected from Examination Papers set to Candidates for admission into Training Colleges, from Papers of Examination of

## Students in Training Colleges, and from Cambridge Examination Papers.

1. Multiply £25, 9s. 7½d. by 51, and divide the product by 42.

2. Find the area of a floor which is 13 ft. 7 in. long, and 11 ft. 9 in. broad, and show by means of a diagram what surface is represented in each term of the answer.

3. How many packets, each weighing 2 lb. 3 oz., can be made up out of a chest of tea weighing 3 cwts. 2 qrs. 9 lb.?

4. Find the nineteenth part of 12 cwts. 3 qrs. 14 lb. 13 oz., and multiply the result by 11.

5. Find the value of 3728 articles at £1, 10s. 4½d. each.

6. Divide £16 among 18 persons. If the same sum were divided among 24 persons, what would be the difference of the share given to each?

7. A rate of 2s. 9d. in the pound produces £352; what is the rental of the parish?

8. A school manager paid £2, 14s. 3d. for the glazier's bill; each pane of glass charged for cost 14½d; how many panes of glass were charged for, and what would have been the additional expense had the glass cost 15¾d. each pane?

9. Multiply £713, 18s. 9¾d. by 29.

10. Twelve rupees, three florins, and six half-crowns amount to £2, 8s. What is the value of a rupee?

11. What is the amount of income tax paid on an annuity of 500 guineas at 7d. in the £1?

12. If a person's estate be worth £1384, 16s. a year, and the land be assessed at 2s. 9½d. in the £1, what is his clear annual income?

13. How many bricks, 9 inches long by 4 wide, will be required to pave a court 13 ft. 9 in. by 11 ft. 7 in.?

14. What would be the cost of 3 tons 17 cwts. 1 qr. 13 lb. of sugar at 4¾d. per lb.? What would be the difference if the cost were 4½d. per lb.?

15. A grocer, by selling sugar at 5d. per lb. gained £5, 13s. 4d. upon the sale of 16 cwts. 2 qrs. 14 lb. What was the prime cost of the whole?

16. From one million guineas, subtract £499, 2s. 10½d., and divide the difference by 99. Explain the reason of each step of the process.

17. A piece of land is required which can be divided into allotments of 3 roods, 2 roods, 1 rood, 35 perches, and 25 perches; there are to be 127 allotments of each size. How many acres must the land contain?

18. What cash must be given with 24 yards of cloth at 8s. 3¾d. per yard to pay for 6 cwts. of sugar at 38s. per cwt.?

19. A coal train consists of 23 trucks. The average weight of a loaded truck is 15 tons 11 cwts. 1 qr., and of an empty truck 3 tons 15 cwts. 3 qrs. What is the weight of the coal in the train?

20. A yacht and its fittings cost £1005, 12s.; the yacht cost 5 times as much as the fittings; how much did the yacht cost?

21. A merchant spends £1636, 5s. on equal quantities of wheat at £2, 2s. per quarter, barley at £1, 1s. per quarter, and oats at 14s. per quarter. What quantity of each will he have?

22. A house and its furniture are worth £6734, 5s. 9d., and the house is worth eight times as much as the furniture. What is the house worth?

23. A person's weekly income is £14, and he spends £128, 10s. per quarter. How much does he save in a year?

24. How many half-guineas are there in 537 half-crowns?

25. How many hours are there in 10 years, of which the first is 1852?

26. If a silver tankard weighing 100 oz. 16 dwts. is made into spoons weighing 2 oz. 16 dwts. each, how many spoons are there?

27. A merchant buys 10 gallons of spirits at 12s., 15 at 4s. 6d., and 18 at 5s. 9d. What will be the price of a gallon of the mixture, so that he may gain £2, 5s. 6d. on his outlay?

28. Multiply 2 cwts. 3 qrs. 11 lb. by 365; explain each step of the process.

29. Find the cost of 250 lb. of tea at 3s. 11½d. per lb. If 10 lb. be spoiled, what will be gained by selling the remainder at 4s. 6d. per lb.?

30. Find the cost of 20 dozen of wine at 4s. 11½d. per bottle. If 3 bottles be spoiled, what will be gained by selling the remainder at 5s. 4d. per bottle?

31. Six dollars, four florins, and four half-crowns are together worth £2, 3s. What is the value of a dollar?

32. Four thalers, six half-crowns, and eight florins are together worth £2. What is the value of a thaler?

33. When will a number divide exactly by 8, 9, or 11, and when by 3, 11, or 12?

34. A sum of money amounting to £7816, free of legacy duty, was left by will to be divided equally amongst a certain number of legatees. The share proved to be £325, 13s. 4d. What was the number of legatees?

35. Find the difference in area between a room which is 12 ft. 6 in. by 10 ft. 3 in., and one which is 15 ft. 8 in. by 11 ft. 4 in.

36. Find the complete area of the walls of a room which is 15 ft. 6 in. by 13 ft. 4 in. by 10 ft. 6 in.

37. Multiply 17 miles 7 fur. 29 poles 3 yds. by 27; and divide 7 tons 2 cwts. 18 lb. 12 oz. by 45.

38. From 43 wks. 5 days 16 hrs. 11 min. 36 secs. subtract 32 wks. 6 days 9 hrs. 11 min. 47 secs.; and explain the reason for what is called "carrying one" in Subtraction.

39. Find the value of 327 articles at £3, 15s. 7½d. each, and explain each step of the process as you would to a class.

40. How many strokes of his legs must a person on a bicycle give in going 26 miles, supposing each wheel to have a circumference of 3¼ yards, and that 2 strokes turn the wheel once round?

41. A farmer sells 20 sheep at £1, 12s. 6d. each, and 24 at £1, 9s. 2d. each. How many can he buy with the proceeds, at £1, 13s. 9d. each?

42. If 8 cubic feet 1161 cubic inches of timber are required to floor a room 13 ft. 6 in. long, and 12 ft. 4 in. wide, what is the thickness of the boards?

43. Find how many days it would take me to lay up £5, 17s. 6d. if I put by 3d. every day.

44. If a child attends for 4 yrs. at school, paying at the rate of 2d. per week, on an average of 36 weeks in a year; what is the whole amount?

45. If a man gets £3, 3s. a week, and puts by £10 a quarter, how much does he spend weekly?

46. If a man pays £749, 18s. 4d. for debts, at 3s. 7d. in the £; how much did he owe?

47. How many ounces of silver will it take to make 10 dozen spoons, each weighing 3 oz. 7 dwts.?

48. How many lbs. of butter at 17d. a lb. will pay for a cow worth £22, 10s.?

49. What is the difference between the product of 18, 19, and 35, and that of 24, 17, and 19?

50. Given that the French franc is worth 9¾d., find how many francs are worth £203, 16s. 3¾d.

51. A cistern is to be made 5 ft. 7 in. long, 4 ft. 4 in. wide, and 3 ft. 6 in. deep. How many square feet of lead will line it, and how many cubic inches of water will fill it?

52. To the product of 130 times £70, 8s. 4d. add £35, 4s. 2d., take £9089, 5s. from the sum, and divide the remainder by 89.

53. Find the greatest common measure and the least common multiple of 128, 384, 768, and 2304.

54. If the food of 82 men for 26 days cost £199, 17s. 6d., what is the cost of the daily food of each man?

55. How many strips of turf 1 ft. 4 in. long and 9 in. wide will be required to turf a playground which is 83 ft. long and 45 ft. broad, there

being a gravel walk 7 ft. broad running through its entire length, and a space in one corner occupied by an out-house which is 10 ft. square?

56. What sum will settle the following bill:—40 chests of cloves at 2s. 1d. each, 35 bags of coffee at £2, 0s. 6d. per bag, 71 bags of saltpetre at £1, 5s. 6d. a bag, 5 casks of sugar at £2, 6s. 6d. a cask?

57. A boy buys a suit of clothes, which he will pay in a year, at so much per week; there are 2¼ yards of cloth at 11s. 4d. per yard, the trimming costs 7s. 1d., and the making 15s. 3d.; what has he to pay per week?

58. Find the amount of the following bill:—20 dozen copybooks at 2s. 6d. per dozen, 15 dozen account-books at 15s. per dozen, 1000 quills at 5s. 6d. per hundred, 125 inkstands at 7½d. each, 24 dozen lead pencils at 2s. 9d. per dozen, 64 boxes of steel pens at 1s. 8d. each box.

59. A person's weekly expenditure is £15, 5s. What must be his daily income that at the end of eleven years he may have saved £425, 18s. 8d., supposing that the first year is leap year?

60. Find the value of (2346784 × 53 ÷ 583) + (107298 ÷ 18 × 79.)

61. Find the Least Common Multiple of 225, 255, 289, 1023, and 4095.

62. How many planks, each 27 ft. long and 21 inches wide, will be required for the construction of a platform which is 54 yds. long and 21 yds. broad; and what will be the cost at 5½d. per square foot?

63. A contractor beginning trade with a capital of £20500, gains for the first six years at the rate of £2109, 13s. 4d. a year, and in the next four years he gains the following sums—viz., £108, 10s. 6d., £3021, 19s. 6d., £710, 19s. 10½d., £4682, 17s. 4½d. In consequence of strikes, and the consequent rise in prices of materials, he loses next year one-half his total profits in the last two years, and in the following year he again loses, but his loss is only one-half of that in the preceding year. What will be his real capital at the end of the twelve years?

## *CHAPTER IV.*

# FRACTIONS.

## DEFINITIONS.

52. **A Fraction** is a quantity which expresses a part or parts of a unit.

Thus, if one apple be divided into five equal parts, each of these parts would be a fraction of the whole apple, and one part would be called *one-fifth*, two parts would be called *two-fifths*, &c.

A fraction in its simplest form consists of two numbers, of which one is placed *above*, and the other *below* a line drawn between them.

The figure placed beneath the line is called

**The Denominator,** *i.e.* the "*name-giver,*" because it gives the name to the parts.

Thus, when, as in the above example, the unit is to be divided into *five* equal parts, each part is called a *fifth*, and two of these parts are called *two-fifths*, which is thus expressed, $\frac{2}{5}$, where the 5, which is the *denominator*, or *name-giver*, gives the name *fifths* to the fraction.

The figure placed above the line is called

**The Numerator,** *i.e.* the "*numberer,*" or "*counter,*" because it indicates how many of the parts named by the denominator are to be taken.

Thus, in the above example, $\frac{2}{5}$, the 2 is used to indicate that two-*fifths* are to be taken.

**53.** By help of the following proposition, which is best explained by an example, we shall be able to obtain another definition of a fraction.

**Ex.** *To prove that* $\frac{2}{5}$ *of* 1 = $\frac{1}{5}$ *of* 2.

Since 1 = *five*-fifths of a unit,
2 = *ten*-fifths of a unit,
∴ $\frac{1}{5}$ of 2 = $\frac{1}{5}$ of *ten*-fifths of a unit = *two*-fifths of a unit = $\frac{2}{5}$ of 1.

**54.** The following definition of a fraction will now be easily understood.

**Def.** *A fraction is a simple manner of indicating that its numerator is to be divided by its denominator; e.g.*—

The fraction $\frac{3}{7}$ = (1.) three-sevenths of one,
= (2.) one-seventh of three, which is found by dividing 3 by 7, *i.e.* the numerator by the denominator.

## VULGAR FRACTIONS.

**55. A Vulgar Fraction,** ***i.e.*** **a Common Fraction,** is one which is expressed by means of a numerator and a denominator placed one above the other.

A Vulgar Fraction is either—

(1.) **A Proper Fraction,** *i.e.* a fraction whose numerator is *less than* its denominator, as $\frac{3}{10}$; or,

(2.) **An Improper Fraction,** *i.e.* a fraction whose numerator is either *equal to,* or *greater than,* its denominator, as $\frac{7}{7}$, $\frac{41}{41}$, $\frac{10}{3}$, $\frac{101}{43}$; both of which may be either—

(1.) **A Compound Fraction,** *i.e.* a fraction of a fraction, as $\frac{3}{7}$ of $\frac{1}{5}$; or,

(2.) **A Complex Fraction,** *i.e.* a fraction whose numerator or denominator, or both, are fractions, or mixed numbers, as $\frac{\frac{2}{3}}{\frac{7}{8}}$, $\frac{6}{13\frac{4}{7}}$, $\frac{4\frac{1}{5}}{2\frac{3}{7}}$.

**A Mixed Number** is one which consists of a whole number and a fraction, as $4\frac{2}{3}$, which is only a shortened form of $4 + \frac{2}{3}$, the sign *plus* being understood.

**56.** *Every integer may be considered as a fraction, having itself for the numerator, and unity for the denominator*—

Thus, $6 = \frac{6}{1}$, for 6 divided by $1 = 6$.

**57.** TO MULTIPLY ANY GIVEN FRACTION BY ANY GIVEN INTEGER.

**Ex.** $\frac{2}{35} \times 7$.

Here we want to make the quotient, found by dividing 2 by 35, *seven* times greater. Now any quotient may be made seven times greater in either of two ways, viz.—

1st. By making the *dividend* seven times *greater*; or,

2nd. By making the *divisor* seven times *less*;

so that $\frac{2}{35}$ may be multiplied by 7 either

1st. By keeping the same denominator, and multiplying its numerator by 7;

Thus, $\frac{2}{35} \times 7 = \frac{2 \times 7}{35} = \frac{14}{35}$; or,

2nd. By keeping the same numerator, and dividing its denominator by 7;

Thus, $\frac{2}{35} \times 7 = \frac{2}{35 \div 7} = \frac{2}{5}$.

Hence we deduce the following rule—

**RULE.—To multiply any given fraction by any given integer, multiply the numerator by it, or divide the denominator by it.**

N.B.—It follows that—

$$7 \times \frac{2}{35} = (1.)\ \frac{7 \times 2}{35} = \frac{14}{35}; \text{ or,}$$

$$(2.)\ \frac{2}{35 \div 7} = \frac{2}{5}.$$

**58. Conversely,** TO DIVIDE ANY GIVEN FRACTION BY ANY GIVEN INTEGER.

**Ex.** $\frac{21}{22} \div 7$.

Here we want to make the quotient, found by dividing 21 by 22, *seven times less*. Now any quotient may be made seven times less in either of two ways, viz.—

1st. By making the *dividend* seven times *smaller*; or,

2nd. By making the *divisor* seven times *greater*;

so that $\frac{21}{22}$ may be divided by 7, either—

1st. By keeping the same denominator, and dividing its numerator by 7;

$$\text{Thus, } \frac{21}{22} \div 7 = \frac{21 \div 7}{22} = \frac{3}{22}; \text{ or,}$$

2nd. By keeping the same numerator, and multiplying its denominator by 7;

$$\text{Thus, } \frac{21}{22} \div 7 = \frac{21}{22 \times 7} = \frac{21}{154}.$$

Hence we deduce the following rule—

**RULE.—To divide any given fraction by any given integer, divide the numerator by it, or multiply the denominator by it.**

**59.** THE VALUE OF A FRACTION IS NOT ALTERED BY MULTIPLYING OR DIVIDING BOTH ITS NUMERATOR AND ITS DENOMINATOR BY THE SAME QUANTITY.

**Ex. 1.** *Multiplying the numerator and the denominator of* $\frac{7}{24}$ *by* 3, *we get*

$$\frac{7}{24} = \frac{7 \times 3}{24 \times 3} = \frac{21}{72}.$$

To establish the truth of this proposition, it is necessary to remark that if a quantity be first multiplied and then divided by the same number, its value is not thereby altered.

Now, when the numerator of a fraction is multiplied by 3, as in the above case, its value is thereby *increased* threefold (§ 57); and when its denominator is multiplied by 3, as in the above example, its value is *decreased* threefold (§ 58); so that the value of a fraction is not altered by *multiplying* its numerator and its denominator by the same number.

Conversely, in the following example it will be noticed that by dividing the numerator of a fraction we *decrease* its value, but by dividing its denominator by the same number we *increase* its value in the same degree that we before decreased it.

**Ex. 2.** *Dividing the numerator and the denominator of* $\frac{8}{24}$ *by* 4, *we get*

$$\frac{8}{24} = \frac{8+4}{24+4} = \frac{2}{6}.$$

N.B.—When the numerator and the denominator are both *divided* by the same number or numbers, the process is called *cancelling*.

## 60. TO REPRESENT AN IMPROPER FRACTION AS A WHOLE OR MIXED NUMBER.

**Ex. 1.** *The value of* $\frac{18}{3}$ *is not altered by dividing its numerator and its denominator by* 3 (§ 59); *therefore, we have*

$$\frac{18}{3} = \frac{18 \div 3}{3 \div 3} = \frac{6}{1} = 6 \text{ (§ 54).}$$

**Ex. 2.** $\frac{963}{7} = \frac{959+4}{7} = \frac{959}{7} + \frac{4}{7} = 137 + \frac{4}{7} = 137\frac{4}{7}$.

In this second example the difficulty is to find what number (in this case 4), less than the denominator 7, is to be taken from the numerator to form the fractional part of the answer, so as to leave a multiple of 7 to give the integral part of the answer. This number can always be found by dividing the numerator by the denominator.

Hence the following rule—

**RULE.—To represent an improper fraction as a whole or mixed number, divide the numerator by the denominator. If there be no remainder, the quotient, which will be a whole number, will be the answer; but if there be a remainder, the answer will be a mixed number, consisting of the quotient as a whole number, and a fraction having the remainder for its numerator and the original denominator for its denominator.**

Compare the following examples—

**Ex. 3.** $\frac{917}{182} = 5\frac{7}{182}$ (§ 60) $= 5\frac{7 \div 7}{182 \div 7}$ (§ 59) $= 5\frac{1}{26}$; or,

**Ex. 4.** $\frac{917}{182} = \frac{917 \div 7}{182 \div 7}$ (§ 59) $= \frac{131}{26} = 5\frac{1}{26}$ (§ 60).

## 61. TO REPRESENT A MIXED NUMBER AS AN IMPROPER FRACTION.

**Ex.** $4\frac{2}{7} = 4 + \frac{2}{7}$;

Now, $1 = \frac{7}{7}$,

$\therefore 4 = \frac{7 \times 4}{7} = \frac{28}{7}$,

$\therefore 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}$.

Hence the following rule—

**RULE.—To represent a mixed number as an improper fraction,**

**multiply the integer by the denominator of the fractional part, to this product add the numerator of the fractional part, and set down this sum as the numerator of a fraction which has the same denominator as the fractional part of the given mixed number.**

### 62. TO REDUCE A COMPOUND FRACTION TO A SIMPLE ONE.

**Ex. 1.** $\frac{3}{7}$ *of* $\frac{4}{5}$ = (1.) 3 *times* ($\frac{1}{7}$ *of* $\frac{4}{5}$) = 3 *times* ($\frac{4}{5} \div 7$) = 3 *times* $\frac{4}{5 \times 7}$ (§ 58).

$\therefore \frac{3}{7}$ *of* $\frac{4}{5}$ = (2.) $\frac{4}{5 \times 7} \times 3 = \frac{4 \times 3}{5 \times 7} = \frac{12}{35} = \frac{\text{the product of the numerators.}}{\text{the product of the denominators.}}$

Hence the following Rule—

**RULE.—To reduce a compound fraction to a simple one, multiply together all the numerators for a new numerator, and all the denominators for a new denominator; the fraction, so expressed, is the answer.**

$$\textbf{Ex. 2.}\ \tfrac{3}{5}\ \textit{of}\ 6\tfrac{1}{3}\ \textit{of}\ \tfrac{7}{8}\ \textit{of}\ \tfrac{16}{19} = \frac{\overset{1}{\cancel{3}}}{5} \times \frac{\overset{1}{\cancel{19}}}{\underset{1}{\cancel{3}}} \times \frac{7}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{19}}}$$

The working is here considerably shortened by cancelling; thus—

(1.) Reduce to unity the 3 and the 19 in the line of numerators, and do the same to the 3 and the 19 in the line of denominators. The value of the answer will not be altered because we have divided the numerator and the denominator thereof by the same number—viz., 3 × 19, or 57. (§ 57).

(2.) Divide the 16 in the line of numerators, and the 8 in the line of denominators, by 8, and put 2 above the 16, because 16 ÷ 8 = 2, and 1 beneath the 8, because 8 ÷ 8 = 1.

(3.) Now, applying the rule, we get

$$\left.\begin{array}{l} \text{The product of the numerators} = 1 \times 1 \times 7 \times 2 = 14 \\ \hline \text{The product of the denominators} = 5 \times 1 \times 1 \times 1 = 5 \end{array}\right\} \therefore \textit{Ans.} = \frac{14}{5} = 2\tfrac{4}{5}.$$

N.B.—1. It is not usual to put down the *ones* when cancelling. The following example should therefore be carefully noticed.

$$\textbf{Ex. 1.}\ \tfrac{3}{8}\ \textit{of}\ \tfrac{4}{9} = \frac{\cancel{3} \times \cancel{4}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}}} = \tfrac{1}{6}.$$

N.B.—2. Mixed numbers must be reduced to improper fractions in finding the value of a compound fraction, thus—

**Ex. 2.** $2\frac{1}{3}$ *of* $4\frac{1}{5} = \frac{7}{\cancel{3}}$ *of* $\frac{\overset{7}{\cancel{21}}}{5} = \frac{49}{5} = 9\frac{4}{5}$. *Ans.*

**63.** TO REDUCE A COMPLEX FRACTION TO A SIMPLE ONE.

**Ex. 1.** $\dfrac{3\frac{1}{8}}{4} = \dfrac{\frac{25}{8}}{4} = \dfrac{25}{8 \times 4}$ (§ 58) $= \dfrac{25}{32}$.

**Ex. 2.** $\dfrac{4\frac{1}{4}}{6\frac{1}{3}} = \dfrac{\frac{17}{4}}{\frac{19}{3}}$.

If we divide $\frac{17}{4}$ by 19, we shall evidently divide by a number which is 3 times too great, and our answer will consequently be 3 times too small, and will therefore require to be multiplied by 3 to bring it to its required value.

Now $\frac{17}{4}$ can be divided by 19 by simply multiplying its denominator by 19, thus—

$\frac{17}{4} \div 19 = \frac{17}{4 \times 19}$;

and $\frac{17}{4 \times 19}$ can be multiplied by 3 by simply multiplying its numerator by 3, thus—

$\frac{17}{4 \times 19} \times 3 = \frac{17 \times 3}{4 \times 19} = \frac{51}{76}$.

A consideration of these two examples enables us to lay down the following rule—

**RULE.—To reduce a complex fraction to a simple one, express its numerator and denominator as fractions, keeping the denominator below the numerator; then multiply together the extremes (*i.e.* the top and bottom numbers) for the numerator of the answer, and the means (*i.e.* the middle numbers) for its denominator.**

**Ex. 3.** $\dfrac{17\frac{1}{4}}{9\frac{1}{6}} = \dfrac{\frac{69}{4}}{\frac{55}{6}} = \frac{69 \times 6}{4 \times 55} = \frac{414}{220} = 1\frac{194}{220} = 1\frac{97}{110}$.

The process of cancelling is thus applied in such cases—

Since the answer $= \dfrac{\text{the product of the extremes,}}{\text{the product of the means,}}$ it follows that if either of the extremes is divisible by the same number as will divide one of the means, we may cancel by this common measure, thus—

$$\frac{17\frac{1}{4}}{9\frac{1}{6}} = \frac{\dfrac{69}{\cancel{4}\ 2}}{\dfrac{55}{\cancel{6}\ 3}} = \frac{69 \times 3}{55 \times 2} = \frac{207}{110} = 1\frac{97}{110}.$$

## 64. To reduce a fraction to its lowest terms.

**Def. 1.**—A fraction is said to be reduced to its lowest terms when its numerator and denominator are prime to each other.

When these are not prime to each other, they are each exactly divisible by a number greater than unity; this number is called their *Common Measure*, and the greatest number which will exactly divide them is called their *Greatest Common Measure*, and may be found by § 32.

When one number is prime to another, they have no common measure greater than unity, so that the following definition is also correct:—

**Def. 2.**—A fraction is reduced to its lowest terms when its numerator and denominator have no common measure greater than unity.

The following rule will now be self-evident—

**RULE.—To reduce a fraction to its lowest terms, find the G.C.M. of its numerator and denominator, divide each of them by it, and set down the quotient of the numerator as a new numerator, and the quotient of the denominator as a new denominator.**

**Ex.** *Reduce* $\frac{1197}{1539}$ *to its lowest terms.*

By § 32, we find that the G.C.M. of 1197 and 1539 is 171.

$\therefore$ The answer $= \frac{1197 \div 171}{1539 \div 171} = \frac{7}{9}$.

N.B.—It is often possible to reduce a number to its lowest terms without finding the G.C.M. of its numerator and denominator, thus—

**Ex.** $\frac{819}{945} = \frac{819 \div 9}{945 \div 9} = \frac{91}{105} = \frac{91 \div 7}{105 \div 7} = \frac{13}{15}$,

which should be worked thus—

$$\frac{819}{945} = \frac{91}{105} = \frac{13}{15}.$$

## 65. To reduce fractions to equivalent ones having a common denominator.

**Ex.** *Let it be required to reduce* $\frac{3}{8}$, $\frac{4}{9}$, $\frac{5}{12}$ *to equivalent fractions having a common denominator.*

1. It is evident that no simple fraction whatever can be equivalent to $\frac{3}{8}$, unless its denominator be some multiple of 8, because the fraction being already in its lowest terms, we cannot *divide* its numerator and denominator by any common measure, and the only other way to obtain an equivalent fraction is to multiply both the numerator and the deno-

minator by some number, thereby obtaining a denominator which will be a multiple of 8.

2. In the same way we may show that no simple fraction equivalent to $\frac{4}{9}$ can be found which has not a multiple of 9 for its denominator, nor any equivalent to $\frac{5}{12}$ not having a multiple of 12 for its denominator.

3. We therefore conclude that any common denominator to the fractions $\frac{3}{8}$, $\frac{4}{9}$, and $\frac{5}{12}$ must be a common multiple of their denominators, *i.e.* must be exactly divisible by 8, 9, 12.

The least common denominator will therefore be the L.C.M. of 8, 9, 12 (§ 34), which is 72, and we take this for the common denominator of the given fractions.

4. Taking the fractions in order, it is evidently necessary to multiply the denominator 8, of $\frac{3}{8}$, by 9, to make it 72; and to keep the value of the fraction unaltered, we multiply 3, its numerator, also by 9,

$$\left.\begin{aligned}\text{and we then have } \tfrac{3}{8} &= \tfrac{3\times9}{8\times9} = \tfrac{27}{72},\\ \text{similarly, } \tfrac{4}{9} &= \tfrac{4\times8}{9\times8} = \tfrac{32}{72},\\ \text{and } \tfrac{5}{12} &= \tfrac{5\times6}{12\times6} = \tfrac{30}{72}.\end{aligned}\right\}\ \text{All these fractions have the same denominator—viz., 72.}$$

From what has now been said, the following rule will be self-evident—

**RULE.—To reduce fractions to equivalent ones having a common denominator, find the L.C.M. of the denominators of the given fractions; divide this L.C.M. by each of the given denominators, and multiply each successive quotient by the numerator which corresponds to the denominator used to find that quotient.**

## Exercise 17.

### Exercises on Articles 52 to 65.

1. Explain, as to a class, the term, "Vulgar Fractions."

2. If the numerator and denominator of a fraction be multiplied by the same number, the fraction thus obtained is equivalent to the former fraction. Prove the truth of this rule by taking the fractions $\frac{2}{3}$ and $\frac{10}{15}$, and showing that they are equivalent.

3. Explain the terms *numerator* and *denominator*, as applied to Vulgar Fractions.

4. Explain the rule for finding the Greatest Common Measure, and bring to its lowest terms the fraction $\frac{30923}{59823}$.

5. Reduce the following improper fractions to whole or mixed numbers:—$\frac{35}{7}$, $\frac{119}{17}$, $\frac{403}{182}$, $\frac{82409}{161}$, $\frac{143020}{172}$, $\frac{101936}{8271}$, $\frac{4290856}{1342}$,

6. Reduce the following mixed numbers to improper fractions:—$4\frac{6}{7}$, $8\frac{4}{5}$, $16\frac{2}{3}$, $141\frac{143}{144}$, $218\frac{109}{5000}$, $4\frac{18143}{50723}$, $1638\frac{201}{54823}$.

7. Find the least common denominator of the following fractions, and reduce the fractions to equivalent ones having that least common denominator:—$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{9}$; $\frac{2}{3}$, $\frac{3}{8}$, and $\frac{5}{16}$; $\frac{5}{7}$, $\frac{4}{21}$, $\frac{3}{8}$, and $\frac{5}{9}$; $\frac{13}{15}$, $\frac{17}{20}$, $\frac{99}{100}$, $\frac{17}{18}$, and $\frac{13}{36}$; $\frac{18}{91}$, $\frac{14}{65}$, $\frac{101}{143}$, $\frac{38}{55}$, $\frac{16}{35}$, $\frac{118}{245}$, and $\frac{49}{77}$; $\frac{19}{34}$, $\frac{20}{119}$, $\frac{67}{105}$, $\frac{17}{30}$, $\frac{181}{357}$, and $\frac{199}{210}$.

8. Reduce the following compound fractions to simple ones, or to mixed numbers:—$\frac{1}{3}$ of $\frac{4}{7}$, $\frac{6}{7}$ of $\frac{14}{15}$, $\frac{18}{19}$ of $\frac{7}{9}$, $\frac{4}{5}$ of $\frac{15}{16}$ of $\frac{7}{24}$, $\frac{19}{108}$ of $\frac{36}{95}$ of $\frac{150}{203}$ of $\frac{4}{7}$, $2\frac{1}{3}$ of $16\frac{1}{2}$, $5\frac{3}{4}$ of $\frac{14}{69}$, $\frac{13}{91}$ of $3\frac{1}{2}$, $\frac{15}{17}$ of $\frac{85}{123}$ of $2\frac{1}{8}$ of $\frac{1}{3}$ of 19, $\frac{16}{19}$ of $\frac{102}{144}$ of $5\frac{1}{7}$ of $81\frac{1}{4}$, $\frac{7}{13}$ of $\frac{6}{35}$ of $1\frac{1}{8}$ of 169, $\frac{3}{17}$ of $2\frac{3}{13}$ of $17\frac{3}{58}$ of 20.

9. Reduce $\frac{8}{15}$ of $\frac{165}{201}$ of $21\frac{3}{13}$ of $\frac{1}{17}$ of 95 to a Vulgar Fraction having 289 for its denominator.

10. Multiply $\frac{4}{9}$ by 3, $\frac{17}{84}$ by 7, $\frac{41}{63}$ by 9, $\frac{201}{221}$ by 5, $\frac{81}{74}$ by 15, $\frac{151}{153}$ by 17, $\frac{131}{134}$ by 81, $\frac{206438}{2119274}$ by 326, $\frac{1911488}{16222767}$ by 8059.

11. Divide $\frac{12}{13}$ by 3, $\frac{16}{21}$ by 7, $\frac{15}{101}$ by 5, $\frac{172}{203}$ by 4, $\frac{219}{1000}$ by 71, $\frac{618}{1249}$ by 17, $\frac{10452}{21935}$ by 804, $\frac{66423}{208719}$ by 413, $\frac{512}{635}$ by 170.

12. Reduce the following fractions to their lowest terms:—$\frac{630}{945}$, $\frac{360}{504}$, $\frac{693}{1287}$, $\frac{129976}{260508}$, $\frac{20748}{161196}$, $\frac{97359}{439070}$, $\frac{112768}{188496}$, $\frac{3336}{4587}$, $\frac{153051}{171037}$, $\frac{475835}{1469443}$, $\frac{15409021}{17221847}$, $\frac{1256671637}{7231427811}$.

## I. Addition of Fractions.

**66. Addition of Fractions** is effected in the following cases—

(1.) WHEN ALL THE FRACTIONS TO BE ADDED TOGETHER HAVE THE SAME DENOMINATOR.

**Ex. 1.** *Add together* $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, *and* $\frac{7}{8}$ (*i.e. Find the value of* $\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$).

Here 1 eighth + 3 eighths + 5 eighths + 7 eighths = 16 eighths = $\frac{16}{8}$ = 2.

The reason of the following rule will now be evident—

**RULE.—To find the sum of a number of fractions having the same denominator, divide the sum of their numerators by their common denominator.**

(2.) WHEN ALL THE FRACTIONS TO BE ADDED TOGETHER HAVE NOT THE SAME DENOMINATOR.

**Ex. 2.** *Add together* $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{8}$, *and* $\frac{7}{9}$ (*i.e. Find the value of* $\frac{2}{3}+\frac{3}{4}+\frac{1}{8}+\frac{7}{9}$).

Since we can only add together quantities of the same kind, we must express these given fractions as equivalent ones having one common denominator before we can find their sum. We find by § 65 that

$$\tfrac{2}{3} + \tfrac{3}{4} + \tfrac{1}{8} + \tfrac{7}{9} = \tfrac{48}{72} + \tfrac{54}{72} + \tfrac{9}{72} + \tfrac{56}{72} = \tfrac{167}{72} = 2\tfrac{23}{72}.$$

Hence the reason of the following rule—

**RULE.—To find the sum of a number of fractions having different denominators, express them first as equivalent fractions having the same denominator (§ 65), and then divide the sum of the numerators of these new fractions by their common denominator.**

### (3.) WHEN SOME OR ALL OF THE FRACTIONS TO BE ADDED TOGETHER FORM PARTS OF MIXED NUMBERS.

**Ex.** 3. *Add together* $3\tfrac{1}{2}$, $\tfrac{2}{7}$, *and* $8\tfrac{1}{4}$ (*i.e. Find the value of* $3\tfrac{1}{2} + \tfrac{2}{7} + 8\tfrac{1}{4}$).

Adding together *the whole numbers* separately, we have

$$3 + 8 = 11,$$

and adding together *the fractions* separately, we have

$$\tfrac{1}{2} + \tfrac{2}{7} + \tfrac{1}{4} = \tfrac{29}{28} = 1\tfrac{1}{28},$$

therefore, $3\tfrac{1}{2} + \tfrac{2}{7} + 8\tfrac{1}{4} = 11 + 1\tfrac{1}{28} = 12\tfrac{1}{28}$, *Answer.*

Hence the reason of the following rule—

**RULE.—To add together mixed numbers, add together the whole numbers separately, and also the fractions separately, and find the sum of the two results.**

### (4.) WHEN ALL OR ANY OF THE FRACTIONS TO BE ADDED TOGETHER ARE IMPROPER FRACTIONS.

**Ex.** 4. *Find the sum of* $\tfrac{143}{12}$, $\tfrac{38}{9}$, $\tfrac{101}{36}$ *and* $\tfrac{48}{6}$ (*i.e. Find the value of* $\tfrac{143}{12} + \tfrac{38}{9} + \tfrac{101}{36} + \tfrac{48}{6}$).

Reducing the improper fractions to whole or mixed numbers, we have

$$\begin{aligned}\tfrac{143}{12} + \tfrac{38}{9} + \tfrac{101}{36} + \tfrac{48}{6} &= 11\tfrac{11}{12} + 4\tfrac{2}{9} + 2\tfrac{29}{36} + 8,\\ &= (11 + 4 + 2 + 8) + (\tfrac{11}{12} + \tfrac{2}{9} + \tfrac{29}{36},)\\ &= 25 + 1\tfrac{34}{36},\\ &= 26\tfrac{34}{36}, \textit{ Answer.}\end{aligned}$$

Hence the reason of the following rule—

**RULE.—To find the sum of a number of improper fractions, express them first as whole or mixed numbers, and then add them together as such.**

N.B.—When the given improper fractions are not expressed in their lowest terms, they should first be reduced to their lowest terms, then to mixed numbers, and then be added together, thus—

**Ex.** The sum of $\frac{180}{27}$, $\frac{637}{28}$, and $\frac{150}{45} = \frac{180 \div 9}{27 \div 9} + \frac{637 \div 7}{28 \div 7} + \frac{150 \div 15}{45 \div 15}$

$= \frac{20}{3} + \frac{91}{4} + \frac{10}{3}$

$= 6\frac{2}{3} + 22\frac{3}{4} + 3\frac{1}{3}$

$= 32\frac{3}{4}$, *Answer.*

### (5.) WHEN ALL OR ANY OF THE QUANTITIES TO BE ADDED TOGETHER CONSIST OF COMPOUND OR COMPLEX FRACTIONS.

The reason of the following rule is self-evident—

**RULE.—To find the sum of a number of compound or complex fractions, first reduce them to simple fractions or mixed numbers, and then add them together as such, thus—**

**Ex. 5.** *The sum of* $\frac{2}{5}$ *of* $\frac{1}{8}$, $2\frac{1}{5}$ *of* 8, *and* $\frac{7}{8}$ *of* $2\frac{1}{2}$ *of* $1\frac{1}{7}$ =

$$\left(\tfrac{2}{5} \text{ of } \tfrac{1}{8}\right) + \left(2\tfrac{1}{5} \text{ of } 8\right) + \left(\tfrac{7}{8} \text{ of } 2\tfrac{1}{2} \text{ of } 1\tfrac{1}{7}\right) =$$

$$\left(\frac{\cancel{2}}{5} \text{ of } \frac{1}{\cancel{8}_{4}}\right) + \left(\frac{11}{5} \text{ of } \frac{8}{1}\right) + \left(\frac{\cancel{7}}{\cancel{8}} \text{ of } \frac{5}{2} \text{ of } \frac{\cancel{8}}{\cancel{7}}\right) =$$

$\frac{1}{20} + \frac{88}{5} + \frac{5}{2} =$

$\frac{1}{20} + 17\frac{3}{5} + 2\frac{1}{2} =$

$20\frac{3}{20}$, *Answer.*

**Ex. 6.** *The sum of* $\dfrac{4\frac{1}{9} \textit{ of } 2\frac{1}{7}}{2\frac{2}{3} + 5\frac{5}{9}}$, $\dfrac{\frac{1}{4} + (\frac{1}{8} \textit{ of } 1\frac{6}{7})}{2\frac{1}{2}}$, $\frac{41}{63}$, *and* $\frac{7}{18}$ *of* 9 =

$$\frac{\dfrac{37}{{}_{3-}\cancel{9}} \text{ of } \dfrac{\cancel{15}^{-5}}{7}}{(2 + 5) + (\frac{2}{3} + \frac{5}{9})} + \frac{\frac{1}{4} + (\frac{1}{8} \text{ of } \frac{13}{7})}{\frac{5}{2}} + \frac{41}{63} + \left(\frac{7}{\cancel{18}_{2}} \text{ of } \frac{\cancel{9}}{1}\right) =$$

$$\frac{\frac{185}{21}}{8\frac{2}{9}} + \frac{\frac{1}{4} + \frac{13}{56}}{\frac{5}{2}} + \frac{41}{63} + \frac{7}{2} =$$

$$\cfrac{\cfrac{\cancel{185}^{-5}}{\cancel{21}^{-7}}}{\cfrac{\cancel{74}^{-2}}{\cancel{9}^{-3}}} + \cfrac{\cfrac{27}{\cancel{56}^{-28}}}{\cfrac{5}{\cancel{2}^{-1}}} + \frac{41}{63} + \frac{7}{2} =$$

$\frac{15}{14} + \frac{27}{140} + \frac{41}{63} + 3\frac{1}{2} =$

$5\frac{523}{1260}$, *Answer.*

## Exercise 18.

### Addition of Fractions.

1. Add together $\frac{3}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$; $\frac{4}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, and $\frac{5}{9}$; $\frac{6}{13}$, $\frac{12}{13}$, $\frac{9}{13}$, $\frac{11}{13}$, and $\frac{10}{13}$; $\frac{121}{171}$, $\frac{15}{171}$, $\frac{98}{171}$, $\frac{117}{171}$, and $\frac{162}{171}$.

2. Find the sum of $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{16}{27}$; $\frac{5}{14}$, $\frac{13}{21}$, and $\frac{19}{30}$; $\frac{8}{35}$, $\frac{19}{28}$, $\frac{20}{21}$, and $\frac{33}{34}$; $\frac{1}{3}$, $\frac{6}{11}$, $\frac{2}{17}$, $\frac{14}{33}$, and $\frac{16}{51}$; $\frac{14}{99}$, $\frac{15}{28}$, $\frac{17}{77}$, $\frac{101}{126}$, $\frac{85}{234}$, and $\frac{1000}{1001}$; $\frac{5}{19}$, $\frac{18}{505}$, $\frac{271}{273}$, $\frac{1}{1008}$, $\frac{4132}{9275}$, $\frac{13}{209}$, and $\frac{1}{19152}$.

3. Find the value of $2\frac{1}{7} + 3\frac{1}{9}$; $6 + 2 + 4\frac{1}{3} + 3\frac{5}{8}$; $\frac{2}{3} + \frac{2}{3} + 1\frac{1}{3} + 4 + 6\frac{1}{5}$; $7\frac{1}{3} + 8 + 16\frac{5}{17} + \frac{16}{51}$; $21\frac{5}{13} + 8\frac{1}{7} + \frac{163}{910} + 8\frac{3}{11} + \frac{48}{383}$; $\frac{17}{27} + \frac{41}{81} + 6\frac{405}{486} + 710\frac{520}{729} + 2\frac{16}{135} + 102$.

4. Simplify the following expressions, $\frac{2}{4} + \frac{14}{3} + \frac{8}{3}$; $\frac{14}{11} + \frac{306}{121} + \frac{519}{77} + \frac{49}{7}$; $\frac{1801}{600} + \frac{1027}{108} + \frac{207}{54} + 3\frac{1}{2} + 19$; $\frac{28}{7} + \frac{36}{11} + 5\frac{1}{6} + 3 + \frac{191}{77}$; $\frac{301}{30} + \frac{919}{10} + \frac{71}{15} + \frac{86}{25} + \frac{101}{75} + \frac{460}{45} + \frac{1880}{10} + 17\frac{3}{4}$.

5. Simplify the following expressions, $\frac{3}{4} + 1\frac{3}{4} + \frac{9}{10} + \frac{13}{16}$; $3\frac{1}{3} + \frac{51}{101} + 904\frac{19}{27} + (2\frac{1}{2} \text{ of } 53)$; $4\frac{1}{7} + \frac{18}{35} + 3\frac{3}{4} \text{ of } 2\frac{9}{25}$; $3\frac{1}{4} \text{ of } \frac{5}{8} \text{ of } 2\frac{2}{143} + 17 + 16\frac{1}{4} \text{ of } 2\frac{3}{7}$ of $\frac{16}{35}$ of $\frac{119}{19}$ of 15; $\dfrac{1\frac{7}{9} \text{ of } 8\frac{2}{3}}{\frac{4}{27} \text{ of } 7\frac{2}{9}}$; $\dfrac{7\frac{1}{6} \text{ of } 33}{19\frac{1}{9} \text{ of } \frac{11}{42}}$; $\dfrac{\frac{15}{16} \text{ of } \frac{13}{25}}{\frac{17}{35} \text{ of } \frac{42}{55}}$; $\dfrac{7\frac{2}{7} \text{ of } \frac{9}{106}}{\frac{39}{49} \text{ of } \frac{7}{9}}$; $\dfrac{4\frac{2}{7}}{1\frac{6}{9}} + \dfrac{8\frac{3}{9}}{6\frac{9}{11}} + \dfrac{5}{9} + \dfrac{1\frac{1}{3}}{1\frac{9}{11}}$; $\dfrac{\frac{2}{3} + 16\frac{2}{3} \text{ of } 31\frac{1}{200} \text{ of } 20}{\frac{5}{9} \text{ of } 2\frac{5}{8} \text{ of } 1\frac{7}{25} \text{ of } 19 + 2\frac{1}{2} + \frac{5}{6} \text{ of } 16} \text{ of } \dfrac{3\frac{1}{2} + 4\frac{2}{3}}{6\frac{1}{2} + 8\frac{2}{3}}$; $\dfrac{3\frac{8}{9} + 4\frac{1}{5}}{6\frac{3}{4} + \frac{3}{7} \text{ of } \dfrac{4\frac{3}{8}}{15}} + \dfrac{16\frac{4}{5} + 5\frac{3}{10}}{17 + \frac{4}{15} \text{ of } \frac{1}{3} \text{ of } \dfrac{4 \times 36}{18 \div 5}}$; $\dfrac{3\frac{1}{4} + 2\frac{3}{8} + \frac{1}{8}}{3\frac{3}{8} \text{ of } 3\frac{1}{5}} \text{ of } \dfrac{\dfrac{2}{4\frac{1}{5}} + \dfrac{3}{2\frac{1}{3}} + \dfrac{2}{3\frac{1}{3}}}{\frac{1}{4} + \frac{1}{6} + \frac{1}{5}} + \dfrac{\frac{3}{8} \text{ of } 7\frac{1}{3}}{\frac{4}{9} \text{ of } 2\frac{1}{4} \text{ of } 3\frac{1}{7}}$.

6. Find the value of

$$\frac{6 \times 7 \div 9}{18 \div 5 \times 3} + \frac{2 + 4 + (19 \div 7) \times 3}{(17 \times 2 \div 11) + (61 \div 9) + (31 \times 17 \div 9 \text{ times } 3)}.$$

7. Simplify

$$\frac{\{(3\times4)\div(5\times2\times7)\} + \{(102\div19)\times14\times3\} + \{(18+101-19)\div7\times3\}}{(19\times6\times7) + \{(18+14-17)\div6\}}.$$

## II. Subtraction of Fractions.

**67. Subtraction of Fractions** is effected, in the following cases, by methods the reason of which may be generally found by referring to the corresponding case of "*Addition of Fractions.*"

(1.) WHEN THE GIVEN FRACTIONS HAVE THE SAME DENOMINATOR.

**Ex. 1.** *From* $\frac{14}{15}$ *take* $\frac{4}{15}$ *(i.e. Find the value of* $\frac{14}{15} - \frac{4}{15}$).

Since $14 - 4 = 10$,

14 fifteenths − 4 fifteenths = 10 fifteenths $= \frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$.

The reason of the following rule is self-evident—

**RULE.—To find the difference of two fractions having the same denominator, divide the difference of their numerators by their common denominator.**

(2.) WHEN THE GIVEN FRACTIONS HAVE NOT THE SAME DENOMINATOR.

**Ex. 2.** *Take* $\frac{2}{3}$ *from* $\frac{14}{19}$ *(i.e. Find the value of* $\frac{14}{19} - \frac{2}{3}$).

As in addition of fractions, we first reduce the given fractions to equivalent ones having one common denominator, and we thus find that

$\frac{14}{19} - \frac{2}{3} = \frac{42}{57} - \frac{38}{57} = \frac{4}{57}$, *Answer.*

Hence the following rule—

**RULE.—To find the difference of two fractions having different denominators, reduce them to equivalent fractions having a common denominator, and divide the difference of their new numerators by their common denominator.**

(3.) WHEN THE GIVEN FRACTIONS FORM PARTS OF MIXED NUMBERS.

The reason of the following rule is self-evident—

**RULE.—To find the difference between two mixed numbers, find the difference of the whole numbers separately, and also of the fractions separately, and add the two results together.**

**Ex. 3.** *From* $8\frac{6}{7}$ *take* $5\frac{1}{3}$ (*i.e. Find the value of* $8\frac{6}{7} - 5\frac{1}{3}$).

$$\left.\begin{array}{l} 8 - 5 = 3 \\ \frac{6}{7} - \frac{1}{3} = \frac{18-7}{21} = \frac{11}{21} \end{array}\right\} \therefore 8\tfrac{6}{7} - 5\tfrac{1}{3} = 3 + \tfrac{11}{21} = 3\tfrac{11}{21},\ \textit{Answer.}$$

**Ex. 4.** *From* $19\frac{1}{6}$ *take* 8 (*i.e. Find the value of* $19\frac{1}{6} - 8$).

$19 - 8 = 11,$

$\therefore 19\frac{1}{6} - 8 = 11 + \frac{1}{6} = 11\frac{1}{6}$, *Answer.*

**Ex. 5.** *From* $14\frac{1}{5}$ *take* $2\frac{5}{12}$ (*i.e. Find the value of* $14\frac{1}{5} - 2\frac{5}{12}$).

$14 - 2 = 12$

$\frac{1}{5} - \frac{5}{12} = \frac{12-25}{60}.$

Here we want 13 *sixtieths* more to enable us to subtract; we are, in fact, in the position of a debtor who owes 25 *sixtieths*, and has only 12 *sixtieths* with which to pay his debt; so that, after paying all he has—*i.e.* 12 *sixtieths*—he will still owe 13 *sixtieths*, which may be expressed by saying that he has a debt of 13 *sixtieths*, which is indicated thus—

$-\frac{13}{60}$ (*minus* $\frac{13}{60}$)

$\therefore 14\frac{1}{5} - 2\frac{5}{12} = 12 - \frac{13}{60} = 11 + 1 - \frac{13}{60} = 11 + \frac{60}{60} - \frac{13}{60} = 11 + \frac{47}{60} = 11\frac{47}{60}.$

In all cases of subtraction of fractions it will be well to remember the following rule, the reason of which is apparent from what we have already said—

**RULE.—When the fractions have been reduced to a common denominator, if it is found—**

(*a.*) *That the fraction in the subtrahend is* **less** *than that in the minuend*, take the difference of the new numerators as the numerator of a fraction which shall have the common denominator of the given fractions for its denominator; set the sign **plus** before this fraction, and, in *Subtraction of Mixed Numbers*, place it with its sign after the difference of the integers, and perform the operation indicated by this expression, thus—

*Ex.* 1. $7\frac{5}{8} - 4\frac{1}{3}.$

The difference of the integers $= 7 - 4 = 3.$

The difference of the fractions $= \frac{5}{8} - \frac{1}{3} = \frac{15-8}{24} = \frac{7}{24}.$

$\therefore$ The *Answer*, $= 3 + \frac{7}{24} = 3\frac{7}{24}.$

(*b*.) *That the fraction in the subtrahend is* **greater** *than that in the minuend*, take the difference of the new numerators as the numerator of a fraction which shall have the common denominator of the given fractions for its denominator; set the sign **minus** before this fraction, and, in *Subtraction of Mixed Numbers*, place it with its sign after the difference of the integers, and perform the operation indicated by this expression, thus :—

*Ex.* 2. $7\frac{1}{3} - 4\frac{5}{8}$.

The difference of the integers $= 7 - 4 = 3$.

The difference of the fractions $= \frac{1}{3} - \frac{5}{8} = \frac{8-15}{24} = -\frac{15-8}{24} = -\frac{7}{24}$.

$\therefore$ The *Answer*, $= 3 - \frac{7}{24} = 2 + 1 - \frac{7}{24} = 2 + \frac{24}{24} - \frac{7}{24} = 2 + \frac{17}{24} = 2\frac{17}{24}$.

## (4.) WHEN EITHER THE SUBTRAHEND OR THE MINUEND, OR BOTH, CONSISTS OF AN IMPROPER FRACTION—

**RULE.—To subtract improper fractions, reduce them to mixed numbers, and then proceed as in** ***Subtraction of Mixed Numbers.***

**Ex.** 6. $2\frac{17}{19} - \frac{16}{15} = 2\frac{17}{19} - 1\frac{1}{15} = 1\frac{236}{285}$.

**Ex.** 7. $\frac{24}{19} - 1\frac{1}{8} = 1\frac{5}{19} - 1\frac{1}{8} = \frac{21}{152}$.

**Ex.** 8. $\frac{65}{7} - \frac{39}{14} = 9\frac{2}{7} - 2\frac{11}{14} = 6\frac{1}{2}$.

## (5.) WHEN EITHER THE SUBTRAHEND OR THE MINUEND, OR BOTH, CONSISTS OF A COMPOUND OR COMPLEX FRACTION—

**RULE.—To subtract compound or complex fractions, reduce them first to their simplest forms, and then proceed as in** ***Subtraction of Simple Fractions or of Mixed Numbers,*** **as the case may be.**

**Ex.** 9. $4\frac{1}{7} - (\frac{3}{8} \text{ of } 5\frac{1}{3}) = 4\frac{1}{7} - \left(\frac{\cancel{3}}{\cancel{8}} \text{ of } \frac{\overset{2}{\cancel{16}}}{\cancel{3}}\right) = 4\frac{1}{7} - 2 = 2\frac{1}{7}$.

**Ex.** 10. $2\frac{14}{15} - \frac{\frac{2}{3} \text{ of } 16\frac{1}{2}}{6\frac{3}{7}} = 2\frac{14}{15} - \frac{\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\overset{11}{\cancel{33}}}{\cancel{2}}}{\frac{45}{7}} = 2\frac{14}{15} - \frac{77}{45} = 2\frac{14}{15} - 1\frac{32}{45} = 1\frac{10}{45} = 1\frac{2}{9}$.

**Ex.** 11. $(1\frac{2}{9} \text{ of } 10\frac{2}{7}) - 2\frac{1}{2} = \left(\frac{11}{\cancel{9}} \text{ of } \frac{\overset{8}{\cancel{72}}}{7}\right) - 2\frac{1}{2} = \frac{88}{7} - 2\frac{1}{2} = 12\frac{4}{7} - 2\frac{1}{2} = 10\frac{1}{14}$.

**Ex. 12.** $\frac{3\frac{1}{3}-\frac{5}{16}}{4\frac{3}{8}}-\frac{4}{7}=\frac{3\frac{1}{48}}{4\frac{3}{8}}-\frac{4}{7}=\frac{\frac{\cancel{145}^{29}}{\cancel{48}_{6}}}{\frac{\cancel{35}^{7}}{\cancel{8}_{1}}}-\frac{4}{7}=\frac{29}{42}-\frac{4}{7}=\frac{5}{42}.$

**Ex. 13.** $3\frac{3}{8}$ of $7\frac{1}{2}-\frac{7}{8}$ of $16\frac{1}{3}$ of $8=\frac{27}{8}$ of $\frac{15}{2}-\left(\frac{7}{\cancel{8}}\text{ of }\frac{49}{3}\text{ of }\frac{\cancel{8}}{1}\right)$
$=\frac{405}{16}-\frac{343}{3}=25\frac{5}{16}-114\frac{1}{3}=\ -(114\frac{1}{3}-25\frac{5}{16})=\ -89\frac{1}{48}.$

**Ex. 14.** $\frac{2\frac{1}{2}-1\frac{1}{3}}{4\frac{1}{7}}-\frac{\frac{4}{5}\text{ of }7}{\frac{1}{2}+4\frac{2}{3}}=\frac{1\frac{1}{6}}{4\frac{1}{7}}-\frac{\frac{28}{5}}{4\frac{5}{6}}=\frac{\frac{7}{6}}{\frac{29}{7}}-\frac{\frac{28}{5}}{\frac{29}{6}}=\frac{49}{174}-\frac{168}{145}=$
$\frac{245-1008}{870}=\ -\frac{763}{870}.$

## Exercise 19.

### Subtraction of Fractions.

1. *Find the value of—*

(*a.*) $\frac{8}{9}-\frac{4}{9}$; $\frac{3}{23}-\frac{1}{23}$; $\frac{17}{101}-\frac{13}{101}$; $\frac{19}{22}-\frac{8}{22}$; $\frac{101}{220}-\frac{79}{220}$; $\frac{413}{1000}-\frac{163}{1000}$.

(*b.*) $\frac{7}{8}-\frac{3}{4}$; $\frac{6}{11}-\frac{9}{22}$; $\frac{19}{20}-\frac{4}{5}$; $\frac{5}{6}-\frac{23}{42}$; $\frac{101}{200}-\frac{49}{50}$; $\frac{21}{29}-\frac{31}{290}$; $\frac{6}{11}-\frac{10}{121}$; $\frac{101}{108}-\frac{14}{27}$; $\frac{250}{919}-\frac{1603}{15628}$; $\frac{2018}{4025}-\frac{26}{805}$; $(\frac{14}{19}+\frac{2}{19}+\frac{3}{38})-\frac{17}{19}$.

(*c.*) $\frac{5}{9}-\frac{4}{11}$; $\frac{18}{103}-\frac{1}{6}$; $\frac{17}{21}-\frac{5}{8}$; $\frac{41}{51}-\frac{2}{31}$; $\frac{16}{19}-\frac{7}{9}$; $\frac{1}{9}-\frac{1}{107}$; $\frac{38}{63}-\frac{7}{22}$; $\frac{6}{29}-\frac{13}{801}$; $\frac{511}{1534}-\frac{171}{1198}$; $\frac{1031}{1084}-\frac{20203}{109008}$; $\frac{63417}{80000}-\frac{63417}{190000}$; $\frac{43}{93}-\frac{715}{7966}$.

(*d.*) $\frac{3}{8}-\frac{5}{22}$; $\frac{7}{9}-\frac{23}{42}$; $\frac{8}{15}-\frac{6}{85}$; $\frac{41}{42}-\frac{15}{133}$; $\frac{20}{33}-\frac{61}{770}$; $\frac{82}{95}-\frac{161}{380}$; $\frac{712}{885}-\frac{783}{5900}$; $\frac{28}{133}-\frac{16}{95}$; $\frac{805}{6657}-\frac{717}{6974}$; $\frac{25816}{302993}-\frac{8496}{87493}$.

(*e.*) $\frac{4}{7}\sim\frac{5}{8}$; $\frac{19}{20}\sim\frac{3}{7}$; $\frac{16}{21}\sim\frac{11}{12}$; $\frac{201}{302}\sim\frac{48}{103}$; $\frac{17}{35}\sim\frac{16}{21}$; $\frac{181}{205}\sim\frac{36}{73}$; $\frac{27}{28}\sim\frac{289}{290}$; $\frac{1}{4}\sim\frac{7}{9}$; $\frac{3}{19}\sim\frac{3}{20}$; $\frac{5}{17}\sim\frac{16}{51}$; $\frac{18}{190}\sim\frac{180}{191}$; $\frac{5132}{6003}\sim\frac{7001}{8002}$.

(*f.*) $\frac{5}{7}-\frac{2}{3}$; $\frac{4}{9}-\frac{8}{13}$; $\frac{7}{12}-\frac{14}{25}$; $\frac{161}{280}-\frac{203}{210}$; $\frac{603}{604}-\frac{222}{244}$; $\frac{7131}{7231}-\frac{3}{4}$; $\frac{321}{454}-\frac{12}{13}$; $\frac{202}{203}-\frac{405}{406}$; $\frac{17}{19}-\frac{171}{191}$; $\frac{305}{307}-\frac{61}{62}$; $\frac{103}{119}-\frac{86}{87}$; $\frac{4032}{5003}-\frac{21}{31}$.

(*g.*) $4\frac{4}{9}-2\frac{1}{3}$; $16\frac{3}{7}-\frac{5}{19}$; $2\frac{1}{4}-1\frac{1}{5}$; $17\frac{14}{19}-3\frac{6}{7}$; $20-4\frac{1}{3}$; $10-\frac{1}{17}$; $16\frac{1}{5}-3$; $8\frac{1}{2}-2\frac{3}{4}$; $7-1\frac{1}{5}$; $2\frac{1}{2}-1\frac{1}{4}$; $3\frac{1}{3}-3\frac{1}{7}$; $2\frac{2}{9}-1\frac{5}{6}$; $2-1\frac{16}{25}$; $20\frac{21}{22}-9\frac{3}{4}$; $1\frac{4}{5}-\frac{3}{17}$; $3\frac{1}{3}-1\frac{21}{10040}$; $16-14\frac{3134}{7001}$; $18\frac{1}{3}-5\frac{5}{10313}$.

(*h.*) $6-6\frac{1}{2}$; $2\frac{2}{3}-2\frac{2}{5}$; $1-1\frac{1}{5}$; $2-1\frac{3}{4}$; $18-6\frac{5}{7}$; $\frac{4}{9}-\frac{3}{5}$; $\frac{7}{9}-\frac{12}{13}$; $82\frac{3}{4}-\frac{101}{108}$; $46\frac{5}{21}-17\frac{13}{15}$; $2-8\frac{1}{2}$; $16-2\frac{3}{17}$; $20-40\frac{4}{19}$; $18-3\frac{21}{1060}$; $14\frac{31}{41}-200\frac{13}{19}$; $2\frac{181}{192}-16\frac{31}{410}$; $8\frac{1}{5}-30\frac{3}{8}$; $10-171\frac{1}{171}$; $18\frac{1}{18}-19\frac{1}{19}$.

2. *Simplify the following expressions—*

(*a.*) $\frac{3}{4}+\frac{5}{6}+\frac{6}{7}-\frac{5}{21}$; $\frac{8}{9}-\frac{3}{5}+\frac{2}{3}-\frac{1}{2}$; $\frac{16}{21}+\frac{3}{4}+\frac{2}{3}-\frac{7}{16}-\frac{5}{9}$; $\frac{17}{180}$ of $1\frac{7}{11}+\frac{23}{55}-\frac{7}{16}-\frac{3}{4}$; $\frac{14}{15}-\frac{30}{261}+\frac{4}{25}+\frac{17}{87}-\frac{1}{9}+\frac{3}{8}$ of $\frac{4}{5}$ of 7.

(*b.*) $\dfrac{18\div7}{16-1\frac{1}{4}}+3\frac{8}{19}-\dfrac{115\div19}{2\frac{2}{3}+2\frac{2}{3}+4\frac{2}{3}}$; $\dfrac{16\times29\div7}{5\times17\div3}$ of $\dfrac{10\times51\div145}{64\div13\times48}+$

$$\frac{8\div7\times101}{13\times16\div5}-\frac{\dfrac{8\times7\times9\div14}{13\,(14-17)}}{\dfrac{35}{4\{\,7\,(6+3)-(8+1)\,\}}};\quad \frac{\dfrac{1}{4\frac{16}{63}}\text{ of }19\frac{1}{7}-\frac{7}{10}}{20\frac{1}{3}-\dfrac{1}{2\frac{10}{23}}\text{ of }5\frac{1}{3}};$$

$$\frac{\frac{7}{8}\text{ of }\frac{6}{7}+\frac{7}{13}\text{ of }3\frac{2}{21}-\{\,(\frac{5}{9}\text{ of }4\frac{1}{2}\text{ of }7+\frac{5}{8}\text{ of }1\frac{1}{7}\text{ of }3\frac{3}{5}\text{ of }16)-55\frac{10}{21}\,\}}{\dfrac{1\frac{3}{5}}{7\frac{1}{35}-4\frac{3}{7}}-\dfrac{16\frac{1}{3}+2\frac{1}{2}-17\frac{7}{12}}{4\frac{3}{4}}+\dfrac{\frac{4}{5}\text{ of }1\frac{1}{12}}{1\frac{3}{10}\text{ of }\frac{3}{4}}}$$

3. *What must be added to* $\left(\dfrac{1\frac{1}{2}\text{ of }3\frac{1}{3}}{3\frac{1}{2}\text{ of }2\frac{2}{3}}\text{ of }\dfrac{1\frac{3}{7}\text{ of }1\frac{1}{6}}{1\frac{3}{7}\text{ of }\dfrac{32\frac{2}{3}}{3\frac{1}{2}}}+\dfrac{2\frac{1}{8}\text{ of }6\frac{2}{3}}{3\frac{1}{3}\text{ of }4\frac{1}{2}}\right)$ *to make it equal to* $\left(\dfrac{1}{28\frac{2}{7}}\text{ of }3\frac{3}{4}\text{ of }3\frac{1}{7}\text{ of }1\frac{5}{7}+\frac{3}{8}\right)$?

## III. Multiplication of Fractions.

**68.** In **Multiplication of Fractions** the same rule is followed as in reducing a compound fraction to a simple fraction. The reason of this rule will be evident from an example—

**Ex. 1.** *Multiply* $\frac{4}{19}$ *by* $\frac{3}{7}$ *(i.e. Find the value of* $\frac{4}{19}\times\frac{3}{7}$.*)*

Here, $\frac{4}{19}\times3=\frac{4\times3}{19}$ (§ 57) $=\frac{12}{19}$.

This result is evidently 7 times too great, because $\frac{4}{19}$ is not to be multiplied by 3, but by $\frac{1}{7}$ of 3;

$\therefore\ \frac{4}{19}\times\frac{3}{7}=(\frac{4}{19}\times3)\div7=\frac{4\times3}{19}\div7=\frac{4\times3}{19\times7}$ (§ 58) $=\frac{12}{133}$, *Ans.*

Hence the following rule—

**RULE.—To find the product of two or more fractions, multiply all their numerators for a new numerator, and all their denominators for a new denominator.**

N.B. 1.—Whenever the quantities whose product is to be found do not all consist of simple fractions, they must first be reduced to simple fractions, and then multiplied together by the rule given above, thus—

**Ex. 2.** *Find the value of* $3\frac{1}{8}\times5\frac{1}{6}$.

$$\left.\begin{aligned}3\tfrac{1}{8}&=\tfrac{25}{8}\\5\tfrac{1}{6}&=\tfrac{31}{6}\end{aligned}\right\}\therefore\ 3\tfrac{1}{8}\times5\tfrac{1}{6}=\tfrac{25}{8}\times\tfrac{31}{6}=\tfrac{25\times31}{8\times6}=\tfrac{775}{48}=16\tfrac{7}{48}.$$

**Ex. 3.** *Find the value of* $\frac{4}{7}$ of $5\frac{3}{5} \times 3\frac{1}{2}$ of $9 \times 1\frac{8}{9}$.

$$\frac{4}{7} \text{ of } 5\frac{3}{5} \times 3\frac{1}{2} \text{ of } 9 \times 1\frac{8}{9} = \frac{4}{\cancel{7}} \times \frac{\overset{14}{\cancel{28}}}{5} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{\cancel{9}}{1} \times \frac{17}{\cancel{9}} = \frac{952}{5} = 190\frac{2}{5}.$$

**N.B. 2.—In finding the value of an involved fraction, it should be remembered that the signs ×, ÷, and "*of*" connect the terms between which they are found, into one quantity, while the signs +, −, and $\sim$, separate the terms between which they occur into distinct quantities, thus—**

**Ex. 4.**
$$\begin{aligned} &2 \times 8 + 14 \div 7 - 3\tfrac{1}{2} \text{ of } 5 + 21 \div 4 - 3 \times 19 = \\ &(2 \times 8) + (14 \div 7) - (3\tfrac{1}{2} \text{ of } 5) + (21 \div 4) - (3 \times 19) = \\ &16 + 2 - 17\tfrac{1}{2} + 5\tfrac{1}{4} - 57 = \\ &23\tfrac{1}{4} - 74\tfrac{1}{2} = \\ &- 51\tfrac{1}{4}, \textit{ Answer.} \end{aligned}$$

**Ex. 5.**
$$\begin{aligned} &\tfrac{2}{3} \text{ of } \tfrac{4}{5} + \tfrac{3}{4} - \tfrac{1}{6} + \tfrac{2}{5} \times \tfrac{1}{9} = \\ &(\tfrac{2}{3} \text{ of } \tfrac{4}{5}) + \tfrac{3}{4} - \tfrac{1}{6} + (\tfrac{2}{5} \times \tfrac{1}{9}) = \\ &\tfrac{8}{15} + \tfrac{3}{4} - \tfrac{1}{6} + \tfrac{2}{45} = \\ &1\tfrac{29}{180}, \textit{ Answer.} \end{aligned}$$

## Exercise 20.

### Multiplication of Fractions.

1. *Find the value of—*

(*a.*) $\frac{3}{8} \times \frac{5}{7}$; $\frac{2}{9} \times \frac{7}{5}$; $\frac{5}{6} \times \frac{7}{9}$; $\frac{3}{8} \times \frac{4}{7}$; $\frac{2}{5} \times \frac{3}{4}$; $\frac{3}{5} \times \frac{4}{7} \times \frac{5}{11}$; $\frac{6}{13} \times \frac{39}{40} \times \frac{4}{15}$.

(*b.*) $\frac{18}{73} \times \frac{3}{14} \times \frac{21}{61} \times \frac{126}{198} \times \frac{73}{654} \times 20$; $\frac{18}{25} \times \frac{4}{5}$ of $9 \times \frac{105}{261}$ of $\frac{7}{18}$ of $2\frac{1}{3}$ of $\frac{18}{49}$ of 30.

(*c.*) $1\frac{1}{7} \times \frac{3}{5}$; $\frac{4}{9} \times 3\frac{1}{3}$; $\frac{2}{5} \times \frac{1}{3} \times 1\frac{8}{9}$; $\frac{16}{21}$ of $1\frac{8}{25} \times 1\frac{10}{11}$ of 75; $\frac{12}{35} \times 2\frac{1}{12} \times 1\frac{7}{19}$ of 101.

2. *Simplify the following expressions—*

$2\frac{1}{5} + (4\frac{1}{3}$ of $2\frac{1}{13}) - \frac{1}{5}$; $7\frac{1}{6} + (3\frac{1}{5}$ of $1\frac{9}{16}$ of $4) - \frac{1}{3}$; $3\frac{3}{8}$ of $\frac{4}{81} \times 4\frac{4}{5} + 21\frac{1}{8} - 3\frac{3}{4}$ of $\frac{7}{15}$.

3. *Find the continued product of* $24\frac{2}{3}$, $\frac{1}{4}$, $11\frac{3}{5}$, $\frac{3}{29}$, *and* $\frac{5}{37}$; *and of* $15\frac{1}{2}$, $\frac{1}{3}$, $16\frac{2}{5}$, $\frac{3}{31}$, *and* $\frac{5}{41}$.

4. *Multiply* $\frac{5}{32}$ *of* $40\frac{1}{3}$ *by* $\frac{6}{55}$ *of* $7\frac{3}{11}$; *and* $[\frac{2}{9}-\frac{2}{7}+\frac{1}{8}-\frac{1}{6}+\frac{1}{5}]$ *by* $\dfrac{1}{\frac{2}{3}-\frac{1}{4}+\frac{3}{5}-\frac{1}{7}}$.

5. *Reduce to simplest terms the expressions—*

$$\frac{\frac{4}{11}+\frac{5}{9}}{\frac{7}{8}-\frac{2}{13}} \times \tfrac{1}{4} \text{ of } \tfrac{3}{8} \text{ of } 5\tfrac{1}{2}\ ;\ [409\tfrac{3}{8}+2\tfrac{2}{9}] \times 15\tfrac{6}{7}\ ;$$

$$\frac{\frac{3}{2}-\frac{2}{7}}{\frac{5}{6}-\frac{3}{5}} \times \frac{\frac{1}{3}-\frac{1}{11}}{1\frac{5}{7}-\frac{13}{49}} + \frac{2\frac{1}{4}}{\frac{3}{8}\times\frac{12}{35}}.$$

6. *Find the value of* $\frac{3}{22} \times \frac{11}{91} \times 2\frac{1}{6}$ of $3\frac{7}{15} \times \{4\frac{1}{8} + 2\,(\frac{3}{7} + 4\frac{6}{7}) + 1\frac{163}{168}\} \times \frac{63}{65}$.

## IV. Division of Fractions.

**69.** The Rule for **Division of Fractions** will be seen from an examination of the following example—

**Ex. 1.** *Divide* $\frac{7}{19}$ *by* $\frac{13}{15}$ (*i.e. Find the value of* $\frac{7}{19} \div \frac{13}{15}$).

Here, $\frac{7}{19} \div 13 = \frac{7}{19\times13}$ (§ 58).

This result is evidently 15 times too small, because $\frac{7}{19}$ is not to be divided by 13, but by $\frac{1}{15}$ of 13;

$$\frac{7}{19} \div \frac{13}{15} = \frac{7}{19\times13} \times 15 = \frac{7\times15}{19\times13}\ (\S\ 57) = \frac{105}{247},\ \textit{Answer.}$$

Hence the following rule—

**RULE.—To divide one fraction by another, invert the divisor and multiply the dividend by the fraction thus inverted.**

**Ex. 2.** $4\frac{1}{6} \div 7\frac{1}{4} = \frac{25}{6} \div \frac{29}{4} = \frac{25}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{29} = \frac{50}{87}$, *Answer.*

**Ex. 3.** $(\frac{7}{16}$ of $\frac{8}{9}$ of $3\frac{1}{2}) \div (\frac{14}{15}$ of $2\frac{7}{8}$ of $1\frac{4}{7}) =$

$$\left(\frac{7}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{9} \times \frac{7}{2}\right) \div \left(\frac{\overset{\cancel{2}}{\cancel{14}}}{15} \times \frac{23}{\underset{4}{\cancel{8}}} \times \frac{11}{\cancel{7}}\right) =$$

$$\frac{7\times7}{2\times9\times2} \div \frac{23\times11}{15\times4} = \frac{7\times7}{\cancel{2}\times\cancel{9}\times\cancel{2}} \times \frac{\overset{5}{\cancel{15}}\times\cancel{4}}{23\times11} = \frac{7\times7\times5}{3\times23\times11} = \frac{245}{759},\ \textit{Ans.}$$

**70.** It may be here explained that *all examples in Division of Fractions may be worked out by the rule given for reducing Complex Fractions to Simple Fractions,* thus—

**Ex. 1.** $\frac{2}{3} \div \frac{5}{7} = \frac{\frac{2}{3}}{\frac{5}{7}} = \frac{14}{15}$, *Answer.*

**Ex. 2.** $4\frac{1}{6} \div 7\frac{1}{4} = \frac{\frac{25}{6} - 3}{\frac{29}{4} - 2} = \frac{25 \times 2}{29 \times 3} = \frac{50}{87}$, *Answer.*

## Exercise 21.

### Division of Fractions.

1. *Find the value of—*

$\frac{2}{3} \div \frac{7}{11}$; $\frac{5}{6} \div \frac{4}{9}$; $\frac{7}{11} \div \frac{5}{8}$; $1\frac{1}{3} \div \frac{3}{5}$; $2\frac{1}{7} \div \frac{3}{4}$; $\frac{8}{9} \div \frac{4}{3}$; $6\frac{1}{4} \div 3\frac{11}{13}$; $\frac{17}{21} \div 4\frac{37}{63}$; $4\frac{11}{13} \div 1\frac{32}{39}$; $(17\frac{1}{9} \div 3\frac{2}{3}) \times 21$; $11\frac{2}{9} \div 1\frac{91}{111} \div 2\frac{135}{136}$.

2. *Simplify the following expressions—*

$$\{(\tfrac{4}{5} \text{ of } 1\tfrac{6}{7} \times \tfrac{4}{17} \text{ of } 2\tfrac{1}{17} \text{ of } 4\tfrac{1}{4} \text{ of } 4\tfrac{1}{4}) + 1\tfrac{1}{8}\} \div 1\tfrac{49}{64};\ \frac{\frac{4}{5} \times \frac{5}{6}}{\frac{7}{8} \text{ of } \frac{9}{10}} \div \frac{\frac{16}{17} \times \frac{11}{14}}{\frac{55}{68} \times \frac{23}{36}};$$

$$\frac{25\frac{1}{3} \div 21}{4\frac{5}{13} \times \frac{1}{63}} + \frac{18 \times 23\frac{5}{9}}{13\frac{1}{7} \div \frac{24}{25}};\quad \frac{\frac{8}{9} - \frac{1}{3}}{\frac{7}{10} + \frac{8}{15}} \times \frac{\frac{11}{12} + \frac{3}{4}}{\frac{2}{3} - \frac{2}{9}} \times \frac{6\frac{8}{11}}{56\frac{9}{11}} \div \frac{\frac{3}{5}}{2};$$

$$\frac{1\frac{7}{9} + 8\frac{1}{4}}{7\frac{5}{6} - 4\frac{2}{9}} \div \frac{2\frac{1}{19} + \frac{7}{38}}{\frac{7}{38} - \frac{7}{76}} \times \frac{45\frac{1}{8}}{21\frac{1}{4}};\ 3\tfrac{1}{125} \text{ of } 3\tfrac{4}{7} \div \tfrac{47}{345} \text{ of } 9;$$

$$\frac{1 + \frac{1}{5}}{1 - \frac{1}{5}} \div \frac{1 + \frac{5}{13}}{1 - \frac{5}{13}};\quad \frac{15\frac{1}{3} - 7\frac{1}{6}}{14\frac{1}{5} + 3\frac{1}{4}} \div \frac{16\frac{1}{3} + 2\frac{5}{12}}{9\frac{1}{4} - 3\frac{1}{3}} + [9\tfrac{12}{49} - 7\tfrac{1}{7}];$$

$$\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} \div \frac{1 + \frac{3}{5}}{1 - \frac{3}{5}};\ \frac{1 - \frac{18}{25}}{3\frac{1}{3} + \frac{2\frac{3}{4}}{\frac{3}{4}}} \times \left(\frac{1 + \frac{2}{3}}{1 - \frac{2}{3}}\right)^{2*};\ \frac{1 - \frac{4}{9}}{3\frac{1}{4} + 1\frac{1}{2} + \frac{1\frac{2}{3}}{6\frac{2}{3}}} \times \left(\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right)^{2};$$

$$\left(\frac{\frac{3}{4} - \frac{2}{5}}{\frac{2}{3} - \frac{1}{12}} \div \frac{\frac{3}{7} - \frac{1}{3}}{\frac{11}{21} - \frac{1}{6}}\right)^{2};\quad \left(\frac{\frac{3}{5} - \frac{2}{7}}{\frac{5}{6} - \frac{3}{5}} \div \frac{\frac{5}{7} - \frac{13}{49}}{\frac{1}{3} - \frac{1}{11}}\right)^{2};$$

$$\frac{(\frac{1}{2} + \frac{1}{5}) \times (\frac{1}{3} + \frac{1}{5}) \times (\frac{1}{4} + \frac{1}{5})}{\{(\frac{1}{2} \times \frac{1}{3}) + (\frac{1}{2} \times \frac{1}{4}) + (\frac{1}{3} \times \frac{1}{4})\} \times \frac{1}{5}};$$

$$\frac{(\frac{3}{5} \times \frac{3}{10} \times \frac{3}{20}) + \{(\frac{3}{5} \times \frac{3}{10}) + (\frac{3}{5} \times \frac{3}{20}) + (\frac{3}{10} \times \frac{3}{20})\}\frac{3}{8} + (\frac{3}{5} + \frac{3}{10} + \frac{3}{20})\frac{9}{64}}{(\frac{3}{5} + \frac{3}{8}) \times (\frac{3}{10} + \frac{3}{8}) \times (\frac{3}{20} + \frac{3}{8})}.$$

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* This figure, 2, indicates that the quantity included within the bracket after which it stands, is to be multiplied by itself once. (See § 126.)

## 71. Miscellaneous Examples in Vulgar Fractions.

1. *Add together* 2*s.* 6$\frac{3}{5}$*d.*, £1, 8*s.* 3$\frac{3}{4}$*d.*, £20, 19*s.* 7$\frac{16}{27}$*d.*, 18*s.* 3$\frac{4}{9}$*d.*

$$\begin{array}{lcllrrl}
\text{The sum of the } \textit{pence} & = & (6\frac{3}{5}+3\frac{3}{4}+7\frac{16}{27}+3\frac{4}{9}) \text{ pence} = 21\frac{209}{540}\text{d.} & & & & \\
 & & & = & \pounds 0 & 1 & 9\frac{209}{540} \\
\text{The sum of the } \textit{shillings} & = & 47\text{s.} & = & 2 & 7 & 0 \\
\text{The sum of the } \textit{pounds} & = & & & 21 & 0 & 0 \\
\therefore \text{ The Answer} & = & & & \pounds 23 & 8 & 9\frac{209}{540}
\end{array}$$

2. *Find the value of* $\frac{3}{5}$ *of* £1.

$$\tfrac{3}{5} \text{ of } \pounds 1 = \tfrac{3}{5} \text{ of } 20\text{s.} = \frac{3}{\cancel{5}} \times \frac{\overset{4}{\cancel{20}}}{1}\text{s.} = 12\text{s., } \textit{Answer.}$$

3. *Multiply* 17 *cwt.* 3 *qrs.* 12 *lb. by* 5$\frac{3}{5}$.

$$17 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lb.} \times 5\tfrac{3}{5} = 17 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lb.} \times \tfrac{28}{5} =$$

$$\frac{17 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lb} \times 28}{5} = \frac{500 \text{ cwt.}}{5} = 100 \text{ cwt.} = 5 \text{ tons, } \textit{Answer.}$$

4. *Subtract four-nineteenths of* £78, 11*s.* 10$\frac{1}{4}$*d.*, *from three-sevenths of* £51, 14*s.* 3*d.*

$$\begin{array}{lclrrr}
\frac{3}{7} \text{ of } \pounds 51, 14\text{s. } 3\text{d,} = \frac{\pounds 51, 14\text{s. } 3\text{d.} \times 3}{7} = \frac{\pounds 155, 2\text{s. } 9\text{d.}}{7} & = & \pounds 22 & 3 & 3 \\
\frac{4}{19} \text{ of } \pounds 78, 11\text{s. } 10\frac{1}{4}\text{d.} = \frac{\pounds 78, 11\text{s. } 10\frac{1}{4}\text{d.} \times 4}{19} = \frac{\pounds 314, 7\text{s. } 5\text{d.}}{19} & = & 16 & 10 & 11 \\
\therefore \text{ The Answer,} & = & \pounds 5 & 12 & 4
\end{array}$$

5. *Divide* 20 *hours* 33 *minutes* 20 *seconds by* 4$\frac{1}{9}$.

$$20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \div 4\tfrac{1}{9} = 20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \div \tfrac{37}{9} =$$

$$20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec.} \times \tfrac{9}{37} = \frac{20 \text{ hrs. } 33 \text{ min. } 20 \text{ sec. } \times 9}{37} =$$

$$\frac{185 \text{ hrs.}}{37} = 5 \textit{ hrs., Answer.}$$

6. *Cut a piece of wood, equal to a cubic foot in bulk, from a plank* 10 *inches wide and* 4$\frac{1}{2}$ *inches thick.*

The surface of one end of the plank $=4\frac{1}{2}$ in. × 10 in. $=45$ sq. inches.

$$\therefore 1 \text{ cubic foot} \div 45 \text{ square inches} = \frac{1728}{45} \text{ inches} = \frac{1728 \div 9}{45 \div 9} \text{ inches}$$

$$= \frac{192}{5} \text{ inches} = 38\tfrac{2}{5} \text{ inches, } \textit{Answer.}$$

7. *Reduce 6s. 7d. to the fraction of £1 (i.e. Find what fraction of £1 is worth 6s. 7d.)*

Since 6s. 7d. = 79 pence, and £1 = 240, it follows that 6s. 7d. is £$\frac{79}{240}$, *Answer.*

The rule for reducing one quantity to the fraction of another (see § 104) may be thus stated—

**RULE.—To reduce a given quantity to the fraction of another given quantity, reduce them both to one denomination, and take the result of the first for the numerator, and that of the second for the denominator, of the required fraction, thus—**

**Ex. 1.** *Reduce 4s. 3d. to the fraction of 6s. 1d.*

$$\left.\begin{array}{l}4\text{s. }3\text{d.} = 51\text{d.}\\ \hline 6\text{s. }1\text{d.} = 73\text{d.}\end{array}\right. \quad \therefore 4\text{s. }3\text{d.} = \frac{51}{73} \text{ of } 6\text{s. }1\text{d.}: \quad \textit{Answer, } \frac{51}{73}$$

**Ex. 2.** *Reduce $\frac{4}{7}$s. to the fraction of £1.*

$$\begin{array}{l}\frac{4}{7}\text{s.} = \\ £1 = \end{array} \frac{\frac{4}{7}\text{s.}}{20\text{s.}} = \frac{\frac{\cancel{4}}{7}}{\frac{\cancel{20}\ 5}{1}} = \frac{1}{35}, \ \textit{Answer.}$$

8. *Which is the greatest, and which is the least, of the fractions* $\frac{7}{15}, \frac{9}{17}, \frac{13}{25}, \frac{11}{21}$?

Reducing these fractions to equivalent ones having a common denominator, thus—

$$\frac{4165,\ 4725,\ 4641,\ 4675}{8925},$$

we see that $\frac{9}{17}$ (*i.e.*, $\frac{4725}{8925}$) is the greatest, and
$\frac{7}{15}$ (*i.e.*, $\frac{4165}{8925}$) is the least, of the given fractions.

9. *Find the value of* $6\frac{5}{9}$ *of* $\dfrac{\dfrac{7}{2+\frac{6}{7}}}{3+4\frac{1}{3}}$.

Since $3 + 4\frac{1}{2} = 7\frac{1}{2} = \frac{15}{2}$, $\therefore \frac{7}{3+4\frac{1}{2}} = \frac{7}{7\frac{1}{2}} = \frac{\frac{7}{1}}{\frac{15}{2}} = \frac{14}{15}$

$$\text{Therefore } \frac{\frac{6}{7}}{\frac{7}{3+4\frac{1}{2}}} = \frac{\frac{6}{1}-3}{\frac{14-7}{15}} = \frac{3\times15}{7} = \frac{45}{7} = 6\frac{3}{7}$$

$$\text{Therefore } \frac{7}{2+\frac{\frac{6}{7}}{\frac{7}{3+4\frac{1}{2}}}} = \frac{7}{2+6\frac{3}{7}} = \frac{7}{8\frac{3}{7}} = \frac{\frac{7}{1}}{\frac{59}{7}} = \frac{7\times7}{59} = \frac{49}{59}$$

$$\text{Therefore } 6\tfrac{3}{7} \text{ of } \frac{49}{59} = \frac{\cancel{59}}{9} \times \frac{49}{\cancel{59}} = \frac{49}{9} = 5\tfrac{4}{9}, \textit{ Answer.}$$

## Exercise 22.

### Miscellaneous Exercises in Vulgar Fractions.

1. How many times does $\frac{1}{3} + \frac{1}{6} + \frac{1}{9}$ contain $\frac{1}{12} + \frac{1}{4} + \frac{1}{8}$?

2. Find the difference between $\frac{2}{3} + \frac{4}{9}$ and $\frac{7}{12} \times \frac{8}{11}$.

3. What fraction divided by $3\frac{5}{13}$ will give $7\frac{1}{4}$?

4. Multiply $\frac{3}{4}$ of $5\frac{1}{3}$ by $7\frac{1}{5}$, and divide the result by $1\frac{4}{5}$.

5. What number added to $2\frac{3}{13}$ will make $4\frac{1}{3}$?

6. By what must $16\frac{1}{3}$ be multiplied to make it $48\frac{1}{5}$?

7. A tunnel, $\frac{7}{16}$ mile long, is excavated at the rate of $\frac{11}{12}$ yard per day; in how many years will it be completed?

8. An estate, worth £10000, is left to A, B, and C; $\frac{3}{8}$ to A, $\frac{2}{5}$ to B, and the remainder to C. Find C's portion, and its value.

9. Find the difference between $\frac{3}{104}$ of $29\frac{1}{3}$, and $\frac{7}{26}$ of $\frac{5}{14}$ of $3\frac{1}{4}$.

10. Express in pecks $\frac{3}{5}$ of 1 qr. 3 bush. 2 pks.

11. Multiply the sum of $3\frac{4}{15}$ and $5\frac{12}{13}$ by the difference of $5\frac{3}{16}$ and $3\frac{10}{11}$, and divide the product by $2\frac{1}{17}$ of $3\frac{12}{13}$ of $1\frac{5}{11}$.

12. Find what fraction 258 days 9 hours 19 minutes 57 seconds is of 446 days 7 hours 23 minutes 33 seconds.

13. Add together three-seventeenths of £91, 7s. 6d., and eleven-twelfths of £40, 12s. 6d.

14. What is the difference between the *sum* of $\frac{4}{7}$ and $\frac{5}{9}$, and the *product* of $\frac{4}{5}$ and $\frac{7}{12}$?

15. What fraction of 3 cwts. 1 qr. 17 lb. 3 oz. is $\frac{4}{35}$ of a ton?

16. Multiply 20 cwts. 3 qrs. 7 lb. by $5\frac{1}{3}$, and divide £38, 2s. 6d. by $3\frac{13}{28}$.

17. Find what fraction 25 days 51 minutes 45 seconds are of 51 days 17 hours 46 minutes 57 seconds.

18. Compare the magnitudes of $\frac{11}{15}$, $\frac{18}{25}$, and $\frac{22}{35}$.

19. What relation does a "proper fraction" bear to unity? Can it be made into a "mixed number?"

Define exactly an "improper fraction," and show the value of $\frac{13}{7}$ of £1, 17s. 2d.

20. Add together $\frac{1}{5}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{6}{40}$, and subtract their sum from 1.

21. Find what fraction £2, 3s. $8\frac{3}{4}$d. is of £2, 6s. $3\frac{1}{2}$d.

22. Add together $\frac{5}{16}$ of $\frac{7}{12}$ of $2\frac{1}{3}$ of £1, $\frac{5}{8}$ of $\frac{7}{10}$ of $1\frac{1}{3}$ of 10s., $\frac{8}{13}$ of $\frac{17}{8}$ of $\frac{11}{15}$ of a crown, $\frac{5}{7}$ of $\frac{11}{12}$ of $\frac{7}{22}$ of a florin, and $3\frac{13}{17}$ of £2, 7s. $9\frac{3}{4}$d.

23. Reduce 4s. $7\frac{1}{2}$d. to the fraction of 5s. 9d., and 4 cwt. 57 lb. to the fraction of $1\frac{3}{4}$ qrs.

24. Reduce to their lowest terms and compare the two fractions $\frac{1818}{2550}$ and $\frac{2052}{2565}$.

25. Divide the sum of $6\frac{3}{7}$ and $1\frac{7}{8}$ by the difference between $3\frac{1}{3}$ and $2\frac{1}{5}$, and find the value of $(12\frac{5}{8} - 8\frac{3}{4} - 1\frac{1}{10} + \frac{8}{15}) \times 4\frac{1}{2} \times (7\frac{5}{12} - 6\frac{1}{2})$; and of $\frac{2}{3} \div 1\frac{4}{7} - \frac{5}{8} \div 3\frac{2}{11}$.

26. Express $\frac{3}{8}$ of 2s. 6d. + $\frac{4}{5}$ of a guinea + $\frac{5}{8}$ of £1 − $\frac{1}{16}$ of a penny, as a fraction of £5.

27. If $\frac{7}{15}$ of a guinea be taken from $\frac{8}{13}$ of $\frac{5}{8}$ of £5, what fraction of £100 will remain?

28. Compare the values of $\frac{1}{13}$ of £1, $\frac{1}{20}$ of 1 guinea, and $\frac{8}{35}$ of a crown.

29. A man leaves to one of his children $\frac{1}{3}$ of his property, to another £200, and the remainder, being $\frac{3}{5}$ of the property, among the rest. How much did he leave?

30. Find the greatest and least of the fractions $\frac{3}{8}$, $\frac{9}{20}$, $\frac{22}{45}$. $\frac{13}{28}$.

31. A person at his first game loses $\frac{1}{3}$ of his money, at the second $\frac{3}{5}$ of the remainder, at the third $\frac{4}{7}$ of the rest, and finds himself with £3, 6s. remaining. What had he at first?

32. Which is the larger amount, and by how much, $\frac{4}{7}$ of $\frac{4}{5}$ of $12\frac{1}{2}$ guineas, or $\frac{3}{4}$ of $\frac{1}{3}$ of $7\frac{1}{8}$ pounds?

33. Reduce $3\frac{5}{8} + \dfrac{6\frac{1}{4} - 1\frac{3}{4}}{2\frac{1}{8} \times 4\frac{2}{3}}$ gallons to the fraction of a tun.

34. Find what fraction 87 lb. 3 oz. 6 dwts. 21 grs. is of 133 lb. 9 oz. 18 dwts. 13 grs.

35. What fraction of 5 m. 5 fur. 7 poles $11\frac{4}{17}$ in. is $\frac{8}{17}$ of a league?

36. What fraction of 6 m. 2 fur. 7 poles 11 yds. 1 ft. 6 in. is $\frac{7}{15}$ of a league?

37. Reduce 14 wks. 6 days 23 hrs. 45 min. 20 secs. to the fraction of a year, consisting of $365\frac{1}{4}$ days.

38. Reduce $\dfrac{3\frac{1\frac{3}{14}}{4}}{20}$ of a ton to cwts., &c.; and $\dfrac{2\dfrac{7\frac{5\frac{1}{2}}{12}}{20}}{25}$ of £50 to £, s. d.

39. Reduce 3 fur. 120 yds. 2 ft. 8 in. to the fraction of 1 mile.

40. Find the value of $\dfrac{1\frac{2}{3} \times 7\frac{1}{5}}{2\frac{1}{7} \text{ of } 2\frac{2}{3}} - \dfrac{4\frac{1}{3} \text{ of } 2\frac{3}{5}}{2\frac{4}{11} \div \frac{7}{39}}$ of £270; of $2\frac{1}{2} \times 3\frac{1}{8}$ of $9\frac{1}{7}$ of £721, 17s. 6d.; of $(9\frac{5}{7}$ of $1\frac{16}{17} \div 3\frac{9}{19}) \times 41$ times 140 tons 12 cwt. 1 qr.; and $5\frac{1}{3}$ times $\frac{3}{8} \div 3\frac{1}{5} \times \frac{132}{144} \times 84$ times $6\frac{3}{135}$ of £972.

41. Find the difference between $\frac{3}{8}$ of $1\frac{7}{8}$ of 16 times £8, 17s. 6d., and $(\frac{4}{5} - \frac{1}{7} + 2\frac{1}{8})$ of $(\frac{41}{42} \times 1\frac{3}{5} \div \frac{41}{225})$ of £100, and find what fraction this difference is of £91, 12s. 8d.

42. What is the sum of $\frac{1}{7}$ of a yard $\frac{1}{7}$ of a foot and $\frac{1}{7}$ of an inch?

43. A man spends $\frac{1}{2}$ of his money, then $\frac{1}{3}$ of the remainder, and then $\frac{1}{4}$ of what was still left. He finally has £25 in hand; how much had he at first?

44. Out of a cistern $\frac{2}{3}$ full, 20 gallons are drawn; 70 gallons are then added, and it is found to be $\frac{5}{6}$ full; how much does it hold when full?

45. If $(\frac{2}{3} + \frac{1}{21})$ of an estate is worth £1003, 17s. 1d., what is the value of $(\frac{1}{3} + \frac{1}{15})$ of it?

46. Add together $\dfrac{2\frac{1}{4} + 3\frac{1}{28}}{3\frac{16}{35} + 12\frac{3}{5}}$ and $\dfrac{6\frac{2}{3} + 3\frac{19}{35}}{12\frac{19}{28} + 2\frac{1}{2}}$, and subtract $\dfrac{3\frac{4}{5} + \frac{29}{45}}{3\frac{3}{4} + 14\frac{1}{36}}$ from $\dfrac{7\frac{1}{2} + 1\frac{17}{22}}{8\frac{2}{3} + 3\frac{23}{33}}$.

47. A man has £3000 in hand, having lost a quarter of his property in speculation, and purchased a partnership with $\frac{3}{4}$ of the remainder. What was he worth at first?

48. A man invests $\frac{1}{2}$ of his fortune in land, $\frac{1}{5}$ in the funds, $\frac{1}{6}$ in Exchequer bills, and loses the remaining £2000 in speculation. What was his original fortune?

49. Simplify the following expression :—

$$\Bigg[\Bigg(8\tfrac{2}{3} \text{ of } 1\tfrac{1}{4} \times \frac{18\frac{3}{5}}{29} \div 6\frac{1 + \frac{3}{4} + 2\frac{3}{5}}{7 + \frac{3}{8} - 2\frac{1}{4}} \times \frac{325728}{578305}\Bigg) + \Bigg\{\frac{19}{168} \text{ of} \Big(17\tfrac{1}{3} - 3\tfrac{3}{19}$$

$$\text{of } 8\tfrac{1}{4}\Big)\, 10\,\frac{3 + 1\dfrac{3 + 7\frac{1}{2} + 5\frac{1}{4} \text{ of } 6}{19 \div 3\frac{4}{7} \text{ of } 12\frac{2}{5}} \text{ of } 2\frac{361}{923}}{101\frac{6}{7} - \frac{3}{13} \text{ of } \dfrac{5\frac{1303}{1440} + \frac{4}{5} + 6\dfrac{2 + 19\frac{1}{4}}{72}}{14\frac{3}{7} - \frac{215}{467} \times 8 \text{ times } \dfrac{3\frac{1}{5} + 8 + 2\frac{1}{7}}{6 - 2\frac{1}{14} + 3\frac{5}{21}}}}\Bigg\}$$

$$+\, 2\tfrac{1}{19}\Bigg] \times \frac{50 + 8\frac{5}{9}}{10\frac{7}{9} + 6}.$$

## DECIMAL FRACTIONS, or DECIMALS.

**72.** In explaining the theory of ordinary notation, it was stated that any digit in the unit's place retained its *intrinsic* value, and that any digit to the left of the unit's place acquired a *local* value which became *ten times greater* for every place that it was advanced to the left. In the same way it may be said that *the value of a digit became ten times less whenever it was moved one place farther to the* right.

Thus in the number 7777, the value of

7 in the thousands' place is $7 \times 1000$,
7 „ hundreds' „ $7 \times 100$,
7 „ tens' „ $7 \times 10$,
7 „ units' „ $7 \times 1$,

and now, continuing the figures to the right, thus, 7777·777, the same relation in the values continues; thus, the value of

7 in the *first* place to the right of the units' place is $7 \times \frac{1}{10} = \frac{7}{10}$,
7 „ *second* „ „ „ „ $7 \times \frac{1}{100} = \frac{7}{100}$,
7 „ *third* „ „ „ „ $7 \times \frac{1}{1000} = \frac{7}{1000}$.

**This system of notation, by which we are enabled to express values less than unity, is called the Decimal System of Notation.**

To indicate the units' figure, a dot is placed on its right, as in the number 237·419, where the 7 is the units' figure. This dot is called the **Decimal Point.**

The whole system of Decimal Notation may therefore be indicated thus—

| | Thousands. | Hundreds. | Tens. | UNITS. | | Tenths. | Hundredths. | Thousandths. | |
|---|---|---|---|---|---|---|---|---|---|
| &c. | 4 | 3 | 2 | 1 | · | 2 | 3 | 4 | &c. |

**73.** Returning now to the values of the figures on the right of the decimal point, it will be observed that each is *a fraction having* TEN, *or* *some power*[*] *of* TEN, *for its denominator*, although in the decimal system of notation these denominators are never expressed.

The following definition is now clear—

**Def.** *A decimal fraction is one whose denominator—never expressed as such—is either ten or some power of ten.*

N.B. 1.—It is not necessary that there should be any whole numbers in order that a fraction may be expressed in the decimal system: for

*one one-hundredth* may be expressed thus, ·01;
*five ten-thousandths* ·0005.

N.B. 2.—Decimal Fractions are also called *Decimals*.

**74.** From observing the following equivalent values of decimal fractions,

$$\text{Ex. 1. } \cdot 2748 = \frac{2}{10} + \frac{7}{100} + \frac{4}{1000} + \frac{8}{10000}$$

(The L.C.M. of their denominators is 10000),

$$\therefore = \frac{2000 + 700 + 40 + 8}{10000}$$

$$= \frac{2748}{10000}.$$

$$\text{Ex. 2. } 23\cdot 014 = \frac{23}{1} + \frac{1}{100} + \frac{4}{1000}$$

(The L.C.M. of their denominators is 1000),

$$\therefore = \frac{23000 + 10 + 4}{1000}$$

$$= \frac{23014}{1000}.$$

$$\text{Ex. 3. } \cdot 00125 = \frac{1}{1000} + \frac{2}{10000} + \frac{5}{100000}$$

(The L.C.M. of their denominators is 100000),

$$\therefore = \frac{100 + 20 + 5}{100000}$$

$$= \frac{125}{100000}.$$

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[*] *A power* of a number is found by multiplying that number by itself a certain number of times. When a number is multiplied by itself *once*, the product is called the *second* power of that number; if the number be multiplied by itself *twice*, the product is called the *third* power of that number, &c.

We see—

1st. *That every* **finite** (see § 76) *decimal may be expressed as a vulgar fraction which shall have for its numerator the significant* figures of the decimal fraction, and for its denominator the denominator of the last figure on the right of the decimal.*

2dly. *That this denominator may also be found by writing a cipher for every figure in the decimal fraction, and placing a* 1 *before it*—

$$\text{Thus, } \cdot 648 = \frac{648}{1000}; \ \cdot 0091 = \frac{91}{10000}; \ 7\cdot 01 = 7\frac{1}{100}.$$

3dly. *That any number composed of integers and decimals may be expressed as a vulgar fraction having all the figures of the number for its numerator, and the denominator of the last decimal on the right for its denominator*—

$$\text{Thus, } 418\cdot 0135 = \frac{4180135}{10000}.$$

**75.** Conversely, *Any vulgar fraction having* 10 *or any power of* 10 *for its denominator may be expressed as a decimal*—

$$\text{Thus, Ex. 1. } \frac{749}{10} = 74\frac{9}{10} = 74\cdot 9 \text{ (§ 72).}$$

$$\text{Ex. 2. } \frac{23849}{100} = 238\frac{49}{100} = 238\cdot 49 \text{ (§ 72).}$$

N.B. 1.—It is evident that the addition of a cipher, or ciphers, to the right of a decimal does not alter its value—

$$\text{Thus, } \cdot 39 = \cdot 390 = \cdot 3900 = \cdot 39000, \text{ \&c.,}$$

$$\text{for } \cdot 39 = \frac{39}{100} = \frac{390}{1000} = \frac{3900}{10000} = \frac{39000}{100000}, \text{ \&c.,}$$

which are respectively equal to ·39, ·390, ·3900, ·39000, &c.

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* *The significant figures* of a decimal fraction are those which, taken by themselves alone, have some real value—thus, in ·0309, the figures 309 are the significant figures.

**But the addition of every cipher to the right of the significant figures of a decimal fraction (*i.e.* between the significant figures and the decimal point), decreases the value of that fraction to one-tenth of itself; thus—**

$$\cdot 39 = \frac{39}{100}\text{; but } \cdot 039 = \frac{39}{1000},\ \textit{i.e.}\ \frac{1}{10} \text{ of } \frac{39}{100}.$$

**The effect of prefixing and affixing ciphers to decimals is thus exactly the opposite of the effect produced by prefixing and affixing ciphers to integers.**

**N.B. 2.—It is also evident that a decimal, or a number consisting of integers and decimals, may be *multiplied* by moving the decimal point one place farther to the right, and may therefore be *divided* by moving it one place farther to the left, thus—**

**Ex. 1.** $24\cdot 63 \div 10 = \frac{2463}{100} \div 10 = \frac{2463}{100 \times 10}\ (\S\ 58) = \frac{2463}{1000} = 2\cdot 463,$

**which result might have been obtained by moving the decimal point one place farther to the *left*.**

**Ex. 2.** $24\cdot 63 \times 10 = \frac{2463}{100} \times 10 = \frac{2463}{100 \div 10}\ (\S\ 57) = \frac{2463}{10} = 246\cdot 3,$

**which result might have been obtained by moving the decimal point one place farther to the *right*.**

**Ex. 3.** $\cdot 016 \div 10 = \frac{16}{1000} \div 10 = \frac{16}{1000 \times 10} = \frac{16}{10000} = \cdot 0016.$

**Ex. 4.** $\cdot 016 \times 10 = \frac{16}{1000} \times 10 = \frac{16}{1000 \div 10} = \frac{16}{100} = \cdot 16.$

**76.** *It is not possible to express every vulgar fraction as a decimal, but it is possible to find a decimal so nearly equivalent to any given vulgar fraction as to differ from it by less than any given quantity*, as will hereafter be explained.

It should, however, be at once understood that—

1. **A Terminate or Finite Decimal** is one which exactly expresses the value of a given vulgar fraction; and that—

2. **An Interminate or Infinite Decimal** is one which expresses only the *approximate* value of a vulgar fraction whose *exact* value cannot be expressed by a decimal.

**N.B.—It must always be remembered that a decimal is a fraction, the difference between a vulgar fraction and a decimal fraction being**

**(*a*.) That the denominator of a decimal fraction is always suppressed, whereas the denominator of a vulgar fraction is always expressed ;**

**(*b*.) That while the denominator of a vulgar fraction may be any number whatever, integral or fractional, that of a decimal fraction is always 10, or some power of 10.**

## I. Addition and Subtraction of Decimals.

**77. The Rule for Addition and Subtraction of Decimals, and Numbers including Decimals,** is essentially the same as that for simple addition and subtraction of integers, and may be stated as follows—

**RULE.—Set down the decimals in a column, care being taken to keep the decimal points in the same vertical line; then add or subtract as with common integers, placing the decimal point in the same vertical line as the other points.**

**Ex.** 1. *Add together* ·304, ·00109, ·7143826, ·00009, ·0701835.

**PROCESS.**
·304
·00109
·7143826
·00009
·0701835
*Answer*, **1·0897461**

**The PROOF is seen by reducing these fractions to equivalent ones having a common denominator, which is most easily done by filling up the blank spaces with ciphers, thus—**

·3040000
·0010900
·7143826
·0000900
·0701835
*Answer*, **1·0897461**

**Each fraction being now expressed with a denominator of 10,000,000, we may state the question thus—**

**Question**—"*Find the sum of* 3040000 *ten-millionths,* 10900 *ten-millionths,* 7143826 *ten-millionths,* 900 *ten-millionths, and* 701835 *ten-millionths.*"

**The Answer** would be found, by Simple Addition, thus—

$$\begin{array}{rl}3040000 & \text{ten-millionths}\\ 10900 & \text{ten-millionths}\\ 7143826 & \text{ten-millionths}\\ 900 & \text{ten-millionths}\\ 701835 & \text{ten-millionths}\\ \hline 10897461 & \text{ten-millionths}\end{array} = \frac{10897461}{10000000} = 1\frac{897461}{10000000} = 1\cdot0897461, \textit{ Answer.}$$

**Ex. 2.** *Find the sum of* 28, 7·0143, ·0096305, 2000·104, 71·35.

$$\left.\begin{array}{r}28\cdot\phantom{0000000}\\ 7\cdot0143\phantom{000}\\ \cdot0096305\\ 2000\cdot104\phantom{0000}\\ 71\cdot35\phantom{00000}\end{array}\right\}\text{ By Simple Addition.}$$

$$\begin{array}{r}\hline 2106\cdot4779305\end{array}\textit{, Answer.}$$

**Ex. 3.** *Find the difference between* 89·63 *and* 7·81.

$$\left.\begin{array}{r}89\cdot63\\ 7\cdot81\end{array}\right\}\text{ By Simple Subtraction.}$$

$$\begin{array}{r}\hline 81\cdot82\end{array}\textit{, Answer.}$$

## Exercise 23.

### Notation, Addition, and Subtraction of Decimals.

1. Write down 1 tenth 8 hundredths, 3 hundredths 8 thousandths 3 ten-thousandths, 4 tenths 1 hundredth 6 thousandths, and find their sum.

2. Express in figures the sum of 3 thousandths, 8 hundredths 5 ten-thousandths 6 one-hundred-thousandths, 9 tenths 2 hundredths 8 one-hundred thousandths, 8 hundredths 3 thousandths 1 ten-thousandth 4 millionths.

3. **Express as decimals—**

(*a.*) $\frac{4}{10}$; $\frac{17}{100}$; $\frac{13}{100}$; $\frac{2001}{10000}$; $\frac{1801}{100}$; $\frac{7}{1000}$; $\frac{28}{1000000}$; $\frac{1041}{1000000}$; $\frac{6}{100000000}$.

(*b.*) $\frac{1}{10}$ of $\frac{4}{10}$; $\frac{3}{5}$ of $\frac{18}{200}$; $\frac{7}{8}$ of $\frac{6}{125}$; $\frac{18}{25}$ of $\frac{171}{40000}$; $\frac{15}{400}$ of $\frac{161}{25000}$.

(*c.*) 181 + 5 hundredths; 170 + 2 tenths + 3 hundredths + 4 ten-thousandths.

4. **Reduce to vulgar fractions—**

(*a.*) ·03; ·0451; ·25; ·005; ·000171; ·0080307; ·71003; ·0625; ·0001.

(*b.*) 28·01; 430·002; 71·21625; 30824·5; 1·0625; 1801·4145.

(*c.*) $\frac{1}{10}$ of 2·214; $\frac{1}{10}$ of 168·145; $\frac{1}{10}$ of 732·45; $\frac{1}{10}$ of 61·832; $\frac{1}{10}$ of 190·012; $\frac{1}{10}$ of 705·21688.

(*d.*) $\frac{1}{100}$ of 2183·7; $\frac{1}{100}$ of 62083·41; $\frac{1}{1000}$ of 7382; $\frac{1}{10}$ of ·6; $\frac{1}{10}$ of 21.

(*e.*) 62·41 × 10; 6·381 × 100; 7281·65 × 10; 2·139746 × 1000; ·006 × 10000.

(*f.*) 839·1082 × 10 × 1000; 10 times ·181 × 10000; 20 times 50 times 6·3814; 25 times 40 × ·032565; 50 times 20 times 40 times 250 times 1·8136491.

5. **Find the sum of the numbers in *a*, *b*, *c*, *d*, *e*, *f*, respectively, in Question 4 above, and find the difference between their total sum and a hundred millions.**

6. **Find the value of—**

(*a.*) 3·01 + 2·68 + ·052 + 7·8.

(*b.*) 180·6 + 20·00043 + 7·89106 + ·800617 + 14000·003 + 28810.

(*c.*) 20·64 + 18·064 + 30·0064 + 7·81064 + ·000064 + 81000·415308.

(*d.*) 19000·00019 + 208·20808 + 64·39007 + 210860·8000341 + 9·637.

(*e.*) (30·81 + 2·791 + ·8604 + 100·3) – 20·9063.

(*f.*) 2186·0031 – 79·9083; 4100·06 – 26·0071; 200 – 10·415.

(*g.*) 1600·0001 – 28; 7132·001 – 7106·008093; 25 – ·01706.

(*h.*) 23·906 + 14·20008 – 23·3192 – 17·010204 + 101·5.

(*i.*) 814·21903 – 1007·612 – 203·05566 + 6002·1918 – 50·1382605.

## II. Multiplication of Decimals.

**78.** The rule for **Multiplication of Decimals** is founded on the same principle as that for Simple Multiplication of Integers, and may be stated as follows—

**RULE.—Multiply the given numbers together as if they were common integers, and mark off in the product as many decimal places as there are in the multiplier and multiplicand together.**

If the number of figures in the product be less than the number of decimal places in the multiplier and multiplicand together, ciphers sufficient to supply the deficiency must be prefixed to the product. (See Ex. 2).

**Ex.** 1. *Multiply* ·943 *by* ·8 (*i.e. Find the value of* ·943 × ·8).

There are *three* decimal places in the multiplicand and *one* in the multiplier, *i.e. four* in all; and, since there are also *four* figures in the product, the whole product is a decimal.

PROCESS.

$$\begin{array}{r} \cdot943 \\ \cdot8 \\ \hline \textit{Ans.},\ \cdot7544 \end{array}$$

The PROOF may be demonstrated thus—

$$\cdot943 \times \cdot8 = \frac{943}{1000} \times \frac{8}{10} = \frac{943 \times 8}{1000 \times 10} =$$

Now, 943 × 8 = 7544.

But this product must be divided by 1000 × 10, *i.e.* by 10000, where it will be noticed that there is, and must necessarily be, a cipher for every decimal place in the multiplier and multiplicand together, from which it follows that the result of dividing 7544 by 10000 will be to produce a decimal containing as many places as are found in the multiplier and multiplicand together, thus—

$$7544 \div 10000 = \frac{7544}{10000} = \cdot7544, \textit{ Answer.}$$

**Ex.** 2. *Multiply* ·001436 *by* ·0192 (*i.e. Find the value of* ·001435 × ·0192).

The number of decimal places in the multiplier and multiplicand together being *ten*, while the product contains but *six* figures, it is necessary to prefix *four* ciphers to the product to give the *ten* places of decimals required.

$$\begin{array}{r} \cdot001436 \\ \cdot0192 \\ \hline 2872 \\ 12924\phantom{0} \\ 1436\phantom{00} \\ \hline \textit{Ans.},\ \cdot0000275712 \end{array}$$

**Ex. 3.** *Find the value of* 19·01 × 4·308; *and of* 71438 × ·00043.

```
   4·308            71438
   19·01            ·00043
   ─────            ──────
    4308            214314
  38772            285752
 4308              ───────
 ────────          30·71834, Answer.
 81·89508, Answer.
```

## Exercise 24.

### Multiplication of Decimals.

Find the value of—

1. 230·015 × 5·002; 521801 × ·801; 507 × ·507; 2920 × ·202.
2. ·05 × ·0086; 7·2 × ·005; 375 × ·00009; ·8814 × 7·05; 1·86 × ·015.
3. ·705 × ·1302; ·3108 × 1·65; 6·28 × ·005; 3840 × ·600005; 182 × ·0015.
4. 6·25 × ·016; 62·5 × ·064; 31·25 × ·128; ·075 × 1·4; 625 × ·128.
5. ·025 × 2·56; 4·096 × ·0025; 1·024 × 2·5; ·175 × ·08; 1·6384 × ·025.
6. 830·5 × 1·006 × 7·25; 7·25 × 1·206 × ·1206; ·1905 × 1·905 × 8·08.
7. 15·005 × ·0015005 × 520 × ·00052; 908 × ·00908 × 3750 × ·003104.
8. 32·014 × ·0056; ·0132 × 5·045; 380·1 × ·08301; 5·274 × ·00145.
9. 3920 × ·5025; ·0895 × 3·064; 7509·325 × 86·09; 381·74 × 26·003.
10. 8·0124 × ·0175 × ·08635 × 1·008; 7·308 × 2·405 × ·00807 × 6·0013.
11. 83·016 × ·612; 730·154 × 61·7 × 36·7 × 3·48 × 9·9073 × ·00176.
12. 832·009 × ·00504 × 6·375 × ·01804; 7·263 × ·02108 × ·0000107.

## III. Division of Decimals.

**79.** The rule for **Division of Decimals, and Numbers including Decimals,** may be variously stated, but it always depends upon the same principle as that for Simple Division of Integers.

The only difficulty experienced in performing division with decimals is in fixing the position of the decimal point in the quotient. Here it will be convenient to remember that *there must always be as many decimal places in the divisor and quotient together as there are in the dividend alone.*

**Ex. 1.** *Divide* 84·7 *by* ·7 (*i.e. Find the value of* 84·7 ÷ ·7).

$$\text{The Answer} = \frac{84\cdot7}{\cdot7} = \frac{84\cdot7 \times 10}{\cdot7 \times 10} \text{ (§ 59)} = \frac{847}{7} \text{(§ 75)} = 121.$$

Hence the reason of the following rule—

**RULE 1.—When the dividend and the divisor contain the same number of decimal places, proceed as in division of common integers, and the quotient thus found will be expressed in whole numbers.**

**Ex. 2.** *Divide* 23·326 *by* 2·18 (*i.e. Find the value of* 23·326 ÷ 2·18).

$$\text{The Answer} = \frac{23\cdot326}{2\cdot18} = \frac{\frac{1}{10} \text{ of } 233\cdot26}{2\cdot18} \text{ (§ 75)} = \tfrac{1}{10} \text{ of } \frac{233\cdot26}{2\cdot18} = \tfrac{1}{10} \text{ of}$$
$$\frac{233\cdot26 \times 100}{2\cdot18 \times 100} \text{ (§ 59)} = \tfrac{1}{10} \text{ of } \frac{23326}{218} = \tfrac{1}{10} \text{ of } 107 = 10\cdot7.$$

Hence the reason of the following rule—

**RULE 2.—When the dividend contains *more* decimal places than the divisor, proceed as in division of common integers, and mark off in the quotient as many decimal places as are required to make the number of decimal places in the divisor and quotient together equal to the number in the dividend alone.**

**Ex. 3.** *Divide* 233·26 *by* ·218 (*i.e. Find the value of* 233·26 ÷ ·218).

$$\text{The Answer} = \frac{233\cdot26}{\cdot218} = \frac{10 \text{ times } 23\cdot326}{\cdot218} \text{ (§ 75)} = 10 \text{ times } \frac{23\cdot326}{\cdot218} =$$
$$10 \text{ times } \frac{23\cdot326 \times 1000}{\cdot218 \times 1000} \text{ (§ 59)} = 10 \text{ times } \frac{23326}{218} = 10 \text{ times } 107 = 1070.$$

Hence the reason of the following rule—

**RULE 3.—When the dividend contains *less* decimal places than the divisor, proceed as in division of common integers, and add to the quotient as many ciphers as there are less decimal places in the dividend than in the divisor.**

**80.** To illustrate the rules more fully, we shall now work a few examples.

**Ex. 1.** *Divide* 9358·22 *by* 2·14.

```
2·14) 9358·22 (4373
      856
       798
       642
       1562
       1498
         642
         642
```

Here we have the same number of decimal places in the dividend as in the divisor; the entire quotient, viz. 4373, will therefore be whole numbers, so that the Answer is 4373.

**Ex. 2.** *Divide* 93·5822 *by* 21·4.

```
21·4) 93·5822 (4·373
      856
       798
       642
       1562
       1498
         642
         642
```

In this case we have *four* decimal places in the dividend and *one* in the divisor; the quotient must therefore contain *three* decimal places. (See Rule 2).

**Ex. 3.** *Divide* 93582·2 *by* ·214.

```
·214) 93582·2 (437300
      856
       798
       642
       1562
       1498
         642
         642
```

In this case the decimal places are *two* less in number in the dividend than in the divisor; *two* ciphers are therefore annexed to the quotient, thus making the Answer 437300.

**Ex. 4.** *Divide* 15·84 *by* 7·68.

```
7·68) 15·84 (2·0625
      1536
        4800
        4608
         1920
         1536
          3840
          3840
```

The method here pursued deserves particular notice. The ciphers brought down to enable us to obtain the figures 0625 in the quotient are not seen in the dividend, but it will be remembered that *the addition of ciphers to the right of a decimal does not alter its value*, and therefore we may suppose the dividend to be 15·840000. In this form it contains *six* decimal places, and therefore, since the divisor contains but *two* decimal places, the quotient must contain *four*.

**Ex. 5.** *Divide* 6018·315 *by* 7500.

$$\begin{array}{l}
7500)\,6018\cdot315\,(\cdot802442 \\
\quad\quad\ \ \underline{600} \\
\quad\quad\ \ 183 \\
\quad\quad\ \ \underline{150} \\
\quad\quad\quad 331 \\
\quad\quad\quad \underline{300} \\
\quad\quad\quad\ \ 315 \\
\quad\quad\quad\ \ \underline{300} \\
\quad\quad\quad\quad 150 \\
\quad\quad\quad\quad 150
\end{array}$$

It will be seen that we have here divided by 75 instead of 7500, therefore our quotient will be 100 times too great. We can remedy this by moving the decimal point *two* places to the left in the quotient.

And since we have *four* decimal places in the dividend and *none* in the divisor, we shall require *four* places in the quotient on this account: therefore we shall require altogether *six* decimal places in the quotient.

An inspection of the dividend and divisor will often enable the student to determine the position of the decimal point in the quotient. Thus, in this example, 6018 ÷ 7500 is evidently a little less than 1, and therefore the decimal point should be placed on the left of the 8, to make ·8, &c.

## Exercise 25.

### Division of Decimals.

1. 86·562 ÷ 9; 166·67 ÷ 7; 864·136 ÷ 8; 252·03708 ÷ 12; ·105376 ÷ 13.
2. ·945 ÷ 2·7; 53·0019 ÷ 8·413; 499758 ÷ 61320; ·60104032 ÷ 3·004.
3. 6188204 ÷ 91·003; 1351193 ÷ ·1093; ·0216300 ÷ 72100.
4. ·00000068234 ÷ ·00313; 75·593812 ÷ 9·17; ·0834082398 ÷ ·6034.
5. 21293·4 ÷ ·625; 8249 ÷ 6·25; 7·1013 ÷ ·00625; 81 ÷ ·000625.
6. 31·01 ÷ ·875; ·4134 ÷ ·0375; 61427 ÷ ·125; 190 ÷ 12·5.
7. If 40·550267 be divided by 2·81, what number must be taken from the quotient to leave a remainder equal to the sum of 8·6034 and 4·91?
8. What is that number which, being multiplied by 7, gives a product greater by 28·00927 than the product of 2·103 and 749?
9. What number is that which, when taken from the product of 706·305 and 28·101, leaves a remainder which is equal to the sum of 700·43, ·0804919, 48·9168, and the quotient found by dividing 40·008 times 30·009 by 12?

**81. To reduce a vulgar fraction to a decimal, without altering its value.**

Since a vulgar fraction may have any number for its denominator, while a decimal has always 10, or some power of 10, for its denominator, the point of difficulty in converting a vulgar fraction into a decimal is *to find an integral number such that, when multiplied by the denominator of the given vulgar fraction, the product shall be* 10, *or some power of* 10.

Thus, in reducing $\frac{4}{5}$ to a decimal, we want to know *by what number the* 5 *must be multiplied to make it* 10, *or some power of* 10.

We first try 10 itself, saying, *Fives in* 10, *two;* and then we have

$$\frac{4}{5}=\frac{4\times 2}{5\times 2}\ (\S\ 59)=\frac{8}{10}=\cdot 8\ (\S\ 74).$$

Again, in reducing $\frac{7}{16}$ to a decimal, we want to know *by what integral number* 16 *must be multiplied to make it* 10, *or some power of* 10.

We may find this number thus—

1st. *Putting down* 10 *itself,* we see that it is *not* exactly divisible by 16; therefore there is no integral number by which 16 can be multiplied to give 10 for the product.

2nd. *Raising* 10 *to the second power* by annexing a cipher to it, we see that this number, *i.e.* 100, is *not* exactly divisible by 16; therefore there is no integral number by which 16 can be multiplied to give 100 for the product.

3rd. *Raising* 10 *to the third power* by annexing two ciphers to it, we see that this number, *i.e.* 1000, is *not* exactly divisible by 16; therefore there is no integral number by which 16 can be multiplied to give 1000 for the product.

4th. *Raising* 10 *to the fourth power* by annexing three ciphers to it, we see that this number, *i.e.* 10000, *is* exactly divisible by 16, for $10000 \div 16 = 625$; so that we have

$$\frac{7}{16}=\frac{7\times 625}{16\times 625}=\frac{4375}{10000}=\cdot 4375\ (\S\ 74).$$

An examination of what we have done leads us to the following conclusion—

The significant figures of the decimal fraction equivalent to $\frac{7}{16}$ are found by solving the expression—

$$(10000\div 16)\times 7,\ i.e.\ \frac{10000\times 7}{16},\ i.e.\ \frac{70000}{16};$$

and now, remembering that *as many ciphers were required as would produce a number exactly divisible by* 16, we are enabled to lay down the following rule—

**RULE.—To reduce a vulgar fraction to an equivalent decimal, after reducing the given fraction to its lowest terms, place a decimal point after the numerator, and divide the numerator by the denominator, setting down for this purpose as many decimal ciphers as may be required. The quotient will be the answer, thus—**

**Ex. 1.** *Convert* $\frac{5}{8}$ *into a decimal.*

$$\begin{array}{r|l} 8 & \underline{5\cdot000} \\ & \cdot625,\ \textit{Answer.} \end{array}$$

**Ex. 2.** *Express* $\frac{7}{16}$, $\frac{5}{64}$, *and* $\frac{18}{128}$ *as decimals.*

$$16\begin{cases} 4 \,|\, \underline{7\cdot0000} \\ 4 \,|\, \underline{1\cdot7500} \end{cases} \quad \cdot4375,\ \textit{Answer.} \qquad 64\begin{cases} 8 \,|\, \underline{5\cdot0000} \\ 8 \,|\, \underline{\cdot6250} \end{cases} \quad \cdot078125,\ \textit{Answer.}$$

$$\frac{18}{128} = \frac{18 \div 2}{128 \div 2} = \frac{9}{64} \therefore \quad 64\begin{cases} 8 \,|\, \underline{9\cdot000000} \\ 8 \,|\, \underline{1\cdot125000} \end{cases} \quad \cdot140625,\ \textit{Answer.}$$

**82.** It is sometimes necessary to refer to the rule for Division of Decimals in order to determine the position of the decimal point, thus—

**Ex.** *Convert* $\frac{313}{512}$ *to a decimal.*

$$\begin{array}{l} 512)\,313\cdot000000000\,(\cdot611328125,\ \textit{Answer.} \\ \quad\ \underline{3072} \\ \qquad 580 \\ \qquad \underline{512} \\ \qquad\ 680 \\ \qquad\ \underline{512} \\ \qquad\ 1680 \\ \qquad\ \underline{1536} \\ \qquad\quad 1440 \\ \qquad\quad \underline{1024} \\ \qquad\qquad 4160 \\ \qquad\qquad \underline{4096} \\ \qquad\qquad\ \ 640 \\ \qquad\qquad\ \ \underline{512} \\ \qquad\qquad\ \ 1280 \\ \qquad\qquad\ \ \underline{1024} \\ \qquad\qquad\quad 2560 \\ \qquad\qquad\quad 2560 \end{array}$$

The same example might have been worked as follows:—

$$512\begin{cases} 8 \,|\, \underline{313\cdot} \\ \ \ 8 \,|\, \underline{39\cdot125} \\ \quad 8 \,|\, \underline{4\cdot890625} \\ \qquad\qquad \cdot611328125,\ \textit{Answer.} \end{cases}$$

**83.** If the denominator of the given vulgar fraction be 10, or some multiple of 10, it may be reduced to a decimal as follows—

**Ex.** *Reduce $\frac{7}{40}$ to a decimal.*

$$\frac{7}{40} = \frac{7 \div 10}{40 \div 10} = \frac{\cdot 7}{4} = \cdot 175 \text{ (§ 79).}$$

**84.** *If the denominator of a vulgar fraction, when reduced to its lowest terms, has other prime factors than 2 and 5, the vulgar fraction cannot be reduced to a decimal which shall exactly express its value.*

In § 81 it has been shown that, to reduce a vulgar fraction to an equivalent decimal, both the numerator and the denominator of the given vulgar fraction must be multiplied by such an integral number as will render the *new numerator* exactly divisible by the *original denominator.*

Now the *new* numerator is the product of the *old* numerator × 10 or some power of 10;

Therefore,

$$\frac{\text{The } \textit{old} \text{ numerator} \times 10 \text{ or some power of } 10}{\text{The old denominator}} = \text{a whole number (I.)}$$

But since the vulgar fraction is always reduced to its lowest terms before we commence its conversion into a decimal, *its old numerator and denominator can have no common factor* (§ 64), so that, simplifying the above equation, we may say that

$$\frac{10\text{, or some power of } 10}{\text{The old denominator}} = \text{a whole number (II.)}$$

*But* 10, *and every power of* 10, *can be divided by no prime factor other than* 2 *and* 5, so that the old denominator can be divisible by no prime factor other than 2 and 5 if the vulgar fraction be reducible to a decimal; and therefore

"*Any vulgar fraction, reduced to its lowest terms, whose denominator contains prime factors other than* 2 *or* 5, *cannot be reduced to a decimal which shall accurately represent its value.*"

Hence the reason of the following rule—

**RULE.—Having reduced the given vulgar fraction to its lowest terms, resolve its denominator into its prime factors. If these**

**prime factors be only 2 and 5, the vulgar fraction is reducible to a terminating decimal; but if any other prime factors be contained in the denominator, the vulgar fraction is not reducible to a terminating decimal.**

**85.** *If the denominator of a vulgar fraction, reduced to its lowest terms, contains other prime factors than 2 and 5, the vulgar fraction may be expressed as a* **Circulating Decimal,** *where the number of figures in the recurring period will be less than the number in the denominator.*

**Ex. 1.** *Reduce $\frac{5}{11}$ to a decimal.*

Here we have a remainder, 5, similar to the figure of the numerator, and it is evident that, if we now bring down other ciphers, and proceed with the operation, we shall repeat the figures in the quotient (as is indicated by the dotted figures), *ad infinitum.*

Because the figures of such a decimal as this occur again and again, the decimal is called a *repeating,* or *recurring,* or *infinite,* or *circulating* decimal, and to denote the figures which circulate a dot is placed on the first and last figures of that part of the decimal; thus, $\frac{5}{11} = \dot{4}\dot{5}$.

11) 50 (·4545, &c.
44
60
55
50
44
60
55
50

**Ex. 2.** *Reduce $\frac{43}{7}$ to a decimal.*

Here we have 7 for a divisor, and the number of figures in the recurring period is *six,* which is the greatest number possible with this divisor; for, since *every remainder must be less than the divisor,* the only possible remainders in this case are

6, 5, 4, 3, 2, 1, 0.

Of these, *the cipher cannot occur, else the decimal would not repeat,* so *that there are only six remainders possible,* and, therefore, *there can only be six figures in the circulating period.*

7) 43 ($6\cdot\dot{1}4285\dot{7}$, &c.
42
10
7
30
28
20
14
60
56
40
35
50
49
1

The manner of working the following example deserves notice—

**Ex. 3.** *Find the value of* ·001776 ÷ ·0784 (Matriculation Examination, London University, 1871).

$$\text{Here } \cdot001776 \div \cdot0784 = \frac{\cdot001776}{\cdot0784} = \frac{\cdot001776 \times 1000000}{\cdot0784 \times 1000000} = \frac{1776}{78400} =$$

$$\frac{1776 \div 16}{78400 \div 16} = \frac{111}{4900} = \frac{1}{100} \text{ of } \frac{111}{49}.$$

The value of $\frac{111}{49}$ may be found thus—

$$49\left\{\begin{array}{l} 7 \,|\, \underline{111\cdot} \\ 7 \,|\, \underline{15\cdot\dot{8}5714\dot{2}\ 857142\ 857142\ 857142\ 857142\ 857142\ 857142} \\ \quad\ \ 2\cdot265306\ 122448\ 979591\ 836734\ 693877\ 551020\ 408163 \end{array}\right.$$

Figures remaining, 1 0 6 5 4 3 2 1

Here it will be observed that, although there is no remainder at the end of the first six figures, the answer does not terminate there, as is apparent from considering that the next thing we say is, "*Sevens in* Eight *hundred and fifty-seven thousand one hundred and forty-two?*" whereas at the beginning of the decimal we said, "*Sevens in* Eighteen *hundred and fifty-seven thousand one hundred and forty-two?*" and it is not till we have had, at the beginning of the period, every remainder less than the divisor 7, that the answer is completed, for, having at last the remainder 1 at the end of a period, the next thing we say is "*Sevens in eighteen hundred and fifty-seven thousand one hundred and forty-two?*" which is exactly what we said at the beginning of the decimal; so that

$$\frac{111}{49} = 2\cdot\dot{2}6530612244897959183673469387755102040816\dot{3},$$

and therefore,

$$\frac{1}{100} \text{ of } \frac{111}{49} = \cdot02\dot{2}6530612244897959183673469387755102040816\dot{3}, \textit{ Answer.}$$

This example may also be worked as follows—

$\frac{111}{49} = 2\frac{13}{49}$.

**First.**—*Find the value of* $\frac{1}{49}$.

$$49)\,\dot{1}00\,(\cdot02\tfrac{2}{49}$$
$$\frac{98}{2}$$

Hence $\frac{1}{49} = \cdot\mathbf{02}\frac{2}{49}$: therefore $\frac{2}{49} = \cdot04\frac{4}{49}$.

Hence $\frac{1}{49} = \cdot02\mathbf{04}\frac{4}{49}$: therefore $\frac{4}{49} = \cdot0816\frac{16}{49}$.

Hence $\frac{1}{49} = \cdot0204\mathbf{0816}\frac{16}{49}$: therefore $\frac{16}{49} = \cdot32653061\frac{11}{49}$.

Hence $\frac{1}{49} = \cdot02040816\mathbf{32653061}\frac{11}{49}$: therefore $\frac{11}{49} = \cdot2244897959183673\frac{23}{49}$.

Hence $\frac{1}{49} = \cdot0204081632653061\mathbf{2244897959183673}\frac{23}{49}$: therefore $\frac{23}{49}$ $= \cdot46938775510204081632653061224489\frac{39}{49}$.

Hence $\frac{1}{49} = \cdot02040816326530612244897959183673\mathbf{4693877551}020408$, &c.

We see by inspection that the value of $\frac{1}{49}$ is now complete, for the figures of the decimal begin to recur at the 43rd figure.

**Second.**—*Find the value of* $\frac{13}{49}$ *by multiplying the value of* $\frac{1}{49}$ *by* 13.

$$\text{Then } \tfrac{13}{49} = \cdot\dot{0}2040816326530612244897959183673469387755\dot{1} \times 13$$
$$= \cdot\dot{2}6530612244897959183673469387755102040816\dot{3}.$$
$$\therefore \tfrac{1}{100} \text{ of } 2\tfrac{13}{49} = \tfrac{1}{100} \text{ of } 2\cdot\dot{2}6530612244897959183673469387755102040816\dot{3}$$
$$= \cdot02\dot{2}6530612244897959183673469387755102040816\dot{3},\ \textit{Ans.}$$

## 86. Circulating Decimals are of two kinds, viz.—

(1.) PURE CIRCULATING DECIMALS, *i.e.* those which recur from the beginning;

Thus, ·444, &c.; 232323, &c.; 040904090409, &c.; which are respectively written thus, $\cdot\dot{4}$; $\cdot\dot{2}\dot{3}$; $\cdot\dot{0}40\dot{9}$.

(2.) MIXED CIRCULATING DECIMALS, *i.e.* those which do not begin to recur till after a certain number of figures;

Thus, ·25343434, &c.; 3425555, &c.; 314200920092009, &c.; which are respectively written thus, $\cdot25\dot{3}\dot{4}$; $\cdot342\dot{5}$; $\cdot314\dot{2}00\dot{9}$.

N.B.—The portion of the decimal which is repeated is called the *Period* or *Repetend.*

## 87. To reduce a pure circulator to a vulgar fraction.

**Ex. 1.** *Reduce* $\dot{7}$ *to a vulgar fraction.*

Let $x =$ the value of the given fraction;
Then $x = \cdot77777$, &c.;
$\therefore\ 10x = 7\cdot7777$, &c.

Subtracting downwards we get

$10x - x = 7\cdot7777$, &c., $- \cdot7777$, &c.,
*i.e.* $9x = 7$;
$\therefore\ x = \frac{7}{9}$, *Answer.*

**Ex. 2.** *Reduce* $\dot{3}0\dot{7}$ *to a vulgar fraction.*

Let $x =$ the value of the given fraction;
Then $x = \cdot307307307$, &c.;
$\therefore\ 1000x = 307\cdot307307307$, &c.;

Subtracting downwards we get—

$1000x - x = 307\cdot307307$, &c. $- \cdot307307$, &c.;
*i.e.* $999x = 307$;
$\therefore\ x = \frac{307}{999}$, *Answer.*

Hence the reason of the following rule—

**RULE.—To reduce a pure circulator to a vulgar fraction, take the repetend for a numerator, and, for the denominator, as many nines as there are figures in the repetend.**

## 88. To reduce a mixed circulating decimal to a vulgar fraction.

**Ex. 1.** *Reduce* $03\dot{4}\dot{2}$ *to a vulgar fraction.*

Let $x =$ the value of the given fraction;
Then $x = \cdot03424242$, &c.;
$\therefore\ 10000x = 342\cdot424242$, &c.;
and $100x = 3\cdot424242$, &c.;
Subtracting, we get
$10000x - 100x = 342\cdot424242, \&c. - 3\cdot424242, \&c. = 342 - 3$;
*i.e.* $9900x = 342 - 3 = 339$;

$$\text{Therefore } x = \frac{342-3}{9900} = \frac{339}{9900}, \textit{ Answer.}$$

**Ex. 2.** *Reduce* $\cdot537\dot{2}1\dot{6}$ *to a vulgar fraction.*

Let $x =$ the value of the given fraction;
Then $x = \cdot537216216216$, &c.;
$\therefore\ 1000000x = 537216\cdot216216216$, &c.;
and $1000x = 537\cdot216216216$, &c.;
Subtracting, we get
$1000000x - 1000x = 537216\cdot216216, \&c. - 537\cdot216216, \&c. = 537216 - 537$;
*i.e.* $999000x = 537216 - 537$;

$$\text{Therefore } x = \frac{537216-537}{999000} = \frac{536679}{999000}, \textit{ Answer.}$$

Hence the reason of the following rule—

**RULE.—To reduce a mixed circulator to a vulgar fraction, take the number expressed by the figures of the decimal which do not recur, and take it from the number expressed by all the figures of the decimal; the difference is the numerator of the required vulgar fraction. For the denominator, take a nine for every figure in the decimal which does recur, and on the right of these nines place a cipher for every figure in the decimal which does not recur—**

$$\text{Thus, } \cdot03\dot{4}\dot{2} = \frac{342-3}{9900} = \frac{339}{9900} = \frac{113}{3300}.$$

$$\cdot537\dot{2}1\dot{6} = \frac{537216-537}{999000} = \frac{536679}{999000} = \frac{19877}{37000}.$$

**N.B.**—The rule here given may be applied to the reduction of a mixed number, consisting of a whole number and a circulating decimal, which may be either pure or mixed, thus—

**Ex. 1.** $3\cdot\dot{4}\dot{9} = \dfrac{349 - 3}{99} = \dfrac{346}{99}.$

**Ex. 2.** $170\cdot3\dot{1}6\dot{4} = \dfrac{1703164 - 1703}{9990} = \dfrac{1701461}{9990}.$

## Exercise 26.

### Reduction of Decimals to Vulgar Fractions, and Reduction of Vulgar Fractions to Decimals.

1. Reduce to vulgar fractions in their lowest denominations—

·4 ; ·003 ; ·043 ; ·318 ; ·03106 ; ·00091 ; ·71436 ; ·00105.

2. Reduce to decimals—

$\frac{2}{5}$ ; $\frac{18}{100}$ ; $\frac{1}{2}$ ; $\frac{3}{20}$ ; $\frac{4138}{400000}$ ; $\frac{601}{10000000}$ ; $\frac{8106}{150000000}$ ; $\frac{27}{25000}$.

3. Reduce to vulgar fractions in their lowest denominations—

2·3 ; 40·09 ; 2000·064 ; 50·31 ; 26·00745 ; ·10080·00801.

4. Reduce to decimals—

$4\frac{1}{5}$ ; $12\frac{3}{4}$ ; $17\frac{1}{20}$ ; $106\frac{1}{4000}$ ; $20\frac{36}{11430}$ ; $10007\frac{4}{160400}$.

5. Reduce to vulgar fractions in their lowest terms—

$7\cdot\dot{4}\dot{8}$ ; $106\cdot\dot{0}\dot{9}$ ; $10\cdot\dot{6}$ ; $7\cdot\dot{9}$ ; $\cdot\dot{9}\dot{5}$ ; $30\cdot\dot{6}\dot{2}$ ; $20\cdot53\dot{1}\dot{7}$ ; $010\dot{1}\dot{6}$ ; $\cdot51\dot{4}1\dot{3}$ ; $20\cdot\dot{7}\dot{3}$ ; $8\cdot1\dot{3}\dot{6}$ ; $2000\cdot0908\dot{6}$ ; $7030\cdot\dot{8}\dot{2}$ ; $106\cdot90\dot{8}0\dot{9}$.

6. Reduce to decimals—

$\frac{3}{7}$ ; $\frac{4}{11}$ ; $\frac{51}{90}$ ; $\frac{8}{900}$ ; $\frac{17}{99}$ ; $\frac{108}{13}$ ; $2\frac{1}{700}$ ; $10\frac{18}{91000}$ ; $1\frac{4}{1900}$ ; $\dfrac{\frac{3}{4}}{1\frac{3}{4}}$ ; $\dfrac{4}{20\frac{1}{4}}$ ; $\dfrac{8\frac{1}{3}}{5\frac{2}{3}}$ ; $\dfrac{1\frac{7}{9}}{9\frac{4}{9}}$ ; $\dfrac{7\cdot1}{9}$ ; $\dfrac{2\frac{15}{19}}{1100}$ ; $\dfrac{517}{25\cdot\dot{6}}$ ; $\dfrac{3\frac{9}{11}}{9\frac{6}{11}}$ ; $\dfrac{21\frac{15}{16}}{292\frac{1}{2}}$ ; $\dfrac{24}{112\frac{16}{17}}$ ; $\dfrac{240 + 60\frac{1}{2} + 14\cdot5}{14 \times 226\cdot\dot{8}}$ ; $\dfrac{813}{512}$ ; $1\frac{1}{252}$ ; $\dfrac{807}{216}$ ; $14\frac{180}{231}$ ; 720 ÷ 4704.

## I. Addition and Subtraction of Circulating Decimals.

**89. Addition and Subtraction of Circulating Decimals** is generally performed by repeating the period as many times as seems sufficient to insure the required degree of accuracy, thus—

**Ex. 1.** *Add together* $1\cdot\dot{7}$, $36\cdot04\dot{9}$, $72\cdot0\dot{6}38\dot{1}$, *and* $1073\cdot248$, *the answer to be correct to five places of decimals.*

$1\cdot7777777$, &c.
$36\cdot0499999$, &c.
$72\cdot0638163$, &c.
$1073\cdot248$

$1183\cdot1395939$, ∴ *Answer*, **$1183\cdot13959$.**

Here the decimal is in each case carried out to *seven* places, in order to insure that *five* places of the sum shall be correct.

**Ex. 2.** *Find the value of* $147\cdot3\dot{4}2\dot{6} \sim 29\cdot\dot{7}2\dot{8}$ *correct to eight places of decimals.*

$$147\cdot3\dot{4}2\dot{6} = 147\cdot3426426426, \text{ \&c.}$$
$$29\cdot\dot{7}2\dot{8} = 29\cdot7287287287, \text{ \&c.}$$

Subtracting, we have $117\cdot6139139139$, ∴ *Answer*, **$117\cdot6\dot{1}3\dot{9}$.**

## II. Multiplication and Division of Circulating Decimals.

**90. Multiplication and Division of Circulating Decimals** may also be performed by carrying out the repetend, but these operations are more usually performed by reducing the decimals to vulgar fractions, then multiplying or dividing as in vulgar fractions, and reducing the results once more to decimals.

**Ex. 1.** *Multiply* $2{\cdot}\dot{3}\dot{4}$ *by* $5{\cdot}4$.

**The value of** $2{\cdot}\dot{3}\dot{4} \times 5{\cdot}4 = \dfrac{234-2}{99} \times 5\frac{4}{10} = \dfrac{232}{\underset{11}{\cancel{99}}} \times \dfrac{\overset{3}{\overset{\cancel{0}}{\cancel{54}}}}{\underset{5}{\cancel{10}}} = \dfrac{696}{55} = 12\frac{36}{55}.$

**Ex. 2.** *Multiply* $107{\cdot}2\dot{2}\dot{5}$ *by* ${\cdot}01\dot{4}3\dot{1}$.

**The value of** $107{\cdot}2\dot{2}\dot{5} \times {\cdot}01\dot{4}3\dot{1} = \dfrac{107225-1072}{990} \times \dfrac{1431-1}{99900}$ **(§ 88)** $=$

$$\dfrac{\overset{2869}{\cancel{106153}}}{\underset{9}{\cancel{990}}} \times \dfrac{\overset{13}{\cancel{1430}}}{\underset{2700}{\cancel{99900}}} = \dfrac{37297}{24300} = 1\frac{12997}{24300} =$$

$$1{\cdot}53\dot{4}855967078189300411522633\dot{7}\dot{4}.$$

**N.B.—The latter part of the above example furnishes a good illustration of § 85, Ex. 3, thus—**

***To express*** $\dfrac{12997}{24300}$ ***as a decimal.***

**Here,** $\dfrac{12997}{24300} = \dfrac{12997}{243 \times 100} = \dfrac{1}{100}$ **of** $\dfrac{12997}{243}$ ;

**Now,** $\dfrac{12997}{243} = \dfrac{12997}{9 \times 9 \times 3}$ ;

$$\therefore 243 \left\{ \begin{array}{l} 9 \,|\, \underline{12997} \\ \quad 9 \,|\, \underline{1444{\cdot}\dot{1}} \\ \qquad 3 \,|\, \underline{160{\cdot}\dot{4}5679012\dot{3} \quad 456790123 \quad 456790123} \\ \qquad\qquad 53{\cdot}485596707 \quad 818930041 \quad 15226337\dot{4} \\ \qquad\qquad 1 \qquad\qquad 2 \qquad\qquad 0 \qquad\qquad 1 \end{array} \right.$$

**We thus find that** $\dfrac{12997}{243} = 53{\cdot}\dot{4}85596707818930041152263374\dot{}$ ;

$$\therefore \dfrac{12997}{243 \times 100} = {\cdot}53\dot{4}85596707818930041152263 3\dot{7}\dot{4}.$$

**Ex. 3.** *Divide* $84{\cdot}332\dot{4}7\dot{6}$ *by* $12{\cdot}734$.

**The value of** $84{\cdot}332\dot{4}7\dot{6} \div 12{\cdot}734 = \dfrac{84332476-84332}{999000} \div \dfrac{12734}{1000} =$

$$\dfrac{\overset{6616}{\cancel{84248144}}}{999\cancel{000}} \times \dfrac{\cancel{1000}}{\cancel{12734}} = \dfrac{6616}{999} = 6\frac{622}{999} = 6{\cdot}\dot{6}2\dot{2}.$$

**Ex. 4.** *Divide* $3{\cdot}41\dot{4}$ *by* $93{\cdot}\dot{1}\dot{2}$.

The value of $3{\cdot}41\dot{4} \div 93{\cdot}\dot{1}\dot{2} = \frac{3414-341}{900} \div \frac{9312-93}{99} = \frac{3073}{900} \div \frac{9219}{99}$

$$= \frac{\cancel{3073}}{\underset{100}{\cancel{900}}} \times \frac{\overset{11}{\cancel{99}}}{\underset{3}{\cancel{9219}}} = \frac{11}{300} = {\cdot}03\dot{6}.$$

## Exercise 27.

### Exercises in Recurring Decimals.

1. Find the sum of $9{\cdot}\dot{6}$, $27{\cdot}08\dot{3}$, $2{\cdot}\dot{0}1\dot{9}$, $160{\cdot}\dot{4}1\dot{0}$, $217{\cdot}5\dot{4}3\dot{8}$.

2. Find the value of $28{\cdot}\dot{7}\dot{1} + 4{\cdot}06\dot{0}7\dot{0} + 10{\cdot}1\dot{1}4\dot{3} + {\cdot}00362\dot{8}\dot{4}$.

3. What number must be added to $41{\cdot}\dot{3}1\dot{9}$ to make it $71{\cdot}\dot{2}\dot{4}$?

4. How many must be taken from $28{\cdot}\dot{1}01\dot{3}$ to leave $9{\cdot}016\dot{2}\dot{4}$?

5. By what number is $16{\cdot}41\dot{0}\dot{3}$ exceeded by $100{\cdot}4$?

6. Find the difference between $19{\cdot}10\dot{9}\dot{3}$ and $19{\cdot}2$.

7. From $29{\cdot}3 + {\cdot}\dot{0}16\dot{3} + 100{\cdot}2\dot{3}\dot{7}$ take $2{\cdot}819 + 16{\cdot}\dot{3}\dot{4} + 17{\cdot}1\dot{0}9\dot{1}$.

8. What number must be added to the sum of $7{\cdot}3\dot{6}\dot{2}$, $14{\cdot}18\dot{2}\dot{4}$, ${\cdot}\dot{5}$, and ${\cdot}74\dot{6}$ to make it equal to the difference between ${\cdot}01\dot{9}7\dot{4}$ and 1974?

9. Find, correct to five places of decimals, the value of $19{\cdot}34\dot{6} \times 7$; $213 \times 83{\cdot}\dot{9}\dot{1}$; $10{\cdot}3 \times 9{\cdot}\dot{4}\dot{8}$; $71 \times 3{\cdot}\dot{4} \times 60{\cdot}78$; $1{\cdot}8 \times 4{\cdot}725 \times 36 \times 9{\cdot}17\dot{1}0\dot{5}$; $18{\cdot}02 \times 71{\cdot}103 \times {\cdot}\dot{4}\dot{9}$.

10. Find, correct to seven places of decimals, the value of $17{\cdot}0\dot{1} \div 3$; $92 \div 4{\cdot}3\dot{5}$; $81{\cdot}9\dot{1} \div 7{\cdot}\dot{6}$; $3{\cdot}\dot{5}\dot{4} \div 385\dot{4}$; $7{\cdot}\dot{1}4285\dot{7} \div 19{\cdot}5$.

11. Reduce $2\frac{27}{35}$ to a decimal form, and $5{\cdot}\dot{3}2\dot{4}$ to a fractional form.

12. Reduce to vulgar fractions $3{\cdot}4\dot{9}$, $3{\cdot}49$, and $3{\cdot}\dot{4}\dot{9}$.

13. Simplify, correct to ten places of decimals,

$$\frac{29{\cdot}314 + {\cdot}063\dot{8} + 19}{27 - 13{\cdot}41\dot{3}};\ \frac{16{\cdot}01 \times 7{\cdot}08 \times {\cdot}09\dot{3}}{81 \div 2{\cdot}\dot{7}};\ \frac{83 - 14{\cdot}\dot{6} + 4{\cdot}\dot{5}}{30{\cdot}\dot{7}4\dot{9}};$$

$$\frac{4{\cdot}\dot{2}1\dot{8} \times {\cdot}55 \times 16{\cdot}72}{{\cdot}\dot{2}0\dot{5} \times {\cdot}88 \times 3{\cdot}6} \div \frac{{\cdot}7 \times {\cdot}\dot{6}\dot{3}}{{\cdot}\dot{0}18\dot{1} \times 40};\ \frac{1\frac{1}{3} + 2\frac{2}{7} + 1\frac{1}{4} - 3}{7 \times 19{\cdot}\dot{4}2857\dot{1}} \times \frac{2\frac{1}{3}}{3}.$$

14. Find a decimal fraction equivalent to each of the following values—

$$\frac{107{\cdot}2\dot{2}\dot{5}}{{\cdot}01\dot{4}3\dot{1}};\ \frac{19}{42};\ {\cdot}23\dot{4}\dot{8} \times {\cdot}0\dot{1}8\dot{5} \times 6;\ \frac{7}{121};\ \frac{430{\cdot}65}{2466{\cdot}45}.$$

### 91. To reduce a given quantity to the decimal of another given quantity.

**Ex. 1.** *Reduce £4, 16s. 2½d. to the decimal of £6.*

By § 83 we find that 2 farthings = $\frac{2}{4}$ of a penny = $\frac{1}{2}$ of a penny = ·5 of a penny;

$$\left.\begin{array}{l}\therefore 2\tfrac{1}{2}\text{d.} = 2{\cdot}5 \text{ of a penny};\\ \text{But 1s.} = 12 \text{ pence};\end{array}\right\} \therefore 2{\cdot}5 \text{ of a penny} = \frac{2{\cdot}5}{12} \text{ of a shilling} = {\cdot}208\dot{3}\text{s.}$$

$$\left.\begin{array}{l}\therefore 16\text{s. } 2\tfrac{1}{2} = 16{\cdot}208\dot{3}\text{s.};\\ \text{But £1} = 20\text{s.};\end{array}\right\} \therefore 16{\cdot}2083\text{s.} = \text{£}\frac{16{\cdot}2083}{20} = \text{£}{\cdot}81041\dot{6}.$$

$$\left.\begin{array}{l}\therefore \text{£4, 16s. } 4\tfrac{1}{2} = \text{£}4{\cdot}81041\dot{6};\\ \text{But £6} = \text{£6};\end{array}\right\} \therefore \text{£}4{\cdot}81041\dot{6} = \frac{4{\cdot}81041\dot{6}}{6} \text{ of £6}$$

$$= {\cdot}801736\dot{1} \text{ of £6, } \textit{Answer.}$$

Hence the reason of the following rule—

**RULE 1.—Take the term in the lowest denomination of the first given quantity, and express it as a decimal of the unit of the next higher denomination. To this decimal prefix the whole numbers, if any, of this higher denomination which occur in the first given quantity, and reduce this mixed number to a decimal of the unit of the next higher denomination, and so on, till the first given quantity is brought to the decimal of the second given quantity.**

The usual method of working is as follows—

**Ex. 1.** *Reduce £4, 16s. 2½d. to the decimal of £6.*

It may here be pointed out—

1st. That the divisors are

**4**, because four farthings = 1 penny;

**12**, because twelve pence = 1 shilling;

**20**, because twenty shillings = 1 pound;

**6**, because £6 is the denomination to which the whole is to be reduced.

2nd. That the integers (indicated by darker figures) express the number of units in each denomination of the first given quantity.

$$\begin{array}{r|l}
4 & \mathbf{2}{\cdot} \text{ farthings} \\ \hline
12 & \mathbf{2}{\cdot}5 \text{ pence} \\ \hline
20 & \mathbf{16}{\cdot}208\dot{3} \text{ shillings} \\ \hline
6 & \mathbf{4}{\cdot}81041\dot{6} \text{ of a pound} \\ \hline
\end{array}$$

*Answer*, ·801736$\dot{1}$ of £6.

When the above rule is impracticable because of the occurrence of several terms in the second given quantity, the following must be used—

**RULE 2.—Express the first quantity as a vulgar fraction of the second (§ 71), and then reduce this vulgar fraction to a decimal.**

**Ex.** *Reduce* 4*s*. 9½*d*. *to the decimal of* 7*s*. 6¼*d*.

$$\frac{4s.\ 9\tfrac{1}{2}d.}{7s.\ 6\tfrac{1}{4}d.} = \frac{230f.}{361f.} = \cdot637, \&c., \textit{ Answer}.$$

## 92. To reduce a decimal of any given denomination to its proper value.

**Ex. 1.** *Find the value of* ·8625 *of* £1.

·8625 of £ = ·8625 of 20s. = ·8625s. × 20.

By the rule for multiplication of decimals, we find the value of 20 times ·8625s. thus—

$$\begin{array}{r} \cdot8625 \text{ of a shilling} \\ 20 \\ \hline 17{\cdot}25\cancel{0}\cancel{0} \text{ of a shilling} = 17 \text{ shillings} + \cdot25 \text{ of a shilling.} \end{array}$$

In a similar manner we have—

·25 of 1s. = ·25 of 12d. = ·25d. × 12.

By the rule for multiplication of decimals, we find the value of 12 times ·25d. thus—

$$\begin{array}{r} \cdot25 \text{ of a penny} \\ 12 \\ \hline 3{\cdot}\cancel{0}\cancel{0} \text{ of a penny} = 3 \text{ pence.} \end{array}$$

Therefore the whole value of ·8625 of £1 = 17s. 3d., *Answer*.

Hence the reason of the following rule—

**RULE.—Multiply the given decimal by the number which expresses the number of units of the next lower denomination which are required to make one unit of the given denomination, and mark off in the product as many decimal places as there are in the given decimal. The figures on the *left* of the decimal point will be integers of the lower denomination, and those on the *right* of it will be decimals of that denomination, and the value of this decimal will be found by a repetition of the former process, thus—**

**Ex. 2.** *Find the value of ·375 of £1 (i.e. of ·375).*

$$\begin{array}{r} \pounds\cdot375\\ 20\\ \hline \text{s. } 7\cdot500\\ 12\\ \hline \text{d. } 6\cdot000 \end{array}$$

*Answer*, **7s. 6d.**

N.B.—Since ciphers on the right of a decimal have no value, they may be cut off as fast as they occur, thus—

$$\begin{array}{r} \pounds\cdot375\\ 20\\ \hline \text{s. } 7\cdot5\cancel{00}\\ 12\\ \hline \text{d. } 6\cdot\cancel{0} \end{array}$$

**Ex. 3.** *Find the value of* 17·03 *of* 2·6 *of* £1.

The value of 17·03 of 2·6 of £1 = (17·03 × 2·6) of £1 = £44·278.

Therefore,
$$\begin{array}{r} \pounds44\cdot278\\ 20\\ \hline \text{s. } 5\cdot56\cancel{0}\\ 12\\ \hline \text{d. } 6\cdot72\\ 4\\ \hline \text{f. } 2\cdot88 \end{array}$$

*Answer*; **£44, 5s. 6½d.** +, or, more correctly, **£44, 5s. 6·72d.**

**Ex. 4.** *Find the value of* 19·2125 *of* £1, 12*s.* 6*d.*

*First method*—

£1, 12s. 6d. = £1⅝ = £1·625.

∴ 19·2125 of £1, 12s. 6d. = 19·2125 of £1·625 = (19·2125 × 1·625) of £1.

Therefore,
$$\begin{array}{r} \pounds19\cdot2125\\ 1\cdot625\\ \hline 960625\\ 384250\phantom{0}\\ 1152750\phantom{00}\\ 192125\phantom{000}\\ \hline \pounds31\cdot2203125\\ 20\\ \hline \text{s. } 4\cdot40625\cancel{00}\\ 12\\ \hline \text{d. } 4\cdot875\cancel{00} \end{array}$$

*Answer*; **£31, 4s. 4·875d.**

*Second method*—

£1, 12s. 6d. = 390d.

Therefore,
$$\begin{array}{r} 19\cdot2125\\ 390\text{d.}\\ \hline 17291250\\ 576375\phantom{0}\\ \hline \text{d. } 7492\cdot875\cancel{0} \end{array}$$

= d. 7492·875
= 624s. 4·875d.
= **£31, 4s. 4·875d.**, *Answer.*

*Third method* (founded on the principle of "**Practice,**" § 96) —

$$\begin{array}{lll}
 & 19{\cdot}2125 \text{ of } £1 & = \qquad\qquad\qquad\quad £19{\cdot}2125 \\
\therefore & 19{\cdot}2125 \text{ of } 10s. & = £19{\cdot}2125 \div 2 = £\ 9{\cdot}60625 \\
\therefore & 19{\cdot}2125 \text{ of } 2s.\ 6d. & = £9{\cdot}60625 \div 4 = \underline{£\ 2{\cdot}4015625} \\
\therefore & 19{\cdot}2125 \text{ of } £1,\ 12s.\ 6d. & = \qquad\qquad\qquad\quad £31{\cdot}2203125
\end{array}$$

$$\begin{array}{r}
£31{\cdot}2203125 \\
\underline{20} \\
s.\ 4{\cdot}4062500 \\
\underline{12} \\
d.\ 4{\cdot}87500
\end{array}$$

*Answer*, £31, 4s. 4·875d.

**Ex. 5.** *Find the value of* $2{\cdot}04\dot{6}$ *of* £1.

$$\begin{array}{r}
£2{\cdot}0466666, \text{ \&c.} \\
\underline{20} \\
s.\ 0{\cdot}9333320 \\
\underline{12} \\
d.\ 11{\cdot}1999840
\end{array}$$

Here it is evident that if the sixes had been carried out farther, the "20" in the line of *shillings* would have been "33," in fact, the "3" would have circulated, and the "840" in the line of *pence* would have been "999," in fact, the "9" would have circulated, as may be seen below—

$$\begin{array}{rl}
£2{\cdot}0466666666 & \\
\underline{20} & \\
s.\ 0{\cdot}93333333 & = £2,\ 0s.\ 11{\cdot}1\dot{9}d. \\
\underline{12} & = £2,\ 0s.\ 11{\cdot}2d., \textit{ Answer.} \\
d.\ 11{\cdot}1999999 &
\end{array}$$

N.B. 1.—*When the figure "9" circulates by itself, it is usual to omit it, and add* 1 *to the figure on its left*—

Thus, $\cdot 7\dot{9}$ is called ·8; $2{\cdot}\dot{9}$ is called 3·, &c., because

$$\cdot 7\dot{9} = \frac{79 - 7}{90}\ (\S\ 88) = \frac{72}{90} = \frac{8}{10} = \cdot 8.$$

$$2{\cdot}\dot{9} = 2\tfrac{9}{9}\ (\S\ 87) = 2 + 1 = 3.$$

N.B. 2.—In reducing such quantities as that in Ex. 5, it is often preferable to reduce the decimal to a vulgar fraction, and then find its value by § 81.

$$\text{Thus, } 2{\cdot}04\dot{6} \text{ of } £1 = \frac{2046 - 204}{900} \text{ of } 20s. = \frac{\overset{614}{\cancel{1842}}}{\underset{\underset{15}{\cancel{45}}}{\cancel{900}}} \times \frac{\cancel{20}}{1}s. =$$

$$\frac{614}{15}s. = 40\tfrac{14}{15}s. = £2,\ 0s.\ 11\tfrac{1}{5}d.$$

## Exercise 28.

1. Reduce the following sums of money to the decimal of £1—
12s. 6d.; 10s. 6d.; 10s. 3d.; 17s. 8d.; 15s. 6d.; 17s. 6d.; £1, 10s. 8d.; 19s. 7½d.; 18s. 4¾d.; £14, 9s. 8d.; £21, 0s. 3d.; £17, 19s. 8¾d.

2. Reduce the following quantities to the decimal of a ton—
2 cwt. 3 qrs.; 14 cwt. 1 qr.; 10 cwt. 1 qr. 14 lbs.; 13 tons 6 cwt. 7 lbs.; 1 ton 13 cwt.; 17 tons 18 cwt. 3½ lbs.; 17 cwt. 1 qr. 10 lbs.

3. Express (correct to *seven* places of decimals) the following fractions as decimals—

$$\frac{\text{£2, 12s. 6d.}}{\text{£5}};\ \frac{\text{£17, 16s. 9d.}}{\text{£31, 10s.}};\ \frac{\frac{3}{4}\text{ of }\frac{8}{13}\text{ of £27, 6s.}}{\text{5 times }\frac{14}{15}\text{ of £47, 5s.}};\ \frac{\text{3 miles 4 fur.}}{\text{1 league}};$$

$$\frac{\text{101 yds. 2 ft. 9 in.}}{\frac{1}{10}\text{ of a degree}};\ \frac{\text{17 tons 18 cwt. 4 lbs.}}{\text{18 tons 17 cwt. 6 lbs.}};\ \frac{\text{13 lbs. 10 oz.}}{\text{1 qr. 14 lbs. 5 oz.}};\ \frac{\text{1 ton}}{\text{16 lbs. 4 oz.}};$$

$$\frac{\text{2 sq. yds. 10 sq. in.}}{\text{1 sq. pole}};\ \frac{\text{the area of a room 18 ft. by }16\frac{1}{2}\text{ ft.}}{\text{the area of a room 101 ft. by }19\frac{1}{4}\text{ ft.}}$$

4. What decimal part of £31, 8s. 6d. is £19, 12s. 6d.?

5. 81 gals. 2 qts. 1⅕ pints is what decimal part of 17 gals. 3⅛ pints?

6. Find the value of the following expressions—
£·8; £2·4; £17·375; £·625; £10·0625; £2·8$\dot{3}$; £101·375; 19·6 of 8·5 of £1; 71 × 4·8 of £5; 9·01 × 50·08 of £600; 3⅓ of 91·8 of £16½; 17·5 × 2·03 of £10, 10s.; 19·5 of £71, 1s. 8d.; ⅘ of $\frac{9}{100}$ of 40 times £8·625; 2·106 × ·2106 × ·375 of a ton; 65·0102 × ·00625 of 1000 guineas; 7·7816 × ·03125 × 8000 florins.

7. Find the value of 8·1$\dot{5}$ of 12s. 2¼d.; 1·$\dot{7}1428\dot{5}$ of 18 days 23 hrs.; 81·$\dot{6}\dot{3}$ of 27 a. 2 r. 33 p.; ·0$\dot{5}\dot{4}$ of 7 times ½ of £8, 5s.; ·5 of ·2$\dot{7}$ of 18 guineas − 15s. 6d.

8. Reduce 18 hrs. 2¼ mins. to the decimal of 2 wks. 4 days 2 hrs. 15 mins.

9. What decimal of 4 lbs. 10 oz. 3 dwt. is 3 lb. 2 oz. 15 dwt. 8 grs.?

10. What decimal fraction of a ton must be added to 17 cwt. 3 qrs. 9 lbs. to make it 18 cwt. 101 lbs.?

11. What decimal fraction of itself must be added to nineteen-twentieths of 1⅓ tons to make it equal to thirteen-thirtieths of ·1$\dot{6}$ of 4·$\dot{8}$ of 20 tons 1 qr.?

12. Reduce 11 tons 7 cwt. 3 qrs. 14 lbs. to the decimal of 5 tons.*

13. Find the value of ·0135 of £25, 10s. 6d.

14. Find the value of 173·25 yards at 5·25s. per yard.

15. What length added to $\frac{7}{8}$ of a mile would amount to ·865 of a league?

16. Express as a simple decimal the difference between $\frac{3}{5}$ of $\frac{11}{12}$ and $\frac{7}{8}$ of $\frac{9}{14}$.

## The Metric System.

**93.** *The advantage possessed by the Metric System* over every other system of weights and measures depends upon the fact that it is entirely a decimal system. This advantage consists chiefly in the following particulars :—

I. *Reduction is not, as in the English system, a long and involved process ; it is performed by merely shifting the decimal point, as in the following examples—*

**Ex. 1.** *Reduce* 19 *dekametres* 4 *metres to metres.*

$$\begin{aligned} 19 \text{ dekametres} &= (19 \times 10) \text{ metres} = 190 \text{ metres} \\ 4 \text{ metres} &= \quad\ \ 4 \text{ metres} \end{aligned}$$

∴ By addition, 19 dekam. 4 met. = **194 metres,** *Answer.*

But this result might have been obtained by the following rule—

**RULE.—To express a given quantity in terms of any unit in the table in which that quantity is given :—Write the names of all the terms of that table in their proper order, and underneath each term place the given number of units. Place a dot (decimal point) after the term in which the given quantity is to be expressed.**

**Ex. 1.** *Reduce* 19 *dekam.* 4 *met. to metres.*

| dekam. | met. |
|---|---|
| 19 | 4· |

*Answer,* **194 metres.**

Here we first write the terms
dekam. met.,
then we place "19" beneath "dekam.," and "4" beneath "met. ;" lastly, we place a dot after the "4," and we have the
*Answer*, **194 metres.**

---

* The following exercises are taken from various examination papers.

**Ex. 2.** *Reduce* 10 *kilograms* 7 *dekagrams* 6 *grams* 3 *decigrams to grams.*

| kilog. | hectog. | dekag. | gr. | decig. |
|---|---|---|---|---|
| 10 | 0 | 7 | 6 · | 3 |

*Answer*, **10076·3.**

Here the dot is placed after the "6," because the given quantity is to be reduced to "grams." It should be noticed also that a cipher is inserted in the place of "hectograms," because there are no hectograms in the given quantity.

The reason of the above rule may perhaps be best explained by an example, thus—

**Ex. 3.** *Reduce* 9 *dekam.* 4 *met.* 6 *decim. to metres.*

May be also set thus—

*Reduce* 9 *dekam.* 4 *met.* 6 *decim. to the decimal of* 1 *met.*

A metre being the unit, it is evident that a "4" being put in the place of units will express 4 met., and therefore the decimal point must be put after "*metres*" in the given quantity; and, a dekam. being ten metres, a "9" in the place of *tens* will express 9 dekam. Also, a decim. being one-tenth of a metre, a "6" in the place of *tenths* will express 6 decim.

II. *The operations known as the Compound Rules are reduced to Simple Rules*, as in the following examples—

**Ex. 4.** *Multiply* 6 *dekam.* 4 *met.* 3 *decim. by* 91, *and express the answer in metres.*

$$\begin{array}{r} \text{6 dekam. 4 met. 3 decim.} = 64{\cdot}3 \text{ met.} \\ 91 \\ \hline 643 \\ 5787\phantom{0} \\ \hline \end{array}$$

*Answer*, **5851·3 met.**

**Ex. 5.** *How many times is* 7 *met.* 3 *decim. contained in* 5 *kilom.* 9 *hectom.* 4 *dekam.* 2 *met.* 2 *decim.?*

$$\frac{\text{5 kilom. 9 hectom. 4 dekam. 2 met. 2 decim.}}{\text{7 met. 3 decim.}} = \frac{\text{59422 decim.}}{\text{73 decim.}} = 814.$$

*Answer*, **814 times.**

**Ex. 6.** *What is the weight of air in a room* 5 *metres long* 3 *metres wide and* 4 *metres high, if* 1 *cubic decimetre of air weighs* ·0018 *kilogrammes?*

The cubic content of the room = 5 met. × 3 met. × 4 met.
= 60 cub. met. = 60000 cub. decim.

Now 1 cub. decim. of air weighs ·0018 kilog.

Therefore, 60000 cub. decim. of air weigh ·0018 kilog. × 60000 =
**108 kilog.**, *Answer.*

**Ex. 7.** *A "florin" being* $\frac{1}{10}$ *of a £, a "cent"* $\frac{1}{100}$ *of a £, and a "mill"* $\frac{1}{1000}$ *of a £, express* £13, 12*s.* 7½*d. in £, fl., c., and m.*

By § 91, £13, 12s. 7½d. = £13·63125
= **£13 6 fl. 3 c. 1·25 m.**, *Answer.*

## Exercise 29.

### Exercises in the Metric System.

1. Add together 6 dekam. 4 met. 3 decim.; 8 met. 4 decim. 9 centim.; and 21 kilom. 3 hectom. 2 dekam. 7 met. 1 decim. 1 centim.

2. Find the sum of 21 myriam. 6 kilom. 3 met. 4 decim.; 17 myriam. 3 kilom. 8 hectom. 5 dekam. 3 met. 9 centim.; 17 met. 6 decim.; and 21 hectom. 6 dekam. 3 decim. 9 centim. 1 millim.

3. How many centim. are equivalent to the sum of 4 met. 6 decim.; 2 hectom. 6 dekam. 8 met. 19 centim.; and 38 dekam. 2 met.?

4. What is the sum of the following areas:—28 hectar. 7 ar. 2 deciar.; 3 ar. 9 deciar.; 17 hectar. 9 dekar. 8 ar. 31 centiar.; 101 ar. 18 centiar.?

5. By how many centigrams does 43 kilog. 8 hectog. 3 dekag. 6 gr. exceed 21 kilog. 7 hectog. 2 dekag. 8 gr.?

6. What weight must be added to 101 kilog. 6 hectog. 4 dekag. 31 centig. to make it equal to 200 myriag. 3 hectog. 6 gr. 7 decig.?

7. Find the difference between 10 dekag. 6 gr. 8 centig. and the sum of 9 gr. 21 centig.; 18 gr. 3 decig. 4 centig. 7 millig.; and 3 gr. 109 millig.

8. Multiply 61 dekast. 8 st. 31 centist. by 1801.

9. If 14 dekast. 3 st. be multiplied by 1725, and the product be reduced by 7 times 19 dekast. 8 st., what must be taken from the difference to leave a remainder of 1025 st.?

10. How many times must 3 lit. 4 decil. be taken from 2125 centil. to leave 11 lit. 5 centil.?

11. From what quantity must ·291226 of a lit. be taken to leave a quantity equal to 6·043 times 200·018 lit.?

12. A regiment of 8400 men was provided with beer. If 73·28 of a decil. was lost by leakage, how much was each man's portion, supposing that the total amount supplied to the regiment was 530 kilol.?

13. How many bottles of wine, each containing 220·11 centil., were exported from a vineyard which produced altogether 18 kilol., of which however 3735·912 decil. were lost by an accident?

## Exercise 30.

### Miscellaneous Exercises in Decimal Fractions. *

1. Multiply 7·686 by ·0084, and divide the product by ·00036.

2. Reduce 118·37 feet (1) to the decimal of a mile, (2) to the decimal of a furlong, and (3) to the decimal of a yard.

3. The population of England and Wales in 1861 was 20061725, and the area 58320 sq. miles. Find as far as three places of decimals the average number of persons living in a square mile.

4. Find the value of £3·89375, in £ s. d.; and reduce 19 cwt. 1 qr. 24 lb. 8 oz. to the decimal of 1 ton.

5. Multiply 70·048 by 3·23, and 2·07 by 3·105; add the products together, and take away ½ of 98·04717; what is the result?

6. Divide 4900 by ·07; multiply the quotient by ·63 and divide by ·049; what is the result?

7. Divide 3·5 by ·007, and multiply the result by ·0049 and then by ·02.

8. Reduce 3 qrs. 21 lb. to the decimal of a ton.

9. Find the sum of 81·7 at £4, 10s. 3¼d., 519·6 at £17·28, and 327·4 at £·68, each.

10. If 1 metre (= 39·3708 English inches) be the ten-millionth part of a quadrant of a meridian, how many miles are there in a meridian?

11. Reduce $3\frac{5}{7}$, $23\frac{13}{50}$, $5\frac{8}{11}$, $14\frac{9}{16}$, $8\frac{17}{25}$ to decimals, and add together the results.

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* These Exercises are taken in great measure from Examination Papers.

12. Reduce £116, 17s. 6d. to the form of a decimal of £1, and multiply the result by $1\cdot\dot{4}\dot{5}$.

13. Find the sum of 7·168 × 36·95 and 23892·128 ÷ 3·25.

14. Multiply 5·384 by ·00723; 20·7 by 500; ·072 by 50·34; add the results together; take $\frac{1}{8}$ of ·462 from the sum, and what will then remain?

15. Divide 500 by ·25, the quotient by ·025, and the second quotient by 50; what is the result?

16. Divide 7·619 by ·0019, and multiply the quotient by $\frac{3}{25}$ of ·00011569.

17. Reduce ·5 of 16s. 4½d. to the decimal of £1, 9s. 10¼d.

18. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375, and 2·46?

19. Reduce 14 gallons 2 quarts to the decimal of a barrel; and 1 min. 2¼ seconds to the decimal of $\frac{1}{28}$ of a lunar month.

20. Add together, 16·05, $3\cdot\dot{5}\dot{7}$ $2\cdot0\dot{6}$, 18·01375.

21. Find the value of 0·296 of 8 cwt. 3 qrs. 20 lb.

22. Multiply 5·178 by $\frac{9\cdot528}{2\cdot508}$.

23. Divide 76·57 by ·0019 and multiply the quotient by $\frac{3}{7}$ of ·0008568; and find the value of $\frac{73\cdot8}{\cdot0018}$ of $\frac{3}{19}$ of ·0009747.

24. Find the value of 39·39 × 78·78; 237·07 × 4·567; 55·6591 ÷ 1813; 230 ÷ ·016; 140·02564 ÷ 1·871; 406·8 ÷ 018; 3·12 × 2·001; ·01 ÷ ·00001; ·1872 ÷ ·0312; ·75 of £52, 6s. 8d.; ·525 of £366, 6s. 8d.; ·312 × ·20001; ·1 ÷ ·000001; 1872 ÷ ·312.

25. Simplify

$$\frac{\cdot315\times\cdot0315}{\cdot013}\quad\frac{\cdot213\times\cdot512}{\cdot0021};\quad\frac{\cdot064+12\cdot4+\cdot006}{\cdot02};\quad\frac{\cdot0001+7\cdot6+\cdot4}{\cdot3};\quad\frac{318}{\cdot4\dot{7}\dot{7}},$$

$$\frac{69\cdot2374\sim46\cdot7238\dot{8}6\dot{4}}{12\cdot5}\times46\cdot25;\quad\frac{153546\cdot12}{1\cdot308}\sim\frac{1535\cdot4612}{13080}.$$

26. When the oz. of gold costs £3·89, what is the cost of ·04 lb.?

27. The circumference of a circle is 3·14159 times its diameter. Find the circumference of circles whose diameters measure 13·7 feet, 1·96 yds., and 28·342 miles, respectively.

28. Find the product of the sum and difference of 21·8 and ·324.

29. What is the difference between the fifteenth and sixteenth parts of 297·6832?

30. What decimal of £1, is 17s. 9¾d.? How much greater or less is it than ·78965 of £1?

31. Reduce $\frac{27}{32}$ to a decimal, and 0·19712 to a vulgar fraction.

32. Find what decimal 10 lb. 8 oz. 16 dwts. 13 grs. is of 17 lb. 4 oz. 6 dwts. 16 grs.

33. Divide the difference of 487·25 and $51{\cdot}8\dot{3}$ by 20·9, and multiply the quotient by 0·192.

34. Find the value of £·00394875 ÷ 1·75.

35. Simplify $\frac{3{\cdot}471 \times 150{\cdot}61}{10}$; $\frac{2{\cdot}013 \times {\cdot}26}{{\cdot}25}$; ·0065 × ·018; $\frac{18{\cdot}93978}{{\cdot}43}$; $\frac{4{\cdot}4}{{\cdot}00011}$; $\frac{{\cdot}00084}{{\cdot}014}$; $\frac{2{\cdot}184 \times {\cdot}039}{{\cdot}0016}$; $\frac{70 - 6{\cdot}15}{{\cdot}005}$; $\frac{30 + 4{\cdot}15 + 7}{3} \times {\cdot}0068$; $\frac{(358{\cdot}8261 + 167{\cdot}2177) \times (167{\cdot}2177 \sim 358{\cdot}8261)}{463{\cdot}2326}$; $\frac{{\cdot}012}{{\cdot}005} + \frac{{\cdot}001}{1 \times {\cdot}01 \times 100}$.

36. Find the value of 3·0125 of 5s. 4½d., and of 7·18 of 5 oz. 15 dwts. 17 gr.

37. A "florin" being £$\frac{1}{10}$, a "cent" £$\frac{1}{100}$, and a "mill" £$\frac{1}{1000}$, express the following sums in £, fl., c., and m.:—£18, 12s. 6½d.; £12, 7s. 4¾d.; £32, 16s. 5¼d.; £6, 14s. 2½d.

38. Find the difference between $\frac{\frac{2}{3} \text{ of } {\cdot}005}{\frac{1}{12} \text{ of } {\cdot}345}$ and $\frac{2}{5} \times (\frac{11}{35} - \frac{1}{130}) \div ({\cdot}72 + \frac{11}{120})$.

39. What must be added to ·356 of £2, 17s. 10d. to make up $\frac{13}{15}$ of £8, 19s. 7½d?

40. Find the cost of covering a roof with lead at 18s. per cwt., on the following data:—the length of the roof is 43 feet, breadth 32 feet; the guttering is 57 feet long, and 2 feet wide; the former requires lead at 9·831 lbs., and the latter at 7·373 lbs. to the square foot.

41. Divide the sum of $(11\frac{13}{100} - {\cdot}35) \div ({\cdot}05 - \frac{1}{75})$ and $\frac{7}{125} + 6{\cdot}007$ by the difference of $\frac{2}{3}$ (1·35 − ·72) and 5·0004, and express the result as a decimal.

42. A cubic inch of water weighs 252·458 grains, and the weight of an imperial gallon of water is 10 lb. avoirdupois. Find (to three places of decimals) the number of cubic inches in an imperial gallon, there being 7000 grains in the lb. avoirdupois.

43. Multiply $\frac{3}{7}$ of ·175 by ${\cdot}\dot{2}8571\dot{4}$, and divide the result by ·00425.

44. Simplify $\frac{1}{3{\cdot}2} - \frac{2{\cdot}25}{9} + \frac{3{\cdot}625}{12} + \frac{{\cdot}\dot{7}1428\dot{5}}{4{\cdot}\dot{7}1428\dot{5}} + \frac{3 - 2{\cdot}\dot{3}}{4 - 3{\cdot}25}$.

45. What is the cost of ${\cdot}\dot{8}$ of 2·25 mile of paving at 3d. a yard?

46. If ${\cdot}\dot{7}1428\dot{5}$ of a stack of hay is worth £19, what is the value of seven such stacks?

47. Find the sum, difference, and product of 86·25 and 39·625, and divide the sum of the three results by 6·25.

48. Simplify

$$(1.)\quad \frac{\cdot17 \text{ of } (\cdot00617 - \cdot0053\dot{2}) + \cdot119 \times \cdot007}{\cdot17 \times \cdot17}.$$

$$(2.)\quad (1508\cdot77439641\dot{1}\dot{8} + 28\cdot0093 + 23\cdot310008\dot{1}) \div 22\cdot817.$$

$(3.)$ $\left[\left(28 \times 6\cdot09 - 31\cdot206 \div \dot{7} + \dfrac{\cdot019 \times \cdot013}{\cdot019 \div \cdot013} - 5 \text{ times } 33\cdot1474338 + \frac{4}{5}\right) \text{ of } 2\frac{1}{4}\right] \div \left\{3\,(8\cdot1 \times 6\cdot3 - 50\cdot72\dot{3} - \cdot16) + (\cdot1\dot{5} + \cdot\dot{2}\dot{5} + \cdot03\dot{5}) \times \cdot071 + 5\cdot1685\dot{0}1\dot{8}\right\}.$

49. Find what decimal multiplied by 8750 will give the sum of 17·5, $\frac{1}{100}$ of 86, $\frac{8}{700}$ of 1801, and 96·812.

50. What number is that which being reduced by $218\cdot019\dot{0}\dot{1}$ gives a remainder one thousand and eleven times greater than the product of 413·64 and 2·73?

51. $\frac{1}{31}\,(814\cdot0416 \times 2\cdot072 + 2\cdot983848 + 22\cdot318 + 37\cdot00176 + 300\cdot68124 - 49\cdot62)$.

## PRACTICE.

**94. Practice** is a compendious method of finding the value of a number of articles when the value of a unit of any denomination is given.

Thus, we may find by this rule the cost of 196 tons of sugar at £3, 10s. per ton; of 17 sq. yds. of carpet at 3s. 6d. per sq. ft.; of 12 tons of coal at 1s. 8d. per cwt., &c.

N.B. 1.—It is evident that all the results obtained by Practice may also be found by Compound Multiplication, but the former rule has the advantage of arriving at these results by shorter methods.

N.B. 2.—Practice is so called because its methods, being generally shorter and easier than those of Compound Multiplication, are usually employed in actual practice to find the cost of goods.

**95. Def. An aliquot part** (*i.e.* a fractional part) of a number is a part which is contained in that number an exact number of times.

Thus—4 is an aliquot part of 20, because it is contained in 20 an exact number of times.

**N.B.—Because, in calculating the prices of articles by practice, we use aliquot parts, it is sometimes said that "*Practice is the rule by which the value of articles is found by means of fractional, or aliquot, parts.*"**

**96.** Practice is divisible into two cases, **Simple Practice** and **Compound Practice.**

I. SIMPLE PRACTICE is the case in which the given number is expressed in the same denomination as the unit whose value is given.

**Thus—"*To find the cost of* 19 *tons of coal at* 17*s.* 6*d. per ton,*" is a sum in Simple Practice.**

**Ex. 1.** *Find the cost of* 324 *articles at* 16*s.* 8$\frac{1}{2}$*d. each.*

**£324 = the cost of 324 articles at £1 each.**

**Therefore,**

| | £ | s. | d. |
|---|---|---|---|
| **The cost of 324 articles at 10s. each = $\frac{1}{2}$ their cost at £1 each =** | **£162** | **0** | **0** |
| **Their cost at 5s. each = $\frac{1}{2}$ their cost at 10s. each =** | **81** | **0** | **0** |
| **Their cost at 1s. 3d. each = $\frac{1}{4}$ their cost at 5s. each =** | **20** | **5** | **0** |
| **Their cost at 5d. each = $\frac{1}{3}$ their cost at 1s. 3d. each =** | **6** | **15** | **0** |
| **Their cost at $\frac{1}{2}$d. each = $\frac{1}{10}$ their cost at 5d. each =** | **0** | **13** | **6** |
| **Therefore, by adding up the vertical columns, we have** | | | |
| **Their cost at 16s. 8$\frac{1}{2}$d. each =** | **£270** | **13** | **6** |

The usual form of writing the process is as follows—

| | | | | |
|---|---|---|---|---|
| | **£324** | | | **= the cost at £1 each.** |
| **10s. = $\frac{1}{2}$ of £1.** | **£162** | **0** | **0** | **= the cost at 10s. each.** |
| **5s. = $\frac{1}{2}$ of 10s.** | **81** | **0** | **0** | **= the cost at 5s. each.** |
| **1s. 3d. = $\frac{1}{4}$ of 5s.** | **20** | **5** | **0** | **= the cost at 1s. 3d. each.** |
| **5d. = $\frac{1}{3}$ of 1s. 3d.** | **6** | **15** | **0** | **= the cost at 5d. each.** |
| **$\frac{1}{2}$d. = $\frac{1}{10}$ of 5d.** | **0** | **13** | **6** | **= the cost at $\frac{1}{2}$d. each.** |
| **Therefore,** | **£270** | **13** | **6** | **= the cost at 16s. 8$\frac{1}{2}$d. each.** |

***Answer*, £270, 13s. 6d.**

**N.B.—In taking the *aliquot parts*, a little judgment should be exercised to prevent an awkward quantity from remaining at last—*e.g.*, we might have taken the aliquot parts in the above example thus—**

**10s. = $\frac{1}{2}$ of £1.**
**6s. 8d. = $\frac{1}{3}$ of £1.**

We should now have had ½d. to calculate for. But ½d. is $\frac{1}{160}$ of 6s. 8d., so that we should have been compelled to divide "the cost at 6s. 8d." by 160 to find "the cost at ½d." This would have been a comparatively awkward operation, and it will be found better to avoid such remainders when it is possible to do so. When a remainder of this kind does occur, however, it is sometimes easier to make a separate calculation for it, thus—

324 articles at ½d. each = 162 pence = 13s. 6d.

**Ex. 2.** *Find the cost of* 750 *cows at* £14, 18*s.* 10¾*d. each.*

The cost of 750 cows at £1 each = £750.

| | | | |
|---|---|---|---|
| Therefore, | £750 | | |
| | 14 | | |
| | £10500 | = cost of the cows at | £14 each. |
| 10s. = ½ of £1. | 375 | = cost of the cows at | 10s. each. |
| 6s. 8d. = ⅓ of £1. | 250 | = cost of the cows at | 6s. 8d. each. |
| 2s. = ⅕ of 10s. | 75 | = cost of the cows at | 2s. 0d. each. |
| 2d. = $\frac{1}{12}$ of 2s. | 6, 5s. 0d. | = cost of the cows at | 2d. each. |
| ½d. = ¼ of 2d. | 1, 11s. 3d. | = cost of the cows at | ½d. each. |
| ¼d. = ½ of ½d. | 15s. 7½d. | = cost of the cows at | ¼d. each. |
| *Answer,* | £11208, 11s. 10½d. | = cost of cows at | £14, 18s. 10¾d. each. |

**Ex. 3.** *Find the cost of* 196⅚ *cwt. at* £3, 7*s.* 6*d. per cwt.*

The cost of 196⅚ cwt. at £1 per cwt. = £196⅚ = £196, 16s. 8d.

| | £ | s. | d. | |
|---|---|---|---|---|
| Therefore, | 196 | 16 | 8 | = the cost at £1 per cwt. |
| | | | 3 | |
| | 590 | 10 | 0 | = the cost at £3 per cwt. |
| 5s. = ¼ of £1. | 49 | 4 | 2 | = the cost at 5s. per cwt. |
| 2s. 6d. = ½ of 5s. | 24 | 12 | 1 | = the cost at 2s. 6d. per cwt. |
| *Answer,* | £664 | 6 | 3 | = the cost at £3 7 6 per cwt. |

N.B. 1.—The above is the most expeditious method of finding the answer, which, however, might have been obtained as follows—

Since 196 cwt. will cost 196 times £3, 7s. 6d.,
and ⅚ cwt. will cost ⅚ of £3, 7s. 6d.,

it follows that 196⅚ cwt. will cost 196 times £3, 7s. 6d. + ⅚ of £3, 7s. 6d.

**Now, 196 times £1 = £196.**

Therefore, £196
3

£588 = cost of 196 cwt. at £3 per cwt.
5s. = $\frac{1}{4}$ of £1. 49 = cost of 196 cwt. at 5s. per cwt.
2s. 6d. = $\frac{1}{2}$ of 5s. 24, 10s. 0d. = cost of 196 cwt. at 2s. 6d. per cwt.

£661, 10s. 0d. = cost of 196 cwt. at £3, 7s. 6d. per cwt.
$\frac{5}{6}$ of £3, 7s. 6d. = £2, 16s. 3d. = cost of $\frac{5}{6}$ cwt. at £3, 7s. 6d. per cwt.

*Answer*, **£664, 6s. 3d.** = cost of **196$\frac{5}{6}$** cwt. at **£3, 7s. 6d.** per cwt.

N.B. 2.—There is another method of finding the value of a number of articles when the given price of one is less than £1, thus—

**Ex. 4.** *Find the cost of* 186 *articles at* 7*s.* 3$\frac{1}{2}$*d. each.*

The cost of 186 articles at 1s. each = 186s. = £9, 6s.

Therefore we say,

£9, 6s. 0d. = the cost at 1s. each.
7

£65, 2s. 0d. = the cost at 7s. each.
3d. = $\frac{1}{4}$ of 1s. 2, 6s. 6d. = the cost at 3d. each.
$\frac{1}{2}$d. = $\frac{1}{6}$ of 3d. 7s. 9d. = the cost at $\frac{1}{2}$d. each.

*Answer*, **£67, 16s. 3d.** = the cost at **7s. 3$\frac{1}{2}$d.** each.

N.B. 3.—The method pursued in the following example differs from that adopted in the preceding examples, in that the answer is found partly by subtraction—

**Ex. 5.** *Find the cost of* 39 *articles at* £6, 18*s.* 4*d. each.*

If the given price of one had been 1s. 8d. greater, it would have been just £7. Hence the reason of the following process—

£39 = cost at £1 each.
7

Therefore, £273 = cost at £7 each.
1s. 8d. = $\frac{1}{12}$ of £1. 3, 5s. 0d. = cost at 1s. 8d. each.

Subtracting, **£269, 15s. 0d.** = cost at **£6, 18s. 4d.** each.

*Answer*, **£269, 15s. 0d.**

II. COMPOUND PRACTICE is that case of Practice in which the given number is not wholly (sometimes not at all) expressed in the same denomination as the unit whose value is given.

Thus, to find the cost of 19 tons 17 cwt. 2 qrs. of coal at $10\frac{1}{2}$d. per cwt., and of 34 bush. of potatoes at 1s. $3\frac{1}{2}$d. per peck, are examples in Compound Practice.

**Ex. 1.** *Find the cost of* 196 *yards of cloth at* 8*s.* $9\frac{1}{2}$*d. per foot.*

Since 3 feet = 1 yard, we have

8s. $9\frac{1}{2}$d. per ft. = 3 (8s. $9\frac{1}{2}$d.) per yd. = £1, 6s. $4\frac{1}{2}$d. per yard.

Therefore,

| | | | |
|---|---|---|---|
| | £196 | = the cost at £1 | per yard. |
| 5s. = $\frac{1}{4}$ of £1. | 49 | = the cost at | 5s. per yard. |
| 1s. 3d. = $\frac{1}{4}$ of 5s. | 12, 5s. 0d. | = the cost at | 1s. 3d. per yard. |
| $1\frac{1}{2}$d. = $\frac{1}{10}$ of 1s. 3d. | 1, 4s. 6d. | = the cost at | $1\frac{1}{2}$d. per yard. |
| *Answer,* | £258, 9s. 6d. | = the cost at £1, 6s. $4\frac{1}{2}$d. per yard. | |

**Ex. 2.** *Find the cost of* 23 *tons* 17 *cwt.* 3 *qrs.* 14 *lb. at* £20, 12*s.* 6*d. per ton.*

Here £20, 12s. 6d. is the cost of 1 ton.

| | | | |
|---|---|---|---|
| Therefore, | £20, 12s. 6d.<br>23 | = the cost of 1 ton. | |
| | £474, 7s. 6d. | = the cost of 23 tons. | |
| 10 cwt. = $\frac{1}{2}$ of 1 ton. | 10, 6s. 3d. | = the cost of | 10 cwt. |
| 5 cwt. = $\frac{1}{2}$ of 10 cwt. | 5, 3s. $1\frac{1}{2}$d. | = the cost of | 5 cwt. |
| 2 cwt. 2 qrs. = $\frac{1}{2}$ of 5 cwt. | 2, 11s. $6\frac{3}{4}$d. | = the cost of | 2 cwt. 2 qrs. |
| 1 qr. = $\frac{1}{10}$ of 2 cwt. 2 qrs. | 5s. $1\frac{7}{8}$d. | = the cost of | 1 qr. |
| 14 lbs. = $\frac{1}{2}$ of 1 qr. | 2s. $6\frac{15}{16}$d. | = the cost of | 14 lb. |
| *Answer,* | £492, 16s. $2\frac{1}{16}$d. | = the cost of 23 tons 17 cwt. 3 qrs. 14 lb. | |

**The above example might also have been worked thus—**

| | | |
|---|---|---|
| | £23 | = the cost of 23 tons at £1 per ton. |
| | 20 | |
| | £460 | = the cost of 23 tons at £20 per ton. |
| 10s. = ½ of £1. | 11, 10s. 0d. | = the cost of 23 tons at 10s. 0d. per ton. |
| 2s. 6d. = ¼ of 10s. | 2, 17s. 6d. | = the cost of 23 tons at 2s. 6d. per ton. |
| 10 cwt. = ½ of 1 ton. | 10, 6s. 3d. | = the cost of 10 cwt. at £20, 12s. 6d. per ton. |
| 5 cwt. = ½ of 10 cwt. | 5, 3s. 1½d. | = the cost of 5 cwt. |
| 2 cwt. 2 qrs. = ½ of 5 cwt. | 2, 11s. 6¾d. | = the cost of 2 cwt. 2 qrs. |
| 1 qr. = $\frac{1}{10}$ of 2 cwt. 2 qrs. | 5s. 1⅞d. | = the cost of 1 qr. |
| 14 lb. = ½ of 1 qr. | 2s. 6$\frac{15}{16}$d. | = the cost of 14 lb. |
| *Answer,* | £492, 16s. 2$\frac{1}{16}$d. | = the cost of 23 tons 17 cwt. 3 qrs. 14 lb. at £20, 12s. 6d. per ton. |

**Ex. 3.** *Find the cost of carpeting a room which contains* 76 *sq. yds.* 5 *sq. ft.* 108 *sq. in., with carpet which is* 3*s.* 9½*d. per sq. ft.*

Here 3s. 9½d. per sq. ft. = 3s. 9½d. × 9, *i.e.* £1, 14s. 1½d. per sq. yd.

Therefore,

| | | |
|---|---|---|
| | £1, 14s. 1½d. | = the cost of 1 sq. yd. |
| | 12 | |
| | £20, 9s. 6d. | = the cost of 12 sq. yds. |
| | 6 | |
| | £122, 17s. 0d. | = the cost of 72 sq. yds. |
| | 6, 16s. 6d. | = the cost of 4 sq. yds. |
| 3 sq. ft. = ⅓ of 1 sq. yd. | 11s. 4½d. | = the cost of 3 sq. ft. |
| 1 sq. ft. = ⅓ of 3 sq. ft. | 3s. 9½d. | = the cost of 1 sq. ft. |
| | 3s. 9½d. | = the cost of 1 sq. ft. |
| 72 sq. in. = ½ of 1 sq. ft. | 1s. 10¾d. | = the cost of 72 sq. in. |
| 36 sq. in. = ½ of 72 sq. in. | 11⅜d. | = the cost of 36 sq. in. |
| *Answer,* | £130, 15s. 3⅝d. | = the cost of 76 sq. yds. 5 sq. ft. 108 sq. in. |

**Ex. 4.** *What is the rent of* 23 *acres* 3 *roods* 16 *poles at* $2\frac{1}{2}$ *guineas per acre?*

Here, $2\frac{1}{2}$ guineas = £2, 2s. + 10s. 6d. = £2, 12s. 6d.

Therefore,

£2, 12s. 6d. = rent of 1 acre.
23

| | | |
|---|---|---|
| | £60, 7s. 6d. | = rent of 23 acres. |
| 2 roods = $\frac{1}{2}$ of 1 acre. | 1, 6s. 3d. | = rent of 2 roods. |
| 1 rood = $\frac{1}{2}$ of 2 roods. | 13s. $1\frac{1}{2}$d. | = rent of 1 rood. |
| 10 poles = $\frac{1}{4}$ of 1 rood. | 3s. $3\frac{3}{8}$d. | = rent of 10 poles. |
| 5 poles = $\frac{1}{2}$ of 10 poles. | 1s. $7\frac{11}{16}$d. | = rent of 5 poles. |
| 1 pole = $\frac{1}{5}$ of 5 poles. | $3\frac{15}{16}$d. | = rent of 1 pole. |
| *Answer,* | £62, 12s. $1\frac{1}{2}$d. | = rent of 23 acres 3 roods 16 poles. |

**Ex. 5.** *Find the weight of* 56 *sacks of wheat, each sack weighing* 1 *cwt.* 2 *qrs.* 23 *lb.*

Here 56 sacks of wheat, each weighing 1 cwt., weigh 56 cwt.

Therefore,

| | | |
|---|---|---|
| | 56 cwt. | = the weight at 1 cwt. to the sack. |
| 2 qrs. = $\frac{1}{2}$ of 1 cwt. | 28 cwt. | = the weight at 2 qrs. |
| 14 lb. = $\frac{1}{4}$ of 2 qrs. | 7 cwt. | = the weight at 14 lb. |
| 7 lb. = $\frac{1}{2}$ of 14 lb. | 3 cwt. 2 qrs. | = the weight at 7 lb. |
| 2 lb. = $\frac{1}{7}$ of 14 lb. | 1 cwt. | = the weight at 2 lb. |
| *Answer,* | 95 cwt. 2 qrs. | = the weight at 1 cwt. 2 qrs. 23 lb. to the sack. |

**Ex. 6.** *A bankrupt owes* £5509, 7*s.* 6*d.*, *and pays* 8*s.* 4*d. in the* £, *what is the value of his assets?*

If he paid 20s. in the £, his assets would be worth £5509, 7s. 6d.

Therefore,

| | | |
|---|---|---|
| | £5509, 7s. 6d. | = assets when 20s. are paid in £. |
| 6s. 8d. = $\frac{1}{3}$ of £1. | £1836, 9s. 2d. | = assets when 6s. 8d. is paid in £. |
| 1s. 8d. = $\frac{1}{4}$ of 6s. 8d. | 459, 2s. $3\frac{1}{2}$d. | = assets when 1s. 8d. is paid in £. |
| *Answer,* | £2295, 11s. $5\frac{1}{2}$d. | = assets when 8s. 4d. is paid in £. |

# Exercise 31.

## Exercises in Practice.

1. Find the cost of—

(1.) 28 articles at 10s. each.
(2.) 36 ,, 5s. ,,
(3.) 101 ,, 4s. ,,
(4.) 92 articles at 5s. each.
(5.) 108 ,, 2s. 6d. ,,
(6.) 222 ,, 6s. 8d. ,,

(7.) 44 articles at £2, 10s. each.
(8.) 68 ,, £6, 5s. ,,
(9.) 805 ,, £10, 4s. ,,
(10.) 723 ,, £3, 6s. 8d. ,,
(11.) 648 ,, £8, 2s. 6d. ,,
(12.) 492 ,, £9, 3s. 4d. ,,
(13.) 2304 ,, £1, 1s. 8d. ,,
(14.) 725 articles at £13, 4s. each.
(15.) 810 ,, £12, 2s. ,,
(16.) 7132 ,, £18, 2s. 6d. ,,
(17.) 482 ,, £15, 5s. ,,
(18.) 752 ,, £19, 10s. ,,
(19.) 10185 ,, £13, 4s. ,,
(20.) 284 ,, £17, 2s. 6d. ,,

(21.) 16 articles at £2, 1s. 3d. each.
(22.) 2412 ,, £17, 1s. 8d. ,,
(23.) 690 ,, £10, 1s. 4d. ,,
(24.) 288 ,, £9, 0s. 10d. ,,
(25.) 48096 ,, £18, 0s. 5d. ,,
(26.) 930 articles at £81, 0s. 8d. each.
(27.) 96080 ,, £17, 0s. 3d. ,,
(28.) 19515 ,, £23, 1s. 4d. ,,
(29.) 3201616 ,, £10, 1s. 3d. ,,
(30.) 7202496 ,, £71, 0s. 10d. ,,

2. Determine the cost of—

(1.) 42 articles at £2, 15s. each.
(2.) 96 ,, £18, 16s. 8d. ,,
(3.) 486 ,, £7, 13s. 4d. ,,
(4.) 820 ,, £3, 19s. ,,
(5.) 228 ,, £7, 14s. 6d. ,,
(6.) 814 ,, £16, 15s. 10d. ,,
(7.) 85 articles at £7, 16s. 9d. each.
(8.) 103 ,, £17, 11s. 6d. ,,
(9.) 415 ,, £3, 18s. 4d. ,,
(10.) 221 ,, £19, 19s. 4d. ,,
(11.) 712 ,, £13, 17s. ,,
(12.) 540 ,, £15, 13s. 6d. ,,

(13.) 160 articles at 17s. 6d. each.
(14.) 280 ,, 13s. 4d. ,,
(15.) 420 ,, 19s. 6d. ,,
(16.) 88 ,, 5s. 8d. ,,
(17.) 968 ,, 10s. 9½d. ,,
(18.) 724 ,, 16s. 10½d. ,,
(19.) 74 articles at 11s. 8¾d. each.
(20.) 901 ,, 13s. 9¼d. ,,
(21.) 251 ,, 12s. 1¼d. ,,
(22.) 829 ,, 15s. 5¼d. ,,
(23.) 713 ,, 9s. 0¼d. ,,
(24.) 217 ,, 3s. 11¾d. ,,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (25.) | 101 articles at | £21, 16s. 10½d. | ea. | (33.) | 1091 articles at | £100, 19s. 3d. | ea. |
| (26.) | 712 ,, | £35, 19s. 11d. | ,, | (34.) | 2865 ,, | £3, 12s. 0¾d. | ,, |
| (27.) | 285 ,, | £312, 17s. 2¼d. | ,, | (35.) | 7145 ,, | £20, 15s. 1¼d. | ,, |
| (28.) | 643 ,, | £80, 1s. 2½d. | ,, | (36.) | 700 ,, | £801, 3s. 6¾d. | ,, |
| (29.) | 171 ,, | £17, 4s. 6¼d. | ,, | (37.) | 528 ,, | £38, 17s. 7¼d. | ,, |
| (30.) | 876 ,, | £18, 16s. 4½d. | ,, | (38.) | 191 ,, | £191, 19s. 10¾d. | ,, |
| (31.) | 371 ,, | £342, 19s. 7¼d. | ,, | (39.) | 275 ,, | £101, 0s. 8½d. | ,, |
| (32.) | 756 ,, | £47, 16s. 5d. | ,, | (40.) | 707 ,, | £70, 7s. 3½d. | ,, |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (41.) | 84 articles at | 2s. 3⅝d. | each. | (46.) | 243 articles at | £1, 17s. $8\frac{6}{11}$d. | ea. |
| (42.) | 104 ,, | £1, 10s. 7⅜d. | ,, | (47.) | 549 ,, | £10, 13s. 8⅝d. | ,, |
| (43.) | 725 ,, | £3, 19s. 9¾d. | ,, | (48.) | 2282 ,, | £5, 9s. $5\frac{5}{7}$d. | ,, |
| (44.) | 1484 ,, | £13, 16s. $8\frac{4}{7}$d. | ,, | (49.) | 7281 ,, | £6, 4s. $9\frac{5}{7}$d. | ,, |
| (45.) | 209 ,, | £4, 10s. $0\frac{10}{11}$d. | ,, | (50.) | 513 ,, | 2s. $4\frac{8}{13}$d. | ,, |

3. Find the cost of

(1.) 28½ cwt. at £6, 10s. per cwt.
(2.) 30¼ tons at £7, 3s. 4d. per ton.
(3.) 8⅓ gallons at £2, 2s. 6d. per gallon.
(4.) 19⅙ quarters at £7, 2s. per quarter.
(5.) 2⅝ bushels at 3s. 10d. per bushel.
(6.) 1⅝ ounces at 7s. 9d. per ounce.
(7.) 3¾ acres at £40, 5s. 6d. per acre.
(8.) 17⅘ kilograms at £2, 2s. 5d. per kilogram.
(9.) $100\frac{7}{10}$ ares at £13, 18s. 4d. per are.
(10.) $28\frac{13}{30}$ tons at £1, 17s. 6½d. per ton.
(11.) 30½ cwt. at £8, 6s. 8d. per ton.
(12.) 17⅗ tons at £1, 4s. 6d. per cwt.
(13.) 81½ acres at £3, 6s. per rood.
(14.) 37⅗ dozen at 18s. 4d. per score.
(15.) $25\frac{1}{15}$ gross at £24, 19s. 6d. per dozen.
(16.) 3⅓ dozen cows at £19, 19s. 6d. each.
(17.) $8\frac{3}{7}$ kilograms at £20, 17s. 6d. per gram.
(18.) $1\frac{8}{15}$ acres at £7, 4s. 10d. per rood.
(19.) 18⅓ quires at £75, 2s. 1d. per ream.
(20.) 4⅗ barrels of beer at £3, 1s. 6d. per barrel.

4. Calculate the cost of

(1.) 7 tons 10 cwt. at £8, 12s. 6d. per ton.
(2.) 10 tons 16 cwt. at £9, 17s. 6d. per ton.
(3.) 5 tons 18 cwt. at £6, 18s. 6d. per ton.

(4.) 7 yards 3 qrs. at £1, 0s. 6d. per yard.
(5.) 13 acres 2 roods at 16s. 8½d. per acre.
(6.) 11 acres 3 roods at £1, 19s. 7½d. per acre.
(7.) 18 cwt. 3 qrs. at £1, 2s. 6d. per cwt.
(8.) 31 cwts. 1 qr. at £5, 17s. 10d. per cwt.
(9.) 28 oz. 13 dwts. at £4, 14s. 6d. per oz.
(10.) 19 oz. 19 dwts. at £3, 17s. 10½d. per oz.
(11.) 16 tons 17 cwt. 3 qrs. at £2, 12s. 8d. per ton.
(12.) 15 tons 19 cwt. 1 qr. at £1, 5s. per ton.
(13.) 27 tons 3 cwt. 2 qrs. 14 lb. at £61, 7s. 8d. per ton.
(14.) 91 tons 3 qrs. at £61, 10s. per cwt.
(15.) 384 tons 19 cwt. 14 lb. at £86, 17s. 4d. per ton.
(16.) 61 acres 3 roods 14 poles at £10 per acre.
(17.) 43 acres 19 poles at £81, 5s. per rood.
(18.) 3 tons 16 cwt. 21 lb. at £7, 16s. 4d. per cwt.
(19.) 40 tons 3 qrs. 19 lb. at £2, 17s. 2d. per cwt.
(20.) 131 cwt. 2 qrs. 20 lb. at £5, 6s. per ton.
(21.) 7 kilograms 3 hectograms 4 grams at £2, 1s. 8d. per kilogram.
(22.) 19 myriag. 14 hectog. 16 centig. at £18, 18s. per hectog.
(23.) 300 metres 21 centimetres at £21, 19s. 6d. per dekametre.
(24.) 501 yards 2 ft. 10 in. at £8, 12s. per mile.
(25.) 1400 yards 1 ft. 9 in. at 16s. 7½d. per furlong.
(26.) 1701 bush. 1 pk. at 10s. 6d. per quarter.
(27.) 3 yrs. 46 wks. at £22, 12s. 6d. per year.
(28.) 5 months 3 wks. 6 days at £18, 10s. a year of 12 lunar months.
(29.) 27 acres 3 roods 19 poles at 7½ guineas per acre.
(30.) 16 tons 14 cwt. 1 qr. 19 lb. 8 oz. at 13¾ guineas per quarter.

5. Find the united area of 15 fields, the average size of which is 7 acres 3 roods 20 poles.

6. Find the weight of 18 trucks of coal, each of which contains 3 tons 16 cwt. 78 lb.

7. Find the cubical contents of 142 rooms, the length of each of which is 72 feet, the height 19 feet, and the width 30 feet.

8. If the taxes on a house be at the rate of 3s. 6½d. to every £ of the rental, what are the taxes on a rental of £50, 7s. 6d.?

9. Having borrowed wheat of a certain quality I agree to repay it in wheat of an inferior quality, at the rate of 3 qrs. 6 bush. of the former for 4 qrs. 1 bush. of the latter. How many qrs. must I send in return for 30 qrs. 3 bush. borrowed?

## Exercise 32.

### Miscellaneous Exercises in Practice.*

1. What is the cost of 16 cwt. 3 qrs. 16 lb. at £1, 4s. 8d. per cwt.?

2. Find by Practice the value of 4074 articles at 2s. 9$\frac{1}{2}$d. per dozen.

3. What is the amount of profit made by a merchant who sells 2 tons 14 cwt. 2 qrs. of sugar so as to gain £1, 12s. 6d. per cwt.?

4. Find the value of 157 tons at 7 guineas per ton, at £2$\frac{5}{8}$ per ton, and at £4, 11s. 8d. per ton, using one aliquot part only. In the same way calculate the cost of 158 tons at 8 guineas per ton, at £1$\frac{11}{12}$ per ton, and at £4, 7s. 6d. per ton.

5. If I give 105 women 10s. 4d. each, and 140 boys 7s. 9d. each, how many men at 12s. 11d. each can I pay with the same money?

6. If a person's income is 500 guineas a year, and he spends one day with another £1, 2s. a day, how much will he lay by at the end of the year?

7. How much cloth will cover a room whose length is 12 ft. 6 in., and breadth 14 ft. 9 in., and what will it cost at 5s. 6d. per sq. yd.?

8. Find by Practice the value of 31·625 of £20, 10s. 6d.

9. What is the cost of 32 gallons 3$\frac{1}{2}$ quarts of ale at 1s. 9d. per gallon?

10. How much property-tax must be paid on £7964, at 2s. 9$\frac{3}{4}$d. in the pound?

11. A man spends £155, 5s. 7d. a year, how much will he lay by in 37 years out of an income of £200 a year?

12. A sells B three chests of tea, each weighing 2 qrs. 12 lb., at £·225 per lb., and receives in return 23 cwt. 2 qrs. of sugar at £1·875 per cwt. How much money will A have to receive in addition?

13. A contractor having commenced a railway, finds that when he has yet 36 miles 3 fur. 22 yds. to finish, his working expenses are increased by a strike of his men by £14, 13s. 4d. per mile, and by a rise in the price of materials, by £9, 13s. 4d. per mile. If he had reckoned on clearing £5000 by the contract, what will he be able to clear after these misfortunes?

14. If a railway train travels at the rate of 22 miles 3 fur. 18 yds. per hour, how far will it travel between a quarter past 6 in the morning and twenty-five minutes to 3 in the afternoon?

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* These Exercises are taken in great measure from Examination Papers.

15. A borrowed of B a certain sum of money, and in payment of his debt returned to B the sum of £1400 in money and 531 lambs valued at £3, 5s. 7½d. each. How much did A borrow?

16. Find the difference between the value of the contents of a truck containing 14 tons 5 cwt. 2 qrs. 24 lb. of potatoes which will sell at 7s. per cwt., and that of another truck which contains 100 barrels of oysters, each barrel containing 138 dozen oysters valued at 3s. 9½d. a hundred.

17. A man's income is £800 a year. He pays income-tax of 5d. in the pound, a rent of £70 a year, and rates of various kinds amounting to 5s. 8d. in the pound on his rental. How much a day has he left for other expenses?

18. A grocer buys tea at 2s. 8d. per lb. and sells it again so as to gain $\frac{6\frac{1}{2}}{32}$ of the cost price. What will be his receipts on 6043 lb?

19. If $\frac{1}{893}$ of an estate be worth £109, 8s. 10d., what is the value of the whole estate?

20. If two acres of land be worth £223, 2s. 8d., what is the united value of two estates, each of which contains 45 acres 3 roods 20 poles?

21. Three coal-waggons weigh respectively, 4 tons 16 lb., 3 tons 19 cwt. 1 qr., 4 tons 3 qrs. 27 lb. Their united weight, together with that of the coal they contain, is 37 tons 9 cwt. 3 qrs. 15 lb. What will be the value of the coal at £1, 4s. 2d. per ton?

22. Find by Practice the cost of papering and carpeting a room which is 63 ft. long, 21 ft. high, and 21 ft. wide; the paper being 1 yard wide, and costing 8d. a yard, and the carpet being 3s. 9½d. per sq. yard.

## RATIO AND PROPORTION.

**97. The Ratio** of one quantity to another is the relation which the first bears to the second, as expressed by a fraction, of which the first quantity is the numerator and the second the denominator.

Thus, the ratio of 3 to 15 is expressed by $\frac{3}{15}$, *i.e.* $\frac{1}{5}$.

The same ratio is more usually expressed thus—3 : 15.

Similarly, the ratio of 107 to 429 is written thus—107 : 429.

In every ratio the first of the two terms forming it is called the **Antecedent**, and the second is termed the **Consequent.**

It is evident that every vulgar fraction expresses the ratio of its numerator to its denominator; thus, $\frac{5}{6}$ expresses the ratio 5 : 6. It follows that when by multiplication or division the antecedent and the consequent of one ratio are made equal to the antecedent and the consequent, respectively, of one or more other ratios, the first ratio is equal to the other, or others; thus—

$$\text{The ratio of } 18 : 4 = \frac{18}{4} = \text{(I.)}\ \frac{18 \times 3}{4 \times 3} = \frac{54}{12} = \text{the ratio of } 54 : 12.$$

$$\text{(II.)}\ \frac{18 \div 2}{4 \div 2} = \frac{9}{2} = \text{the ratio of } 9 : 2.$$

*Ratio can exist only between quantities which are of the same kind.*

Thus a ratio may exist between 3 oranges and 7 oranges, the ratio in this case being $\frac{3}{7}$, but we cannot establish a ratio between 3 *oranges* and 7 marbles.

It must be remembered that although the quantities between which a ratio is established may be both *concrete* quantities, the ratio itself is always an *abstract* number, integral or fractional.

*The ratio between two quantities of the same kind cannot be expressed by an abstract number unless both are expressed in one denomination.*

Thus, the ratio 9 tons : 7 cwt. can only be expressed as an abstract number by first reducing the quantities to one denomination; thus, $\frac{180 \text{ cwt.}}{7 \text{ cwt.}}$, *i.e.* $\frac{180}{7}$.

**98. Proportion** is the relation of equality subsisting between ratios.

If two ratios are equal (*i.e.* "If the fraction expressed by the antecedent and consequent of the first ratio be equal to the fraction expressed by the antecedent and consequent of the second ratio"), they are said to form a **Proportion**, and the four terms of which it is composed are called **Proportionals**,* or are said to be **proportional to one another.**

Thus, since $14 = \frac{2}{3}$ of 21, and $18 = \frac{2}{3}$ of 27

the ratio of 14 : 21 = the ratio of 18 : 27,

and these four terms 14, 21, 18, 27, form a proportion which is usually written thus—

$$14 : 21 :: 18 : 27,$$

and read thus—

As 14 is to 21, so is 18 to 27,

or—

14 is to 21 as 18 is to 27.

N.B. 1.—The meaning of this proportion is, that—

14 is the same fraction of 21 that 18 is of 27.

This idea of a proportion should be carefully remembered.

N.B. 2.—In this example the two antecedents are 14 and 18, and the two consequents are 21 and 27.

N.B. 3.—The *first* and *last* terms of a proportion are called its **extremes**, and the *second* and *third* terms are called its **means.**

**99.** *In every proportion the product of the extremes is equal to the product of the means.*

**Ex.** $3 : 7 :: 12 : 28,$

*i.e.* $\frac{3}{7} = \frac{12}{28}$.

Reducing these fractions to the common denominator (7 × 28, *i.e.* 196), we have—

$$\frac{3 \times 28}{7 \times 28} = \frac{12 \times 7}{28 \times 7}.$$

And now, multiplying each fraction by the common denominator, we have—

$$3 \times 28 = 12 \times 7, \textit{ i.e.}$$
$$84 = 84$$

*The product of the extremes = the product of the means.*

It follows from this proposition, that—

*Having any three terms of a proportional given, we may always find the remaining term, for—*

---

* The following definition should also be remembered—

"Four quantities are said to be proportionals when the first is the same number of times greater or less than the second that the third is greater or less than the fourth."

Let the four terms be signified by $a$, $b$, $c$, $d$,

and let $a : b :: c : d$

then $a \times d = b \times c$ (§ 99),

and therefore—

$$a = \frac{b \times c}{d}$$

$$b = \frac{a \times d}{c}$$

$$c = \frac{a \times d}{b}$$

$$d = \frac{b \times c}{a}$$

Applying these formulæ to the above example, we have—

$$3 = \frac{7 \times 12}{28}$$

$$7 = \frac{3 \times 28}{12}$$

$$12 = \frac{3 \times 28}{7}$$

$$28 = \frac{7 \times 12}{3}$$

N.B. 1.—When the second and third terms of a proportional are alike, the number constituting each of these terms is called a **Mean Proportional** to the other two.

Thus, $a : b :: b : c$.

Here $b$ is a mean proportional to $a$ and $c$.

Since, also, $a \times c = b \times b = b^2$, we have the following rule—

**RULE.—To find a mean proportional to two given numbers, multiply them together, and extract their square root** (§ 127).

**Ex. 1.** *Find a mean proportional to* 4 *and* 9.

*Answer*, $= \sqrt{4 \times 9} = \sqrt{36} = 6$.

N.B. 2.—The following is but another form of the mean proportional.

**Ex. 2.** *Find a third proportional to* 27 *and* 63.

The problem may be thus expanded—

*Find a number bearing the same relation to* 63 *that* 63 *bears to* 27.

Let $d$ be the number required.

Then $27 : 63 :: 63 : d$

Therefore $27d = 63 \times 63 = (63)^2$

Therefore $d = \frac{(63)^2}{27} = 147$, *Answer.*

Hence the reason of the following rule—

**RULE.—To find a third proportional to two given numbers, square the second number, and divide the product by the first number.**

**100.** Conversely, *When the product of any two quantities is equal to the product of two others, the four quantities are proportional, the factors of either one of the products being the extremes, and those of the other the means of the proportional.*

From which it follows, that—

*If four quantities are proportional in any one given order, they will also be proportional in any other order, provided that the same pair of terms are kept together either as extremes or means.*

Thus, since $6 \times 12 = 8 \times 9$, we have the following proportions—

(1.) $6 : 8 :: 9 : 12$; or, $9 : 12 :: 6 : 8$;
(2.) $8 : 6 :: 12 : 9$; or, $12 : 9 :: 8 : 6$;

but $6 : 12 :: 8 : 9$ is not correct, because $6 \times 9$ is less than $12 \times 8$.

N.B.—The sign $>$ is used to express "*decreases to,*" or, "*is greater than;*" the sign $<$ is used to express "*increases to,*" or, "*is less than;*" and the sign $\propto$ is used to signify "*varies as.*"

**101. Simple Proportion,** or, **The Rule of Three,** is the name given to the method by which, when we have three terms of a proportion given, we can find the fourth.

The following problem forms a case of simple proportion—

**Ex. 1.** *If 6 men earn 42s., how much will 36 men earn?*

It is here supposed that the money earned *varies directly* with the number of men engaged in earning it; *i.e.* that when *more men* are at work *more money* will be earned, and when *fewer men* are engaged *less money* will be earned.

With this supposition we are enabled to establish the ratio, that—

*As* 6 *men* : 36 *men* :: *the money earned by the* 6 *men* : *the money earned by the* 36 *men;*

in other words, that—

6 *men* : 36 *men* :: 24s. : *the Answer;*

therefore—

$$\textit{The Answer} = \frac{42\text{s.} \times 36}{6} = 252\text{s.} = \pounds 12, 12\text{s.}$$

In this example we have the three terms 6 men, 36 men, and 42s. given, and we are required to find the fourth term, which will also be expressed in shillings; for the third term being the antecedent to the fourth, must necessarily be of the same kind and of the same denomination as the fourth.

In the above example we have two things to do, viz.—

1st. *To state the problem as a proportion*, and
2nd. *To solve the proportion*,

for each of which we require a separate rule.

Before proceeding to the enunciation of these rules it will be as well to remark—

1st. That although we generally describe simple proportion as *the method of finding a fourth proportional to three given proportionals*, it may more accurately be defined as *the method of finding the four terms of a proportional when any three of them are given*, which is performed by the help of § 99.

Thus, in the example taken above, the following ratio is correct—

$$\text{(I.)}\quad \textit{As } 36 \textit{ men} : 6 \textit{ men} :: \begin{matrix}\textit{the money earned}\\ \textit{by 36 men}\end{matrix} : \begin{matrix}\textit{the money earned}\\ \textit{by 6 men.}\end{matrix}$$

in other words, that—

$$36 \textit{ men} : 6 \textit{ men} :: \textit{the Answer} : 42\text{s.}$$

therefore—

$$\textit{The Answer} = \frac{42\text{s.} \times 36}{6} = 252\text{s.} = £12,\ 12\text{s.}$$

In this case, we found the *third* term of the proportional, and in the following proportions we find the *first* and *second* terms respectively, the answer being in every case the same.

$$\text{(II.)}\quad \begin{matrix}\textit{The money earned}\\ \textit{by 36 men}\end{matrix} : \begin{matrix}\textit{the money earned}\\ \textit{by 6 men}\end{matrix} :: 36 \textit{ men} : 6 \textit{ men.}$$

$$\textit{i.e. The Answer} : 42\text{s.} :: 36 \textit{ men} : 6 \textit{ men.}$$

$$\text{(III.)}\quad \begin{matrix}\textit{The money earned}\\ \textit{by 6 men}\end{matrix} : \begin{matrix}\textit{the money earned}\\ \textit{by 36 men}\end{matrix} :: 6 \textit{ men} : 36 \textit{ men.}$$

$$\textit{i.e. } 42\text{s.} : \textit{the Answer} :: 6 \textit{ men} : 36 \textit{ men.}$$

2nd. That there are two cases of **Variation**, which are usually described as (1.) "**Direct**" and (2.) "**Inverse.**"

The example taken above is a case of direct variation, because the *money earned* increases when the *number of men* increases, and diminishes when the *number of men* diminishes.

The following example will furnish an instance of inverse variation—

**Ex. 2.** *If the sixpenny loaf weighs* 3 *lb. when wheat is at* 6*s. a bushel, how much ought it to weigh when wheat is at* 8*s. a bushel?*

Here the weight of the loaf varies inversely as the price of flour, *i.e.* the loaf weighs *less* when the price is *greater*, and *vice versa.*

This inverse variation is expressed thus—

If $a$ varies inversely as $b$, the ratio is thus stated—

$$\frac{1}{a} = \frac{1}{b}$$

The above problem may now be stated as a proportion—

$$6\text{s.} : 8\text{s.} :: \frac{1}{\textit{the weight at } 6\text{s.}} : \frac{1}{\textit{the weight at } 8\text{s.}}$$

$$\textit{i.e. } 6\text{s.} : 8\text{s.} :: \frac{1}{3 \text{ lb.}} : \frac{1}{\textit{the Answer.}}$$

$$\text{Therefore, } 6 \times \frac{1}{\textit{the Answer}} = \frac{1}{3 \text{ lb.}} \times 8.$$

$$\textit{i.e. } \frac{6}{\textit{The Answer}} = \frac{8}{3 \text{ lb.}}$$

$$\therefore\ 6 \times 3 \text{ lb.} = 8, \textit{ Answer.}$$

$$\therefore\ \textit{Answer} = \frac{6 \times 3 \text{ lb.}}{8} = \frac{18 \text{ lb.}}{8} = 2\tfrac{1}{4} \text{ lb.}$$

The preceding examples and remarks will now render intelligible the following rules—

**I. RULE FOR STATING THE PROPORTION.—Of the three given quantities which are involved in the proportion, set that which is of the same kind as the answer in the *third* term.**

**If now the nature of the question demands that the fourth term of the proportion (*i.e.* the answer) should be of a value *greater* than that of the third term, put the greater of the two remaining quantities in the second term, and the other in the first; but, if the fourth term is to be *less* than the third, put the less of the two remaining quantities in the second term and the other in the first.**

If the first and second terms be expressed in different denominations, reduce them to one denomination before proceeding any further.

II. RULE FOR SOLVING THE PROPORTION.—Multiply the second and third terms together, and divide the product by the first.

**Ex. 1.** *If 6 men earn 42s., how much will 36 men earn?*

Here the answer must evidently be in wages, and 42s., being the wages of 6 men, will be the third term.

Again, it is evident that the wages of 36 men will exceed the wages of 6 men, therefore "36" is put in the second term because it exceeds 6.

Therefore—

$$6 \text{ men} : 36 \text{ men} :: 42\text{s.} : \text{wages of } 36 \text{ men.}$$

Therefore—

$$\text{The wages of } 36 \text{ men} = \frac{42\text{s.} \times 36}{6} = \frac{1512}{6}\text{s.} = 252\text{s.}$$

Or, by cancelling—

$$\frac{42\text{s.} \times \overset{6}{\cancel{36}}}{\underset{1}{\cancel{6}}} = 252\text{s.} = £12, 12\text{s.}$$

N.B.—It will here be noticed that we *apparently* do what it is really impossible to do—viz., multiply 42 men by 36s. But, *in reality*, we do nothing whatever of the kind; for, as we have already stated, a ratio itself is always an abstract number, from which it follows that, in the above example, the 6 and 36 (representing, as they do, the antecedent and the consequent of the ratio 6 : 36) are merely abstract numbers, so that in multiplying by the second term of a proportion, and dividing by the first, we *really* multiply and divide by abstract numbers in every case.

**102. "First Principles"** is a name applied to that method of solving arithmetical problems by means of no fixed rule, but by the use of a clear and concise method of reasoning.

This method has the advantage of being somewhat easier to understand than that proposed by the "*Rule*," but for facility of working the rule is by far the more preferable.

The rule, or "**Proportion by Statement,**" as it is sometimes called, has been condemned as being too mechanical, but this objection can only be urged when the subject is *taught* mechanically. There is certainly as much exercise of the mind in intelligently stating a proportion on the principle of equality of ratios as in reasoning out any one of the puny problems suggested by the mode of working "**Proportion by First Principles.**"

The following is an example of "*Proportion by First Principles,*" and although the answer is found by the use of a certain principle of "*Proportion,*" it will be seen that it scarcely recognises at all the mathematical theory of *Equality of Ratios* and *Proportion.*

**Ex. 1.** *If* 6 *men earn* 42*s., how much will* 36 *men earn?*

The wages of 6 men = 42s.

Therefore, the wages of 1 man $= 42s. \div 6 = \frac{42s.}{6}$

Therefore, the wages of 36 men $= \frac{42s.}{6} \times 36 = \frac{42s. \times \overset{6}{\cancel{36}}}{\cancel{6}} = 252s.$

The above remarks are not intended to apply to the method of working by "First Principles" in a general way. They are intended rather *to defend the rule* against the charge of being too mechanical. Other exercises worked by "First Principles" will be given hereafter, and the student will do well to work a number of exercises by both methods, as they will thus explain and illustrate each other.

N.B.—It may be as well to point out that, relying on § 57 and 58, in arranging the fraction when working by "First Principles," to make the value of that fraction *less*, we put the number under consideration *below* the line; to make the value of the fraction *greater*, we put the number *above* the line.

Thus, in the above example, when we wished to make the 42s. *less*, we placed the "6" *beneath* the line, thus, $\frac{42s.}{6}$; and when we desired to *increase* the value of this fraction, we put the "36" *above* the line, thus, $\frac{42s. \times 36}{6}$.

**Ex. 2.** *If I pay* 7*s. for the use of* 100*s., how many shillings must I pay for the use of* 75*s.?*

**Here all the terms are in *shillings;* but it is easily seen that the third term must be 7s., because the answer is not only to be in *money,* but in *money paid for the use of other money.***

**Therefore, by the rule—**

**100s. : 75s. :: 7s. : *the Answer.***

$$\therefore \textit{ The Answer} = \frac{7\text{s.} \times \overset{3}{\cancel{75}}}{\underset{4}{\cancel{100}}} = \frac{21\text{s.}}{4} = 5\text{s. } 3\text{d.}$$

**Ex. 3.** *If* 4 *tons* 3 *cwt. of coal cost* £6, 4*s.* 6*d., how much coal may be had for* £4, 16*s.?*

**(I.) By the rule—**

**£6, 4s. 6d. : £4, 16s. :: 4 tons 3 cwt. : Answer.**

**Reducing the first and second terms to one denomination, and the third term to its lowest denomination, we have—**

**249 *sixpences* : 192 *sixpences* :: 83 *cwt.* : *Answer.***

**Therefore—**

$$\textit{Answer} = \frac{\overset{1}{\cancel{83}}\text{ cwt.} \times \overset{64}{\cancel{192}}}{\underset{\cancel{3}\ 1}{\cancel{249}}} = \frac{64\text{ cwt.}}{1} = 64\text{ cwt.}$$

**(II.) By First Principles—**

**For (£6, 4s. 6d., *i.e.* for) 249 sixpences we can purchase (4 tons 3 cwt., *i.e.*) 83 cwt. of coal;**

**Therefore, for 1 sixpence we can purchase $\frac{83\text{ cwt. of coal;}}{249}$**

**And for (£4, 16s., *i.e.* for) 192 sixpences, we can purchase $\frac{83\text{ cwt.} \times 192}{249}$ of coal.**

$$\frac{\overset{1}{\cancel{83}}\text{ cwt.} \times \overset{64}{\cancel{192}}}{\underset{\cancel{3}\ 1}{\cancel{249}}} = \frac{64\text{ cwt.}}{1} = 64\text{ cwt., } \textit{Answer.}$$

**Ex. 4.** *If* $(\frac{2}{3} + \frac{1}{21})$ *of an estate is worth* £1003, 17*s.* 1*d., what is the value of* $(\frac{1}{3} + \frac{1}{15})$ *of it?*

(Examination for Certificates. Males, 1st year. 1870.)

$$\text{Here } \frac{2}{3} + \frac{1}{21} = \frac{14+1}{21} = \frac{15}{21} = \frac{5}{7}.$$

$$\text{And } \frac{1}{3} + \frac{1}{15} = \frac{5+1}{15} = \frac{6}{15} = \frac{2}{5}.$$

Therefore, the question stands thus—
"*If $\frac{5}{7}$ of an estate is worth* £1003, 17*s.* 1*d.*, *what is the value of $\frac{2}{5}$ of it?*"
By the rule—

$$\tfrac{5}{7} : \tfrac{2}{5} :: £1003,\ 17\text{s. } 1\text{d.} : \textit{the Answer.}$$

$$\begin{aligned}\therefore \textit{The Answer} &= £1003,\ 17\text{s. } 1\text{d.} \times \tfrac{2}{5} \div \tfrac{5}{7}\\ &= £1003,\ 17\text{s. } 1\text{d.} \times \tfrac{2}{5} \times \tfrac{7}{5}\\ &= £1003,\ 17\text{s. } 1\text{d.} \times \tfrac{14}{25}\\ &= \frac{£1003,\ 17\text{s. } 1\text{d.} \times 14}{25}\\ &= \frac{£14053,\ 19\text{s. } 2\text{d.}}{25}\\ &= £562,\ 3\text{s. } 2\text{d.}\end{aligned}$$

**Ex.** 5. *If a man pays* £749, 18*s.* 4*d.* *for debts at* 3*s.* 7*d.* *in the* £, *how much did he owe?*

(Examination for Certificates. Males, 1st year. 1871.)

By the rule—

$$\begin{aligned}3\text{s. }7\text{d.} &: 20\text{s.} :: £749,\ 18\text{s. } 4\text{d.} : \textit{the Answer.}\\ \textit{i.e.}\ 43\text{d.} &: 240\text{d.} :: £749\tfrac{11}{12} : \textit{the Answer.}\end{aligned}$$

$$\therefore \textit{The Answer} = \frac{£749\frac{11}{12} \times 240}{43} = £\frac{8999 \times \overset{20}{\cancel{240}}}{\cancel{12} \times 43} = £\frac{179980}{43}$$

$$= £4185\tfrac{25}{43} = £4185,\ 11\text{s. } 7\tfrac{23}{43}\text{d.}$$

**Ex.** 6. *A person, after paying* 6*d.* *in the* £ *income-tax, had* £457, 1*s.* 5¼*d.* *left; what was his yearly income?*

Every pound of his yearly income became 19s. 6d. after paying income-tax; from which it follows that—

$$\begin{aligned}19\text{s. }6\text{d.} &: 20\text{s.} :: £457,\ 1\text{s. } 5\tfrac{1}{4}\text{d.} : \textit{his yearly income.}\\ \textit{i.e.}\ 39\ \textit{sixpences} &: 40\ \textit{sixpences} :: £457,\ 1\text{s. } 5\tfrac{1}{4}\text{d.} : \textit{his yearly income.}\end{aligned}$$

$$\begin{aligned}\therefore \text{His yearly income} &= £457,\ 1\text{s. } 5\tfrac{1}{4}\text{d.} \times \tfrac{40}{39}\\ &= £468,\ 15\text{s. } 10\text{d.},\ \textit{Answer.}\end{aligned}$$

**Ex.** 7. *If a pair of blankets, each measuring* 6 *ft.* 9 *in. by* 5 *ft.* 3 *in., cost* 10*s.* 6*d.*, *what will be the cost of a pair, of like thickness and quality, each measuring* 9 *ft.* 9 *in. by* 8 *ft.* 3 *in.?*

(Examination for Certificates. Females, 1st and 2nd years. 1869.)

It is evident that the cost of the second pair of blankets will vary as their *area*, since the *number*, *thickness*, and *quality* are the same.

Therefore—

*The area of the first pair* : *the area of the second pair* :: *the cost of the first pair* : *the cost of the second pair ;*

*i.e.* (6 ft. 9 in. × 5 ft. 3 in.) : (9 ft. 9 in. × 8 ft. 3 in.) :: 10s. 6d. : *the cost of the second pair ;*

*i.e.* ($6\frac{3}{4}$ ft. × $5\frac{1}{4}$ ft.) : ($9\frac{3}{4}$ ft. × $8\frac{1}{4}$ ft.) :: 10s. 6d. or, £$\frac{21}{40}$. : *the cost of the second pair.*

Therefore—

$$\text{The cost of the second pair} = £\frac{21}{40} \times \frac{9\frac{3}{4} \times 8\frac{1}{4}}{6\frac{3}{4} \times 5\frac{1}{4}}$$

$$= £\frac{21}{40} \times \left(\frac{39}{4} \times \frac{33}{4}\right) \div \left(\frac{27}{4} \times \frac{21}{4}\right)$$

$$= £\frac{\cancel{21}}{40} \times \frac{\overset{13}{\cancel{39}}}{\cancel{4}} \times \frac{\overset{11}{\cancel{33}}}{\cancel{4}} \times \frac{\cancel{4}}{\underset{\underset{3}{\cancel{9}}}{\cancel{27}}} \times \frac{\cancel{4}}{\cancel{21}} = £\frac{143}{120}$$

$$= £1\frac{23}{120} = \text{£1, 3s. 10d., } \textit{Answer.}$$

**Ex. 8.** *If* 5·25 *yards of calico cost* 3s. 8·625*d., what will be the cost of* 367·5 *yards at the same rate ?*

(Examination for Certificates. Females, 1st and 2nd year. 1867.)

By the rule—

5·25 *yards* : 367·5 *yards* :: 3s. 8·625d. or, 44·625d. : *cost of* 367·5 *yards.*

Therefore—

$$\text{The cost of 367·5 yards} = \frac{44·625\text{d.} \times 367·5}{5·25}$$

$$= 3123\tfrac{3}{4}\text{d.} = \text{£13, 0s. } 3\tfrac{3}{4}\text{d., } \textit{Answer.}$$

**103. "Compound Proportion," or, "The Double Rule of Three,"** is the name given to the method by which the fourth term of a proportion is found when the antecedent and the consequent consist each of more than one quantity.

The following example furnishes a case of compound proportion.

**Ex. 1.** *If* 6 *men in* 24 *days of* 8 *hours each set up* 456 *yards of telegraph wire, how many yards will be set up by* 5 *men in* 20 *days of* 10 *hours each ?*

**Here we have the *seven quantities*—6 men, 24 days, 8 hours, 456 yards; 5 men, 20 days, 10 hours.**

**We shall presently see that these *seven quantities* will form but *three "terms,"* and that our business will be to find a fourth term to these three given terms.**

As in Simple Proportion, we shall have two rules to find, viz.—

**1st. A rule for stating the problem as a proportion.**

**2nd. A rule for solving this proportion.**

These rules may be found by the help of the rule for Simple Proportion and the following considerations—

**(1.) 456 yds. of wire were set up by 6 men in 24 days of 8 hours each;**
***i.e.* by 6 men working (24 × 8) hours each;**
***i.e.* in (24 × 8 × 6) working hours.***

**(2.) The number of yards in the answer were set up by 5 men in 20 days of 10 hours each;**
***i.e.* by 5 men working (20 × 10) hours each;**
***i.e.* in (20 × 10 × 5) working hours.**

**Therefore, we may state the question thus—**

**"*If* 456 *yds. of wire were set up in* (24 × 8 × 6) *working hours, how many yards can be set up in* (20 × 10 × 5) *working hours?*"**

**And by the rule for stating Simple Proportion, we have—**

| *Working hrs.* | | *working hrs.* | | *yds.* | | |
|---|---|---|---|---|---|---|
| (24 × 8 × 6) | : | (20 × 10 × 5) | :: | 456 | : | ***Answer.*** |

$$\therefore \textit{Ans.} = \frac{\overset{5}{\cancel{20}} \times \overset{5}{\cancel{10}} \times 5 \times \overset{\cancel{76}\ 19}{\cancel{456}}}{\underset{6}{\cancel{24}} \times \underset{\cancel{4}}{\cancel{8}} \times \cancel{6}} \text{ yds.} = \frac{5 \times 5 \times 5 \times 19 \text{ yds.}}{6} = 395\tfrac{5}{6} \text{ yds.}$$

**We have now arrived at the answer by considering the question as a whole. Let us now work it out by First Principles, preparatory to reasoning out the rule from a more individual consideration of the several parts of the problem.**

---

*** "*A working hour*" is used to signify "*one man working one hour.*"**

**By First Principles**—

1. *The wire set up by* 6 *men in* 24 *days of* 8 *hours each* = **456** yds.

2. „ 1 *man* 24 „ 8 „ $= \frac{456}{\mathbf{6}}$ yds.

3. „ **5** *men* 24 „ 8 „ $= \frac{456 \times \mathbf{5}}{6}$ yds.

So far we have obtained the same result as in section 2 of the rule for stating, page 175.

4. „ 5 *men* 1 *day* 8 „ $= \frac{456 \times 5}{6 \times \mathbf{24}}$ yds.

5. „ 5 „ 20 *days* 8 „ $= \frac{456 \times 5 \times \mathbf{20}}{6 \times 24}$ yds.

So far we have obtained the same result as in section 3 of the rule for stating.

6. „ 5 „ 20 „ 1 *hour* $= \frac{456 \times 5 \times 20}{6 \times 24 \times \mathbf{8}}$ yds.

7. „ 5 „ 20 „ 10 *hours* $= \frac{456 \times 5 \times 20 \times \mathbf{10}}{6 \times 24 \times 8}$ yds.

Which is the same result as that obtained in section 4 of the rule for stating.

We will now consider each part of the question individually—

1st. "*If* 6 *men set up* 456 *yds. of wire, how many yards will be set up by* 5 *men?*"

**By Simple Proportion we have**—

$$6 \textit{ men} : 5 \textit{ men} :: 456 \textit{ yds.} : \textit{Answer}.$$

$$\textit{Answer} = \frac{456 \text{ yds.} \times 5}{6}.$$

2nd. "*If* $\frac{456 \textit{ yds.} \times 5}{6}$ *of wire are set up in* 24 *days, how many yards will be set up in* 20 *days?*"

**By Simple Proportion we have**—

$$24 \textit{ days} : 20 \textit{ days} :: \frac{456 \times 5}{6} \textit{ yds.} : \textit{Answer}.$$

$$\textit{Answer} = \frac{456 \times 5}{6} \text{ yds.} \times \frac{20}{24} = \frac{456 \times 5 \times 20}{6 \times 24} \text{ yds.}$$

3rd. "*If* $\frac{456 \times 5 \times 20}{6 \times 24}$ *yds. of wire are set up when the men work* 8 *hours a day, how many yards will be set up when the men work* 10 *hours a day?*"

**By Simple Proportion we have—**

$$8 \textit{ hrs.} : 10 \textit{ hrs.} :: \frac{456 \times 5 \times 20}{6 \times 24} \textit{ yds.} : \textit{Answer.}$$

$$\textit{Answer} = \frac{456 \times 5 \times 20}{6 \times 24} \text{ yds.} \times \frac{10}{8} = \frac{456 \times 5 \times 20 \times 10}{6 \times 24 \times 8} \text{ yds.}$$

The reason of the following rule will now be evident—

**I. RULE FOR STATING THE PROPORTION—**

**1. For the third term, write the given quantity which is most nearly connected with the answer, and which must always be** *of the same kind as the answer.*

In the above example the third term will therefore be "456 yds."

**2. From the remaining quantities select two which are of the same kind, and which evidently compose one of the several ratios involved in the question, and leaving all the other given quantities out of consideration, state these two quantities as the first and second terms of the proportion, according to the rule for Simple Proportion.**

In the above example consider thus—

"*If* 6 *men can set up* 456 *yards of wire, how many yards can be set up by* 5 *men?*"

This proportion would be stated thus—

*As* 6 *men* : 5 *men* :: 456 *yds. of wire* : *the yards set up by* 5 *men.*

**3. From the remaining quantities select two others, and proceed with them as with the last two. Having determined which shall be the antecedent of this new proportion, place it beneath the antecedent of the last proportion, and deal similarly with the consequent.**

In the above example consider thus—

"*If* 456 *yds. of wire are set up in* 24 *days, how many yards will be set up in* 20 *days?*"

This proportion would be stated thus—

*As* 24 *days* : 20 *days* :: 456 *yds.* : *the yards set up in* 20 *days.*

And therefore, resuming the statement obtained above, we have—

$$\left.\begin{array}{l} 6 \textit{ men} : 5 \textit{ men} \\ \textbf{24 days} : \textbf{20 days} \end{array}\right\} :: 456 \textit{ yds.} : \textit{the Answer.}$$

**4. If any given quantities yet remain, dispose of them as of the last two.**

In the above example consider thus—

"*If* 456 *yds. are set up when the men work* 8 *hours a day, how many yards will be set up when the men work* 10 *hours a day?*"

This proportion would be stated thus—

*As* 8 *hours* : 10 *hours* : : 456 *yds.* : *the yards set up in days of* 10 *hours each.*

And therefore, resuming the statement obtained above, we have—

$$\left.\begin{array}{rcl}6 \textit{ men} & : & 5 \textit{ men}\\ 24 \textit{ days} & : & 20 \textit{ days}\\ 8 \textbf{ hours} & : & 10 \textbf{ hours}\end{array}\right\} :: 456 \textit{ yds.} : \textit{the Answer.}$$

**II. RULE FOR SOLVING THE PROPORTION—**

**Having reduced the antecedent and the consequent in each case to the same denomination, multiply the third term by the product of all the quantities in the second term, and divide the result by the product of all the quantities in the first term.**

In the above example—

$$\frac{5 \times \overset{5}{\cancel{20}} \times \overset{5}{\cancel{10}} \times \overset{\overset{19}{\cancel{57}}}{\cancel{456}}}{\underset{\cancel{2}}{\cancel{6}} \times \underset{6}{\cancel{24}} \times \cancel{8}} \text{ yds.} = \frac{5 \times 5 \times 5 \times 19}{6} \text{ yds.} = \frac{2375}{6} \text{ yds.} =$$

**$395\frac{5}{6}$ yds.,** *Answer.*

**Ex.** 2. *If* 10,000 *persons, each travelling* 30 *miles weekly, give a railway company a receipt of* £1200 *a week, how many persons, travelling each* 25 *miles weekly, will bring an income of* £130,000 *a year, the cost of travelling being reduced one-third?*

(Examination for Certificates. Males, 2nd year. 1865.)

Here £1200 a week = (£1200 × 52) a year = £62400 a year.

We may therefore take the first part of the question thus—

(1.) "*If* 10,000 *persons give a receipt of* £62,400, *how many persons will give a receipt of* £130,000?"

By the rule for Simple Proportion we get the following proportion—

£62400 : £130000 : : 10000 *persons* : *the number of persons giving a receipt of* £130000 (I.)

We may next consider the second part of the question thus—

(2.) "*If* 10,000 *persons, travelling* 30 *miles each, give a certain weekly receipt, how many persons, travelling* 25 *miles each, will give the same receipt?*"

By the rule for Simple Proportion we get the following proportion—

$$25\ \textit{miles} : 30\ \textit{miles} :: 10000\ \textit{persons} : \begin{array}{c}\textit{the number of persons travelling}\\ 25\ \textit{miles.}\end{array}$$

Therefore, resuming the statement obtained before, we have—

$$\left.\begin{array}{lcl}\pounds62400 & : & \pounds130000\\ \mathbf{25\ miles} & : & \mathbf{30\ miles}\end{array}\right\} :: 10000\ \textit{persons} : \textit{the Answer}.\ \text{(II.)}$$

N.B.—The last proportion established involved a case of inverse variation.

The remainder of the question may now be considered thus—

(3.) "*If* 10,000 *persons travel a certain distance for a certain sum of money, how many persons may travel the same distance for the same amount of money, if the cost of travelling be reduced one-third, i.e. if the cost of travelling be in the ratio of* 2 : 3?

By the rule for Simple Proportion we get the following proportion—

$$2 : 3 :: 10000\ \textit{persons} : \textit{the number travelling at the reduced rate.}$$

Therefore, resuming the statement obtained before, we have—

$$\left.\begin{array}{lcl}\pounds62400 & : & \pounds130000\\ 25\ \textit{miles} & : & 30\ \textit{miles}\\ 2 & : & 3\end{array}\right\} :: 10000\ \textit{persons} : \textit{the Answer}.$$

Therefore—

$$\textit{The Answer} = \frac{\overset{\overset{2}{\cancel{50}}}{\cancel{10000}}\ \text{persons} \times \overset{\overset{625}{\cancel{10000}}}{\cancel{130000}} \times 30 \times \cancel{3}}{\underset{\underset{\cancel{16}}{\cancel{208}}}{\cancel{62400}} \times \cancel{25} \times \cancel{2}}$$

$$= 2 \times 625 \times 30\ \text{persons} = \mathbf{37500\ persons.}$$

**Ex. 3.** *A can do a piece of work in* 20 *days, B in* 12 *days; A and B work at it together for* 6 *days, and then C finishes it in* 2 *days; in how many days could C have done it alone?*

(Examination for Certificates. Males, 1st year. 1869.)

(1.) In 20 days A can do all the work.
$\therefore$ in 1 day A can do $\frac{1}{20}$ of the work.
$\therefore$ in 6 days A does $\frac{6}{20}$, *i.e.* $\frac{3}{10}$ of the work.

(2.) In 12 days B can do all the work.
$\therefore$ in 1 day B can do $\frac{1}{12}$ of the work.
$\therefore$ in 6 days B does $\frac{6}{12}$, *i.e.* $\frac{1}{2}$ of the work.

Therefore—

(3.) In 6 days A and B, working together, do $(\frac{3}{10} + \frac{1}{2})$ of the work, *i.e.* $\frac{4}{5}$ of the work.

And therefore, when A and B cease working, $\frac{1}{5}$ of the work yet remains to be done.

This remaining fifth is done by C in 2 days.

Therefore C could do all the work in 2 days × 5, *i.e.* in 10 days.

*Answer*, 10 days.

**Ex. 4.** *If* 3 *horses are worth* 7 *cows, and* 5 *cows cost as much as* 30 *sheep, and* 16 *sheep cost* £165, *what is the value of* 8 *horses?*

By First Principles we have—

16 sheep cost £165.

$\therefore$ 1 sheep ,, £$\frac{165}{16}$.

$\therefore$ 30 sheep ,, £$\frac{165 \times 30}{16}$.

But by the question 5 *cows cost as much as* 30 *sheep*—

$\therefore$ 5 cows cost £$\frac{165 \times 30}{16}$.

$\therefore$ 1 cow ,, £$\frac{165 \times 30}{16 \times 5}$.

$\therefore$ 7 cows ,, £$\frac{165 \times 30 \times 7}{16 \times 5}$.

But by the question 3 *horses cost as much as* 7 *cows.*

$\therefore$ 3 *horses cost* £$\frac{165 \times 30 \times 7}{5 \times 16}$.

$\therefore$ 1 *horse* ,, £$\frac{165 \times 30 \times 7}{5 \times 16 \times 3}$.

$\therefore$ 8 *horses* ,, £$\frac{165 \times 30 \times 7 \times 8}{5 \times 16 \times 3}$ = £1155, *Answer.*

Such problems may more easily be solved by the following process—

(1.) **State the several equations each under each,** as below—

Let $x$ = the number of £ in the Answer.
Then the value of 3 *horses* = the value of 7 *cows*,
" " 5 *cows* = " 30 *sheep*,
" " 16 *sheep* = £165,
and £$x$ = the value of 8 *horses*.

(2.) ***Divide* the product of the numbers in the right-hand column *by* the product of the numbers in the left-hand column.**

Thus—

$$x = \frac{8 \times 165 \times 30 \times 7}{3 \times 5 \times 16} = \pounds 1155, \textit{ Answer.}$$

**104.** The proposition stated in § 71, "*To reduce one quantity to the fraction of another given quantity,*" becomes more intelligible when viewed by the light of Ratio and Proportion.

Thus, the following example may be stated in several ways—

**Ex.** *Reduce 2 tons 3 cwt. to the fraction of 5 tons.*

The same problem may be stated thus—

*What fraction of 5 tons is 2 tons 3 cwt.?*

Or, *Find the ratio subsisting between 2 tons 3 cwt. and 5 tons.*

In the last case, we have—

$$2 \textit{ tons } 3 \textit{ cwt.} : 5 \textit{ tons} :: 43 \textit{ cwt.} : 100 \textit{ cwt.};$$

$$\textit{i.e.} \ \frac{2 \text{ tons } 3 \text{ cwt.}}{5 \text{ tons.}} = \frac{43}{100}, \textit{ Answer.}$$

## Exercise 33.

### Exercises in Simple Proportion.

1. Give the meaning of the terms "ratio" and "proportion," and show how the rule called "proportion" is connected with your definition of proportion.

2. Find the value of $x$ in each of the following proportions—

$3 : 7 :: 21 : x$
$19 : 63 :: 95 : x$
$91 : 84 :: 325 : x$
$7 : x :: 21 : 90$
$2 : 9 :: x : 117$
$15 : x :: 14 : 154$
$x : 102 :: 21 : 9$
$x : 71 :: 3 : 355$
$81 : x :: 14 : 42$
$38 : x :: 19 : 200$
$21 : 71 :: x : 284$
$100 : 91 :: x : 910$

3. If 6 lb. of sugar cost 3s. 9d., what will be the cost of 13 lb.?

4. If 13 lb. of tea cost 52s., how much tea may be bought for £3, 8s.?

5. If 18 cows be purchased for £1827, how many cows may be bought for £1319, 10s.?

6. What will be the cost of 16 cwt. of coals, when 13 cwt. cost 16s. 9d.?

7. How many sheep are worth as much as 18 cows, if 4 cows are worth 10 sheep?

8. If 36 bushels of wheat cost £40, how many bushels of wheat may be bought for £38, 10s.?

9. If 8 men can do a piece of work in 15 days, in how many days could 12 men do the same work?

10. What weight of sugar may be bought for 7s. 9½d., when 13 lb. may be purchased for 5s. 11½d.?

11. If the cost of 11 gas-burners be £33, 16s. 6d. a year, what will be the cost of 14 gas-burners a year?

12. If the price of 18½ yds. of cloth be £17, 2s. 3d., how much cloth may be bought for £41, 12s. 6d.?

13. A bankrupt's debts are £1300, his assets are £800; after payments of costs his creditors receive 6s. 5d. in the £1; what are the costs?

14. If 27 cows are worth 350 sheep, how many sheep are worth 81 cows?

15. If 81 lb. of black tea are worth as much as 90 lb. of green tea, what weight of green tea is equal in value to 108 lb. of black tea?

16. If a clock, which was right at 12 o'clock A.M., points 3.15 P.M. when it is 3 o'clock P.M., what time will it be when the clock points 6 P.M.?

17. What will be the cost of mowing a field of 23 acres 3 roods, when £1, 4s. 9d. is charged for mowing 3 acres?

18. If ⅗ of an estate be worth £1005, 7s. 6d., what will be the value of ⅘ of the estate?

19. If 38 bales of cotton cost £2850, how many bales may be bought for £1200?

20. The rent of 19 acres of land being £14, 5s., what will be the rent of 13 acres?

21. If the railway fare for 76 miles be 14s. 3d., how far may a person travel for £2, 16s. 7½d.?

22. If the railway fare for 77½ miles be 12s. 11d., what ought to be the fare for 93 miles?

23. Find a quantity which bears the same ratio to 5 yds. 3 qrs. 3 nails that 30 days 3 hrs. 29 min. 29 sec. bears to 13 days 16 hrs. 51 min. 35 sec.

24. Find the value of 7 tons 8 cwt., when 5 cwt. are worth £18, 13s. 4d.

25. A bankrupt's debts are £1350, his assets are £850; how much does he pay in the £?

26. A servant is engaged at £7 a year, but leaves at the end of 17 wks. and 4 days; how much ought she to receive?

27. If a cottager spends 8s. a week on flour, what difference will it make to him if the price of wheat rises from £2, 10s. 4d. to £2, 15s. 1⅜d. per quarter, supposing the price of flour to rise in the same proportion?

28. If a bankrupt pays 13s. 7d. in the pound, what will he pay on a debt of £4572? Work the sum also by Practice.

29. If 1½ cwt. of sugar cost £4, 2s. 6d., what will be the cost of 32 tons 18 cwt. 21 lb.?

30. If a man's earnings for 51 days be £19, 19s. 6d., what will be his earnings for 85 days at the same rate?

31. A rate of 1s. 5d. in the pound was levied upon a parish, and produced £4814, 18s. 7½d.; but the collector becoming a defaulter to the amount of £708, 1s. 6¾d., an additional rate was levied to make up this deficiency; how much was this in the pound?

32. A tank contains 406 gallons of water when $\frac{2}{7}$ full; how many gallons does it contain when quite full?

33. If one pound Troy of sterling gold is worth £46, 14s. 6d., what is the united weight of 1246 sovereigns and the same number of half-sovereigns?

34. The rental of a parish is £6720, 12s., and the assessment to the poor-rate £700, 1s. 3d.; how much will the rate be on a farm of 63⅛ acres which is rented at £1 to the acre?

35. A poor man sells his pig for £2, 7s. 4d., and gains thereby a sum of money which stands in the same ratio to his outlay as £17 stands to £100; how much did the pig cost him?

36. The cost of covering a roof 9 yds. by 8 yds. amounted to £27; what would be the cost of covering another roof which is 3 yds. longer and 5 yds. wider than the other, but which, however, has a trap-door in it which is 1 yd. square, and is not to be included in the calculation?

37. If 17 yds. of flannel cost £3, 10s. 10d., how many yards of this flannel must be given, with £14, 14s. 9d., in return for 18 yds. of cloth at 19s. 6d. a yard?

38. In a parish, whose rental is estimated at £16528, 3s. 4d., the total amount of the rate collected is £2169, 6s. 5¼d.; how much is that in the pound? and what will be the rate payable by a person whose rental is £50?

39. The ratio of the *vibrations* of a pendulum to *the number of seconds in which the vibrations are made* is $\frac{1\frac{2}{3}}{\frac{2}{3}}$; how many times will it vibrate in 24 hours.

40. If a silver tankard; weighing 1 lb. 10 oz. 10 dwt., cost £6, 3s. 9d., what will be the value of the silver in the handle of a silver tea-kettle, if the weight of the handle be ⅕ of the weight of the kettle itself, the kettle and handle together weighing 13 lb. 4 oz.?

41. An army is besieging a town in which are 1000 men with provisions for 3 months; how many men must leave it at once that the rest may be able to subsist for a year?

42. A block of stone is 4 ft. long, 2½ ft. broad, and 1¼ ft. thick; it weighs 27 cwt.; find the weight of 100 cubic inches of the stone.

43. A man has an income of £200 a year, and an income-tax is established of 7d. in the £, while a duty is taken off sugar, so that the price of that commodity is reduced by 3s. 6d. on 28 lb.; what must be his yearly consumption of sugar that he may just save his income-tax?

44. If 7 cwt. 3 qrs. 27 lb. of sugar cost £13, 19s. 8¼d., what will be the cost of 12 cwt. 3 qrs. 14 lb. at the same rate? Explain the working of this example.

45. If $\frac{11}{12}$ of a truck of coal weighs 26 cwt. 1 qr. 19 lb., what is the weight of all the coal in the truck?

46. After paying an income-tax of 5d. in the £, there remains £66, 1s. 10½d.; what was the amount on which the tax was paid?

47. The circumference of the driving-wheel of an engine is 16 ft. 6 in., and the train is going at the rate of a mile in 4 minutes; in what time will the wheel make one revolution?

48. Two pieces of cloth, of the same length, cost £11, 3s. 6d. and £14, 8s. respectively. The price of the first was 6s. 2½d. per yard; what was the price of the second?

49. A person builds a house with ⅓ of his money, with ⅖ of the remainder he stocks his farm, and now has £81, 7s. 9d. left; how much had he at first?

50. The cost of carpeting a room, which contains 148 sq. yds., being £90, 13s., what would be the cost of carpeting an ante-room, which is 8 yds. long and 7 yds. wide?

51. If 50 lb. of a certain kind of tea are worth 112 lb. of coffee, how much coffee should be received in exchange for 91 lb. of tea of a quality four times as good as the first?

52. If 2·625 tons cost £36·75, what is the value of 1·5 lb.?

53. A hop merchant exchanges 6 cwt. 3 qrs. 12 lb. of hops at £13 per lb., for sugar at £·03 per lb.; what quantity did he receive?

54. What is the expense of plastering a ceiling at £·075 per yard, when the length is 18 ft. 4 in., and the breadth 14 ft. 3 in.?

55. If 2 cwt. 3 qrs. 13 lb. of cheese cost £5, 13s. 8¼d., how much cheese of a quality twice as good as the last can be bought for £17, 1s. 0¾d?

56. A gallon contains 277·274 cubic inches. How many gallons can there be in a tank 8½ ft. long, 5 ft. broad, and 2½ ft. deep?

57. Paid £43, 15s. for 35 sacks of oats, each weighing 20 stone; what must I give for 4 cwt. 42 lb.?

58. If 2 cwt. 3 qrs. 21 lb. of sugar cost £12, 3s. 4d., what is the value of 17 cwt. 2 qrs. 14 lb.?

59. If 5/7 of a stack of hay is worth £19, what is the value of seven such stacks?

60. A piece of cloth, when measured with a yard measure which is two-thirds of an inch too short, appears to be 10½ yds. long; what is its true length?

61. If a pound weight of standard gold were worth £46, 14s. 6d., how much should one sovereign weigh?

62. A bankrupt owes £900 to three creditors, and his whole property amounts to £675. The claims of two of the creditors are £125 and £375 respectively. What sum will the remaining creditor receive for his dividend?

# Exercise 34.

## Exercises in Compound Proportion.

1. If 8 men can build 33 ft. of wall in 11 days, in how many days will 12 men build 36 ft.?

2. If 10 men can do a piece of work in 6 days, working 10 hours a day, how many hours a day must 8 men work to complete the same undertaking in 15 days?

3. If the carriage of 8 cwt. for 60 miles be 19s., what ought to be the charge for sending 36 cwt. a distance of 38 miles?

4. A railway company charges £1, 8s. 6d. for the carriage of 19 cwt. for 30 miles; what weight ought to be carried 21 miles for 10s. 6d. at the same rate?

5. If 180 soldiers are kept 14 days for £175, how many soldiers may be kept 27 days for £900?

6. If a family of 9 people spend £120 in 8 months, how much will serve a family of 24 people for 16 months at the same rate?

7. If I pay £1, 4s. for the use of £80 for 6 months, how much must I pay for the use of £95 for 8 months at the same rate?

8. A railway train, travelling at 30 miles an hour, travels from one station to another in 150 minutes; how long would it be travelling 3 times as far if the speed be increased to 45 miles an hour?

9. If 18 trucks of coal, each containing 7½ tons, be worth £200, what is the value of 31 trucks of the same coal if each truck contains 9 tons?

10. If the expense of paving a court-yard, which is 21 ft. long and 17 broad, be £36, 12s. 6d., what will be the cost of paving one side of a street which is 680 ft. long and 10½ ft. wide? (Work this sum (1.) as an exercise in Compound Proportion, and (2.) as an exercise in Simple Proportion.)

11. A wall, which was 39 ft. long and 2 yds. high, was built for £25; what was the height of another wall, which was built for £26, and was 75 ft. long?

12. If the rent of a farm of 17 acres 3 roods 3 poles be £39, 4s. 7d., what would be the rent of another farm containing 26 acres 2 roods 23 poles, if 6 acres of the former be worth 7 acres of the latter?

13. A farmer sells a flock of 120 sheep for £1200, 10s.; how many sheep were there in a second flock for which he received £1640, 6s. 8d., the price of each sheep in the first flock being to that of each sheep in the second flock as £60, 0s. 6d. : £61, 10s. 3d.?

14. If 18 apples are worth 14 pears, and 1600 pears are sold for £2, 17s. 6d., what is the price of 14400 apples?

15. The cost of supplying a town with matches for 39 weeks being £17, 1s. 8d., what would have been the cost of the matches used in the same town in 1 year if Mr Lowe's proposed tax had been passed, and the consumption of matches had been thereby reduced by one-fifth of the original consumption?

16. If the sixpenny loaf weighs 36 oz. when wheat is at 55s. per quarter, what should be the weight of an eightpenny loaf when wheat is 60s. per quarter?

17. If the second-class railway fare for 35 miles be 4s. 4½d., what must a first-class passenger pay for travelling 63 miles, if the first-class fare for 3 miles be the same as the second-class fare for 5 miles?

18. If a page of a newspaper, consisting of 6 columns of large print, contain 205000 letters, how many letters will be contained on the page of another newspaper which consists of 7 columns, each as large as one of the former, there being on an average 48 letters in the second newspaper occupying the same space as 41 in the first?

19. The price of 38 bags of oatcake, each bag of which contained 40 lb., was £7, 6s. 8d., what was the weight in each of 27 other bags, if the total cost of the 65 bags was £11, 14s. 5d.?

20. If 40 men require £20 worth of bread in 10 days when wheat is at 63s. per quarter, how long would £90 worth serve 54 men when wheat is at 56s. per quarter?

21. If 24 oxen require 6 acres of turnips to supply them for 10 weeks, how many acres would supply 6 score sheep for 15 weeks, on the supposition that 3 oxen eat as much as 10 sheep?

22. If £25, 11s. 3¾d. pay the carriage of 15 tons 16 cwt. 14 lb. for 240 miles, what weight should be carried 180 miles for £76, 13s. 11¼d., if the rate per mile in the second case be $\frac{7}{6}$ of the rate in the first case?

23. If 35 men, working 8 hours a day, can build a wall 1700 yds. long, 13 ft. high, and 2 ft. thick, in 130 days, how many hours a day must 40 men work in order to construct a wall 1⅓ miles long, 17 ft. high, and 2½ ft. thick, in 210 days?

24. A canal, which was 14⅓ miles long, 21 yds. wide, and 19½ ft. deep, was dug by 1200 men in 8 months. What was the depth of another canal, which was dug by 1500 men in a year, it being 17½ miles long and 72 ft. wide, remembering that the time employed in completing a mile of the second canal, with a given number of men, was (owing to greater difficulties in cutting the rock) to the time employed over a mile of the first as 13 : 8.

## Exercise 35.

### Miscellaneous Exercises involving Proportion.

1. If 2·625 lb. of butter cost 3s. 8·625d., what will be the cost of 183·75 lb. at the same rate?

2. An orange-girl buys 100 oranges for 3s. 6d. Of these 4 turn out bad. How many ought she to sell for a shilling, so that when all are sold she may have her first outlay and a profit of 2s. 6d.?

3. A railway train, going at the rate of 16⅘ miles per hour, travels a certain distance in 4 7/12 hours; how long would a train, going at the rate of 20⅜ miles an hour, be in travelling ⅜ of the distance?

4. A man pays a house-rate of 1s. 6d. per £1 on his rental; a water-rate of 1s. per £1; a poor-rate of 1s. 10½d. per £1. Rent and rates amounting to £85, 6s. 3d., what is the rent?

5. A farmer paid £780 for cows and sheep. Of this sum he paid £350 for 25 cows; if a cow cost seven times as much as a sheep, how many sheep did he buy with the rest of his money?

6. Which is cheaper—a piece of cloth yard-wide, at 4s. 6d. per yard, or one 27 inches wide, at 3s. 9d. per yard? If 12 yards of the former are sufficient for a dress, how much will be required of the latter?

7. If 3 bushels cost 1·1 of a £, what will 33·4 qrs. cost at the same rate?

8. How many yards of stuff ¾ yd. wide will line a cloak that is 5½ yds. in length, and 1¼ yds. wide?

9. If a person were to count 80 sovereigns a minute, and continue counting 12 hours each day, how many days would he be occupied in

counting one instalment of the French indemnity, supposing the instalment to be 200000000 francs, paid in English money, and that four English shillings are worth five French francs?

10. If 3 men or 4 women can do a piece of work in 56 days, in what time will one man and one woman (working together) do the work?

11. The annual deaths in a town being 1 in 45, in the country 1 in 50, in how many years will the number of deaths, out of 18675 persons living in the town, and 79250 persons living in the country, amount to 10000?

12. If A can do as much work in 5 hours as B can do in 6 hours, or as C can do in 9 hours, how long will it take C to complete a piece of work, one-half of which has been done by A, working 12 hours, and B, working 24 hours?

13. The income-tax being 7d. in the £, a person has to pay £63 less than when the tax was 11d., although his income has decreased by £300. What was it at first?

14. If 15 horses and 148 sheep can be kept for 9 days for £75, 15s., what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses eat as much as 84 sheep?

15. Thirty-five women can do as much as twenty boys, and sixteen boys as much as seven men; how many women can do the work of thirteen men?

16. A pond, when full, can be emptied by one pipe in 72 minutes, and can be filled again by another pipe, which discharges 36 gallons a minute, in 22 minutes; if the pond have 17 gallons of water in it, and both pipes be opened, how long will it take to fill?

17. A and B fire at targets, having 35 cartridges each. A fires twice in 3 minutes, and B three times in five minutes; how many times will B have to fire after A has finished?

18. The weight of a cubic inch of water is 253·17 grains, that of a cubic inch of air ·310017 grains; how many cubic inches of air are equal in weight to one cubic foot of water?

19. If 9000 persons, travelling 20 miles weekly, give a railway company a net receipt of £900 in one week; how many persons, travelling 30 miles weekly, will give a net receipt of £62400 per annum, when the cost of travelling per mile is reduced one-half?

20. A gallon of water weighs 10 lb., and a cubic foot of water weighs 1000 ounces; how many gallons are there in a cubic foot?

# INTEREST.

**105. Interest** is the sum of money paid for the use of some other sum of money borrowed for a certain time, and at a fixed rate.

Money is generally borrowed at so much for £100 for one year.

"**The Principal**" is the name given to the money borrowed.

"**The Amount**" is the name given to the principal and interest together.

"**The Rate per Cent.**" is the name given to the interest on £100 for one year.

**Ex.** *If* £100 *be borrowed for* 1 *year at* 5 *per cent.*—
*The principal* is £100.
*The interest* will be £5.
*The amount* will be £100 + £5 = £105.

N.B. 1.—"*The rate per cent.*" always means "The rate per cent. *per annum*"—*i.e.* "*for one year*," unless the contrary be expressed.

N.B. 2.—In commerce several abbreviations are used—viz., % for "*per cent.;*" @ for "*at;*" a/c for "*account;*" o/c for "*on account.*"

When interest is reckoned on the original principal only (*i.e.* only on the sum of money first borrowed), it is called **Simple Interest.**

Sometimes the interest is not paid as soon as it is due, but is added to the original principal to form a new principal, on which interest is again calculated for a certain period, and then this interest is again added to its principal, thus forming a third principal, &c. In such cases the interest is called **Compound Interest.**

Thus, if £100 be borrowed for 2 yrs., at 5 per cent. per annum, the *simple interest* will be £10—*i.e.* £5 for the first year, and the same for the second year; but the *compound interest* will be £5 for the first year, and 5 per cent. on (£100 + 5—*i.e.* on) £105 for the second year, *i.e.*—

£5 + £5, 5s., *i.e.* £10, 5s., *i.e.* 5s. more than the simple interest.

## I. Simple Interest.

### 106. To find the Simple Interest on a given sum of money at a given rate per cent.

**Ex. 1.** *Find the Simple Interest on £925 at 4 per cent.*

(1.) The question may be expressed as one in Simple Proportion, thus—

*If the interest on £100 be £4, what will be the interest on £925?*

From which we have—

£100 : £925 :: £4 : *the interest on* £925.

$$\therefore \text{ The interest on £925} = \frac{\text{£}4 \times 925}{100} = \text{£}37, \textit{ Answer.}$$

Or—

(2.) We may reason thus—

Since the interest is £4 for every "cent.," *i.e.* for every £100 borrowed, the total interest will be—

£4 × the number of cents. borrowed.

Now, the number of cents. borrowed = £925 ÷ 100 = 9¼.

Therefore the total interest = £4 × 9¼ = £37, *Answer.*

Hence the reason of the following rule—

**RULE.—Multiply the Principal by the Rate per cent., and divide the product by 100.**

**Ex. 2.** *What is the Simple Interest on £368, 6s. 8d. at 5 per cent.?*

Here the principal × the rate per cent. = £1841, 13s. 4d. Marking off two decimal places in *the pounds* (*i.e.* dividing by 100), we have £**18**·41. Multiplying the ·41 by 20, and adding in the 13 shillings, we have 833 shillings, which, being divided by 100, gives **8**·33 *shillings.* Again, multiplying the ·33 by 12, and adding in the 4d., we have 400 pence, which, being divided by 100, gives **4** *pence.*

$$\begin{array}{r} \text{£}368,\ 6\text{s.}\ 8\text{d.} \\ 5 \\ \hline \text{£}\mathbf{18}\cdot41,\ 13\text{s.}\ 4\text{d.} \\ 20 \\ \hline \text{s.}\ \mathbf{8}\cdot33 \\ 12 \\ \hline \text{d.}\ \mathbf{4}\cdot00 \end{array}$$

*Answer*, £18, 8s. 4d.

### 107. To find the Simple Interest on a given sum of money at a given rate per cent. for any number of years or any portion of a year.

By § 106 we can find *the Simple Interest on any given sum at any given rate* **for one year**; it is clear, therefore, that *the Simple Interest on any given sum at any given rate* **for any number of years** may be found by first finding the interest for one year, and then multiplying it by the given number of years, thus—

**Ex. 1.** *Find the Simple Interest on* £925 *for* $2\frac{1}{3}$ *yrs. at* 4 *per cent. per annum.*

By § 106, the simple interest on £925 for one year at 4 per cent. =

$$\frac{£925 \times 4}{100};$$

Therefore, the simple interest on £925 for $2\frac{1}{3}$ yrs. at the same rate =

$$\frac{£925 \times 4}{100} \times 2\frac{1}{3} = \frac{\overset{37}{\cancel{925}} \times \cancel{4}}{\underset{\cancel{25}}{\cancel{100}}} \times \frac{7}{3} = £86,\ 6\text{s. } 8\text{d.}$$

Hence the reason of the following rule—

**RULE.—Multiply the Principal by the Rate per cent. and by the number of years, and divide the product by 100.**

**Ex. 1.** *Find the Simple Interest on* £925 *for* $2\frac{1}{3}$ *yrs. at* 4 *per cent. per annum.*

$$\begin{array}{r} £925 \\ 4 \\ \hline 3700 \\ 2\frac{1}{3} \\ \hline 7400 \\ 1233,\ 6\text{s. } 8\text{d.} \\ \hline £\mathbf{86}{\cdot}33,\ 6\text{s. } 8\text{d.} \\ 20 \\ \hline \text{s. }\mathbf{6}{\cdot}66 \\ 12 \\ \hline \text{d. }\mathbf{8}{\cdot}00 \end{array}$$

Or, dividing by 100 earlier in the process, we have—

$$\begin{array}{r} £925 \\ 4 \\ \hline £37{\cdot}00 \\ 2\frac{1}{3} \\ \hline £74 \\ 12,\ 6\text{s. } 8\text{d.} \\ \hline \mathbf{£86,\ 6s.\ 8d.} \end{array}$$

*Answer*, £86, 6s. 8d.

**Ex. 2.** *Find the Simple Interest on* £925 *from January* 6*th till June* 17*th*, 1871, *at* 4 *per cent. per annum.*

From January 6th till June 17th is 162 days. (January 6th does not reckon as a day).

Therefore—

The simple interest $= £\frac{\overset{3\,7}{\cancel{9}\cancel{2}\cancel{5}} \times \cancel{4}}{\underset{\cancel{2}\cancel{5}}{\cancel{1}\cancel{0}\cancel{0}}} \times \frac{162}{365} = £\frac{37 \times 162}{365} = £16,\ 8s.\ 5\frac{19}{73}d.$

**Ex. 3.** *Find the Simple Interest on £925 for 3 yrs. 73 days at* $4\frac{1}{3}$ *per cent. per annum.*

Here 73 days $= \frac{73}{365}$ of a year (§ 71) $= \frac{1}{5}$ of a year.
Therefore, 3 yrs. 73 days $= 3\frac{1}{5}$ yrs.

$$\therefore \text{ The simple interest} = £\frac{925 \times 4\frac{1}{3} \times 3\frac{1}{5}}{100} = £\frac{\overset{\cancel{1}\cancel{8}\cancel{5}\ 37}{\cancel{9}\cancel{2}\cancel{5}} \times 13 \times \overset{4}{\cancel{1}\cancel{6}}}{\underset{\cancel{2}\cancel{0}\ 5}{\cancel{1}\cancel{0}\cancel{0}} \times 3 \times \cancel{5}} =$$

$$£\frac{37 \times 13 \times 4}{5 \times 3} = £\frac{1924}{15} = £128\tfrac{4}{15} = £128,\ 5s.\ 4d.$$

**Ex. 4.** *Find the Amount of £1850 for* $7\frac{1}{5}$ *yrs. at* $3\frac{3}{4}$ *per cent. per annum.*

$$\text{The simple interest} = £\frac{1850}{100} \times 3\tfrac{3}{4} \times 7\tfrac{1}{5} = £\frac{\overset{3\,7}{\cancel{1}\cancel{8}\cancel{5}\cancel{0}}}{\underset{2}{\cancel{1}\cancel{0}\cancel{0}}} \times \frac{\overset{3}{\cancel{1}\cancel{5}}}{\cancel{4}} \times \frac{\overset{9}{\cancel{3}\cancel{6}}}{\cancel{5}} =$$

$$£\frac{37 \times 3 \times 9}{2} = £\frac{999}{2} = £499,\ 10s.$$

Therefore—

The principal + the interest = £1850 + £499, 10s.
= £2349, 10s., *Answer.*

**Ex. 5.** *Find the Simple Interest on £850, 11s. 3d. for 249 days at £3, 16s. 6d. per cent. per annum.*

Here 249 days $= \frac{249}{365}$ of a year;
And £3, 16s. 6d. $= £3\frac{33}{40}$;
And £850, 11s. 3d. $= £850\frac{9}{16}$.

∴ The simple interest on £850, 11s. 3d. for 249 days at £3, 16s. 6d. per cent. =

The simple interest on $£850\frac{9}{16}$ for $\frac{249}{365}$ yr. at $£3\frac{33}{40}$ per cent. =

$$£3\tfrac{33}{40} \times £\frac{850\frac{9}{16}}{100} \times \frac{249}{365} = £\frac{153}{40} \times \frac{13609}{1600} \times \frac{249}{365} = £\frac{518462073}{23360000} =$$

$$£22\frac{4542073}{23360000} = £22,\ 3s.\ 10\tfrac{1}{2}d. + \textit{ Answer.}$$

**Or, Decimals and the method of Practice—**

£850, 11s. 3d. = £850$\frac{9}{16}$ = £850·5625.

The simple interest on £850·5625 for 1 yr. at 1 per cent. = $\frac{1}{100}$ of £850·5625 = £8·505625

3

| | | | |
|---|---|---|---|
| | £25·516875 | = Interest at £3 per cent. | |
| 10s. = £$\frac{1}{2}$. | 4·2528125 | = ,, | 10s. per cent. |
| 5s. = $\frac{1}{2}$ of 10s. | 2·12640625 | = ,, | 5s. per cent. |
| 1s. = $\frac{1}{5}$ of 5s. | ·42528125 | = ,, | 1s. per cent. |
| 6d. = $\frac{1}{2}$ of 1s. | ·212640625 | = ,, | 6d. per cent. |

Therefore, £32·534015625 = Total interest for 1 yr.

30

365 | £976·020468750

£2·67402868, &c. = Interest for 30 days.

73 days = $\frac{1}{5}$ of 1 yr. {
6·50680312, &c. = ,, 73 ,,
6·50680312, &c. = ,, 73 ,,
6·50680312, &c. = ,, 73 ,,

Therefore the total = £**22**·19443804

20

s. **3**·88876080

12

d. **10**·6651296

*Answer*, £22, 3s. 10$\frac{1}{2}$d. +

**108.** In the following cases the charges are usually made at so much per cent., and the questions therefore resolve themselves into questions on Simple Interest, and may be worked by the rule for Simple Interest given above (§ 107)—

1. **Commission,** which is the sum of money charged by one merchant for buying or selling goods for another.
2. **Brokerage,** which is that particular kind of commission which relates to money transactions rather than to dealings in goods or merchandise.

3. **Insurance**, which is a contract by which one party, in consideration of being paid a certain sum of money (called a *premium*), engages to restore to the payer the value of certain property in case that property should be lost.

**109.** It will be noticed that questions in interest involve the consideration of *four* items—viz., 1. **Principal**; 2. **Rate per Cent.**; 3. **Time**; 4. **Total Interest**; and it may now be shown that when any *three* of these are given the fourth may always be found, as in the following examples—

### I. To find the TIME, when the Principal, Interest (or Amount), and Rate are given.

**Ex.** *In what time will £380 amount to £433, 8s. 9d. at 6¼ per cent. per annum?*

The interest = the amount − the principal
= £433, 8s. 9d. − £380
= £53, 8s. 9d.

Now the interest on £380 *for one year* at 6¼ per cent. =

$$£6\tfrac{1}{4} \times \frac{380}{100} = £\frac{\overset{5}{\cancel{25}}}{4} \times \frac{\overset{19}{\cancel{380}}}{\underset{\cancel{5}}{\cancel{100}}} = £\frac{95}{4} = £23,\ 15s.$$

We have now reduced the question to one of Simple Proportion, thus—

"*If a sum of money at a given rate produce an interest of £23, 15s. in 1 year, in what time will the same sum at the same rate produce £53, 8s. 9d.?*"

From which we get the following statement—

£23, 15s. : £53, 8s. 9d. :: 1 year : Answer,

*i.e.* $£23\tfrac{3}{4}$ : $£53\tfrac{7}{16}$ :: 1 year : Answer.

$$\therefore \text{The time} = \frac{1 \text{ yr.} \times 53\tfrac{7}{16}}{23\tfrac{3}{4}} = \frac{\overset{\overset{9}{\cancel{171}}}{\cancel{855}}}{\underset{4}{\cancel{16}}} \times \frac{\cancel{4}}{\underset{\cancel{19}}{\cancel{95}}} \text{yr.} = \frac{9}{4} \text{ yr.} = 2\tfrac{1}{4} \text{ yrs.}$$

### II. To find the PRINCIPAL, when the Rate, Time, and Interest (or Amount) are given.

**Ex.** *What sum of money put out to interest for $2\frac{1}{4}$ yrs. at $6\frac{1}{4}$ per cent. per annum will amount to £433, 8s. 9d.?*

The simple interest on £100 for the given time at the given rate will be £$6\frac{1}{4} \times 2\frac{1}{4}$.

∴ £100, put out to simple interest for the given time at the given rate, will amount to £100 + £$6\frac{1}{4} \times 2\frac{1}{4}$ = £100 + £$14\frac{1}{16}$ = £$114\frac{1}{16}$.

We have now reduced the question to one of Simple Proportion, thus—

"*If* £100 *put out to interest for a given time at a certain rate per cent. amount to* £$114\frac{1}{16}$, *what sum of money must be put out for the same time at the same rate to amount to* £433, 8*s.* 9*d.*?"

From which we get the following statement—

£$114\frac{1}{16}$ : £$433\frac{7}{16}$ :: £100 : the principal which amounted to £$433\frac{7}{16}$.

Therefore, the principal which amounted to £$433\frac{7}{16}$ = £$\frac{433\frac{7}{16} \times 100}{114\frac{1}{16}}$ =

$$£\frac{\overset{95}{\cancel{6935}} \times \overset{4}{\cancel{100}}}{\cancel{16}} \times \frac{\cancel{16}}{\underset{\cancel{73}}{\cancel{1825}}} = £380, \textit{ Answer.}$$

N.B.—£100 is taken for the principal because of its convenience, but any principal will obviously give the answer by the above method.

### III. To find the RATE, when the Principal, Time, and Interest (or Amount) are given.

**Ex.** *At what rate per cent. per annum will* £380 *amount to* £433, 8*s.* 9*d.* *in* $2\frac{1}{4}$ *yrs.?*

As in Case I., by subtracting the principal from the amount, we first find that the interest is £53, 8s. 9d., *i.e.* £$53\frac{7}{16}$.

Next, we assume any rate per cent., 1 per cent. being the most convenient, and proceed thus—

Simple interest on £380 for $2\frac{1}{4}$ yrs. at 1 per cent. = £$2\frac{1}{4} \times \frac{380}{100}$ =

$$£\frac{9}{\underset{2}{\cancel{4}}} \times \frac{\overset{19}{\cancel{380}}}{10\cancel{0}} = £\frac{171}{20} = £8, 11s.$$

We have now reduced the question to one of Simple Proportion, thus—

"*If* £380 *put out to simple interest for a certain time at* 1 *per cent. produce* £8, 11*s. interest, at what rate must it be put out to produce* £53, 8*s.* 9*d. in the same time?*"

From which we get the following statement—

$$\pounds 8\tfrac{11}{20} : \pounds 53\tfrac{7}{16} :: 1 \text{ per cent.} : \text{the rate which gives } \pounds 53\tfrac{7}{16}.$$

Therefore the rate which gives $\pounds 53\frac{7}{16} = \pounds\frac{53\frac{7}{16}\times 1}{8\frac{11}{20}} = \pounds\frac{\overset{5}{\cancel{855}}}{\underset{4}{\cancel{16}}} \times \frac{\overset{5}{\cancel{20}}}{\cancel{171}}$

$$= \pounds\frac{5\times 5}{4} = \pounds\frac{25}{4} = \pounds 6\tfrac{1}{4}, \textit{ Answer.}$$

N.B.—The following particular cases of the above rules deserve attention—

## 1. A PARTICULAR CASE OF RULE 1.

**To find the time in which a sum of money will double itself if put out to Simple Interest at any given rate per cent. per annum.**

**Ex.** 1. *In what time will £8, 7s. double itself (i.e. gain £8, 7s.) if put out to Simple Interest at* $6\frac{2}{3}$ *per cent. per annum?*

Here we have—

$$\text{Interest} : \text{principal} :: 6\tfrac{2}{3} : 100$$

$$\therefore \text{Principal} \times 6\tfrac{2}{3} = \text{interest} \times 100$$

$$\therefore \text{Principal} = \text{interest} \times \frac{100}{6\frac{2}{3}} = \text{interest} \times 15.$$

Therefore, the principal must gain interest 15 times to double itself, *i.e.* it must be put out to interest for 15 yrs.

Hence the reason of the following rule—

**RULE.—Divide 100 yrs. by the number expressing the rate per cent.**

The advantage of remembering this rule will be better understood from the following application of it.

**Ex. 2.** *In what time will £48, 12s. gain £12, 3s. at $6\frac{1}{4}$ per cent. per annum simple interest?*

Here £12, 3s. is $\frac{1}{4}$ of £48, 12s.

And £48, 12s. will double itself at $6\frac{1}{4}$ per cent. per ann. in $\left(\frac{100}{6\frac{1}{4}}\right.$ *i.e.* in$\left.\right)$ 16 yrs.

∴ £48, 12s. will gain (£12, 3s. *i.e.*) $\frac{1}{4}$ of itself in $\frac{1}{4}$ of 16 yrs., *i.e.* in 4 yrs.

2. A PARTICULAR CASE OF RULE 3.

**To find the rate per cent. per annum at which a sum of money will double itself if put out to Simple Interest for any given number of years.**

**Ex. 1.** *At what rate per cent. per annum will £71, 4s. 6d. double itself in $12\frac{1}{2}$ yrs.?*

Let T = the time in which } a sum of money will double itself.
and R = the rate at which }

Then, as above, we have

$$T = \frac{100}{R} \text{ (I.)}$$

From which it follows, that

$$T \times R = 100$$

and therefore, that

$$R = \frac{100}{T} \text{ (II.)}$$

Hence the reason of the following rule—

RULE.—Divide 100 per cent. by the given number of years.

Ex. 1 may be worked thus by the rule—

$$\textit{Answer} = \frac{100}{12\frac{1}{2}} = 8 \text{ per cent.}$$

Therefore £71, 4s. 6d. must be lent at 8 per cent. simple interest in order that it may double itself in $12\frac{1}{2}$ years.

This rule may also be advantageously applied thus—

**Ex. 2.** *At what rate per cent. simple interest must £96 be invested in order to gain £32 in $6\frac{2}{3}$ yrs.?*

By the rule £96 will gain £96 in $6\frac{2}{3}$ yrs. at $\left(\frac{100}{6\frac{2}{3}}, \text{ i.e. at}\right)$ 15 per cent. simple interest.

∴ £96 will gain (£32, *i.e.*) $\frac{1}{3}$ of itself in $6\frac{2}{3}$ yrs. at ($\frac{1}{3}$ of 15 per cent., *i.e.* at) 5 per cent.

## II. Compound Interest.

### 110. To find the Compound Interest on a given sum of money at a given rate per cent. for a given number of years.

From what we have already said (§ 105 and 107), the following rule for the above will be obvious—

**RULE.—Find the *Simple* Interest on the given sum at the given rate for one year, add this interest to the original principal, and take the amount as the principal for the next year. Repeat this process as many times as required by the number of years in the question. The sum of the interests found for each successive year will be the Compound Interest required.**

**Ex. 1.** *Find the Compound Interest of £850 for 3 yrs. at 5 per cent. per annum.*

| | |
|---|---|
| The first year's principal | = £850, 0s. 0d. |
| ,, ,, interest = $\frac{5}{100}$ of £850 | = 42, 10s. 0d. |
| ∴ The second year's principal | = £892, 10s. 0d. |
| ,, ,, interest = $\frac{5}{100}$ of £892, 10s. | = 44, 12s. 6d. |
| ∴ The third year's principal | = £937, 2s. 6d. |
| ,, ,, interest = $\frac{5}{100}$ of £937, 2s. 6d. | = 46, 17s. 1½d. |
| ∴ The principal at the end of the third year | = £983, 19s. 7½d. |
| But the original principal | = 850, 0s. 0d. |
| ∴ The compound interest | = £133, 19s. 7½d. *Ans.* |

Or, by the rule—

The Compound Interest = the sum of the interests gained in each year
= £42, 10s. + £44, 12s. 6d. + £46, 17s. 1½d.
= £133, 19s. 7½d. *Answer.*

It is often easier to work out the answer by decimals, thus—

| | | |
|---|---|---|
| The first year's principal | = | £850 |
| „ „ interest = $\frac{5}{100}$ of £850 | = | 42·5 |
| ∴ The second year's principal | = | £892·5 |
| „ „ interest = $\frac{5}{100}$ of 892·5 | = | 44·625 |
| ∴ The third year's principal | = | £937·125 |
| „ „ interest = $\frac{5}{100}$ of £937·125 | = | 46·85625 |
| ∴ The principal at the end of the third year | = | £983·98125 |
| But the original principal | = | 850· |
| ∴ The compound interest | = | £133·98125 |
| | | 20 |
| | | s. 19·62500 |
| | | 12 |
| | | d. 7·500 |
| | | 4 |
| ∴ *Answer*, £133, 19s. 7½d. | | f. 2·0 |

Sometimes interest is payable half-yearly or quarterly, and in such cases the Compound Interest is different from that obtained when it is payable yearly, because in the former case, the interest being sooner added to the principal, the Compound Interest increases faster. In all such cases the interest must be added to the principal as soon as it is due as in the following example—

**Ex. 2.** *Find the amount of £800 at Compound Interest for 2 yrs. at 5 per cent. per annum, the interest being payable half-yearly.*

The operation will evidently be the same as that for finding the *compound interest on £800 for 4 half-years at 2½ per cent. per half-year.*

**Therefore—**

**The principal for the first half-year = £800**

**The interest ,, ,, $= \frac{2\frac{1}{2}}{100}$ of £820 = 20**

**∴ The principal for the second half-year = £820**

**The interest ,, ,, $= \frac{2\frac{1}{2}}{100}$ of £820 = 20·5**

**∴ The principal for the third half-year = £840·5**

**The interest ,, ,, $= \frac{2\frac{1}{2}}{100}$ of £840·5 = 21·0125**

**∴ The principal for the fourth half-year = £861·5125**

**The interest ,, ,, $= \frac{2\frac{1}{2}}{100}$ of £861·5125 = 21·5378125**

**∴ The amount at the end of the fourth half-year = £883·0503125**

20

s. 1·0062500

12

d. 07500

*Answer*, £883, 1s. ·075d.

**Ex. 3.** *What sum of money put out at 5 per cent. Compound Interest will amount to £8372, 2s. 2¼d. in 2¼ yrs.?*

*First*—£8372, 2s. 2¼d. = £8372·109375.

*Second*—Taking £100 as a "trial-principal," we find that £100 in 2¼ yrs. at 5 per cent. Compound Interest amounts to £111·628125, from which we get the following question in Simple Proportion—

"*If £100 amounts to £111·628125 when put out to Compound Interest for the given time and at the given rate, what sum of money will amount to £8372·109375 in the same time and at the same rate?*"

Which may be resolved into the following proportional statement—

£111·628125 : £8372·109375 :: £100 : the Answer.

Therefore,

$$\textit{The Answer} = £\frac{8372{\cdot}109375 \times 100}{111{\cdot}628125} = £7500.$$

**111.** Before leaving the subject of Compound Interest, it may be as well to explain another method by which the "AMOUNT" of a sum of money may be calculated at Compound Interest.

In finding the Compound Interest at any rate per cent. the second principal is found by adding to the original principal a fraction of itself, which fraction has the rate per cent. for its numerator, and 100 for its denominator.

Thus, in calculating Compound Interest at 6 per cent. per annum, we add to the first year's principal $\frac{6}{100}$ of itself to make the second year's principal.

The same thing is repeated to find the third principal, and again to find the fourth, &c.; so that, to take an example, in calculating the Compound Interest at 6 per cent. on £850 for three years, we have—

The *first* principal = the given sum = £850.
The *second* principal = the first principal + $\frac{6}{100}$ of the first principal
= $1\frac{6}{100}$ of the first principal
= $1{\cdot}06$ of the first principal
= $£850 \times 1{\cdot}06$.
The *third* principal = the second principal + $\frac{6}{100}$ of the second principal
= $1\frac{6}{100}$ of the second principal
= $1{\cdot}06$ of the second principal
= $(£850 \times 1{\cdot}06) \times 1{\cdot}06$
= $£850 \times (1{\cdot}06)^2$.
The *amount*, which is another name for the *fourth* principal } = the third principal + $\frac{6}{100}$ of the third principal
= $1\frac{6}{100}$ of the third principal
= $1{\cdot}06$ of the third principal
= $\{ £850 \times (1{\cdot}06)^2 \} \times 1{\cdot}06$
= $£850 \times (1{\cdot}06)^3$.

Now, it will be noticed that, in the above formula, *the decimal fraction is raised to the same power as the number of years for which compound interest is to be calculated*, from which we may deduce the following rule—

**RULE.—To find the sum to which any principal will amount if put out to Compound Interest at a given rate in a given number of years, add a hundredth of the rate to 1, raise the sum to that power which is denoted by the given number of years, and multiply the result by the number of pounds in the given principal.**

**Ex. 1.** *Find* **the Amount** *of* £850 *in* 3 *yrs. at* 6 *per cent. Compound Interest.*

**By the rule—**

$$\begin{aligned}\text{The amount} &= £850 \times (1·06)^3\\ &= £850 \times 1·191016\\ &= £1012·3636\\ &= £1012,\ 7s.\ 3·264d.,\ \textit{Answer.}\end{aligned}$$

It will be seen that the difficulty here is to find the third power of 1·06, which difficulty would have been considerably increased if the given number of years had been greater. This difficulty may be overcome by the use of *Compound Interest Tables*, or by the use of *Tables of Logarithms*, or it may be obviated by the following arithmetical process—

(1.) *Calculate the amount at Compound Interest on £1 for the given time at the given rate.*

Thus, to take the above example—

£1 = the *principal* for the first year;
Therefore £1·06 = the *amount* at the end of the first year.
And $\frac{6}{100}$ of £1·06
= $\frac{1}{100}$ of £6·36 = ·0636
Therefore £1·1236 = the *amount* „ second „
And $\frac{6}{100}$ of £1·1236 = ·067416
Therefore £1·191016 = the *amount* „ third „

(2.) *Multiply the amount of £1 by the number of pounds in the principal*, thus—

£1·191016 × 850 = £1012·3636 = £1012, 7s. 3·264d., *Answer.*

[If the Compound Interest were required, it would be found thus—

The amount = £1012, 7s. 3·264d.
The principal = 850, 0s. 0d.
The interest = £162, 7s. 3·264d., *Answer.*]

**Ex. 2.** *Let it now be required to find what* **Principal** *will amount to* £1012·3636 *in* 3 *yrs. at* 6 *per cent. per annum Compound Interest.*

From the preceding explanations, we see that—

$$\begin{aligned}\text{The pounds in the principal} \times (1·06)^3 &= £1012·3636\\ \therefore \text{The pounds in the principal} &= £1012·3636 \div (1·06)^3\\ &= £1012·3636 \div 1·191016\\ &= £850,\ \textit{Answer.}\end{aligned}$$

**Ex. 3.** *Let it now be required to find at what* **Rate** *of Compound Interest* £850 *will in* 3 *yrs. amount to* £1012·3636.

Let R = the number expressing the rate per cent. per annum. Then we have—

$$£850 \times \left(1\frac{R}{100}\right)^3 = £1012·3636.$$

$$\therefore £1012·3636 \div £850 = \left(1\frac{R}{100}\right)^3;$$

$$i.e.\ 1·191016 = \left(1\frac{R}{100}\right)^3;$$

$$\therefore \sqrt[3]{1·191016} = 1\frac{R}{100};$$

$$i.e.\ 1·06 = 1\frac{R}{100}.$$

Subtracting 1 from each side of the equation, we have—

$$·06 = \frac{R}{100}.$$

Multiplying each side of the equation by 100, we have—

$$6 = R.$$

Therefore the rate per cent. = 6, *Answer.*

[It is evident that when the number of years is greater or more complicated, the use of logarithms is required to solve the equation.]

**Ex. 4.** *Let it now be required to find the* **Time** *in which* £850 *will amount to* £1012·3636 *when put out to Compound Interest at* 6 *per cent. per annum.*

Let T = the time in years.

Then we have—

$$£850 \times (1·06)^T = £1012·3636.$$

$$\text{Therefore } £1012·3636 \div £850 = (1·06)^T;$$

$$i.e.\ 1·191016 = (1·06)^T.$$

By Logarithms we find that "T" must be 3, because 1·06 must be raised to the third power to make 1·191016.

*Answer*, 3 years.

**112. A few examples involving questions of Interest are here given—**

**Ex. 1.** *What is the amount of an annuity of £150 left unpaid for 4 years, Compound Interest at 4 per cent. per annum being allowed thereon?*

**Note.**—When an annuity is payable at present, it is said to be *in possession;* but between entering on possession and the first payment of the annuity, one of the equal intervals at which the annuity is payable is always supposed to elapse.

*The Amount* of an annuity, the payment of which has been forborne for a certain number of terms, is the sum of all the payments, each improved at Compound Interest, from the period of its becoming due until the whole is paid.

| | |
|---|---|
| At the *beginning* of the *second* year there is due . . | £150· |
| ∴ The interest due at the *end* of the second year is . . | 7·5 |
| The annuity due at the end of the second year for that year is | £150· |
| ∴ The principal for the *third* year will be . . . . . | £307·5 |
| The interest ,, ,, . . . . . | 15·375 |
| The annuity ,, ,, . . . . . | 150· |
| ∴ The principal for the *fourth* year will be . . . | £472·875 |
| The interest ,, ,, . . . | 23·64375 |
| The annuity ,, ,, . . . | 150· |
| ∴ *The Answer* = £646·51875 = **£646, 10s. 4½d.** | £646·51875 |

**Ex. 2.** *The value of a vessel and its cargo being £35588, for what sum must it be insured at the rate of 12½ per cent. so that, if lost, its owners may recover both the value of the ship and cargo, and also the premium paid as insurance?*

The following process will show the *reason* as well as the *method* by which exercises of this kind may be worked—

*The insured value = the real value + the per-centage on the insured value.*
∴ *The insured value − the per-centage on the insured value = the real value;*
*i.e. The insured value − 12½ per cent. on the insured value = the real value;*
*i.e. The insured value − ⅛ of the insured value = the real value;*
*i.e.* *⅞ of the insured value = the real value;*
*i.e.* 7 times *the insured value* = 8 times *the real value*
= 8 times £35588
= £284704;
∴ The insured value = ⅐ of £284704 = **£40672**, *Answer.*

*The reason* is not so apparent, but the labour is less, in the following method—

£100 − *the per-centage* : £100 :: £35588 : *the insured value;*
*i.e.* £87½ : £100 :: £35588 : *the insured value.*

∴ *The insured value* = $£\frac{35588 \times 100}{87\frac{1}{2}}$ = £40672, *Answer.*

*The reason of the "statement"* made in this method is that *every* £87½ must be insured as £100 in order to recover this £87½ and the premium of £12½ upon the £100 insured.

N.B.—If the question had been put thus—

"*At how much above its real value must a ship worth* £35588 *be insured at a rate of* 12½ *per cent., so that if lost the owner may recover the value of the ship and cargo, and also the premium paid upon the insurance?*"

The answer would be found thus—

*The insured value*, as found above = £40672
*The real value* = 35588
∴ By subtracting we get . . £5084, *Answer.*

**Ex. 3.** *What is the premium of a policy of life assurance for* £4000 *upon the life of a person aged* 40, *when it is found by the tables that for every* £100 *assured such a person must pay* £3, 7*s.* 6*d. a year?*

£3, 7s. 6d. = £3⅜.

∴ The premium = £3⅜ × $£\frac{4000}{100}$ = £3⅜ × 40 = £135, *Answer.*

**Ex. 4.** *The rent of an estate is* £46, 10*s.* 7½*d.; if the estate were bought at* 20 *yrs. purchase at a rent which was to the other rent as* 4 : 3, *what was the price of the estate?*

(1.) The second rent : the first rent :: 4 : 3;
*i.e.* The second rent : £46, 10s. 7½d. :: 4 : 3;

∴ The second rent = $\frac{£46,\ 10s.\ 7\frac{1}{2}d. \times 4}{3}$ = £62, 0s. 10d.

(2.) If 20 yrs. purchase be given for an estate, it follows that—
The price = 20 times the rental
= 20 times £62, 0s. 10d. = £1240, 16s. 8d., *Answer.*

## Exercise 36.

### Exercises in Simple Interest.

1. Find the simple interest on £600 at 3 per cent.; on £800 at 4 per cent.; on £1200 at 2 per cent.; on £150 at 4 per cent.; on £1720 at 5 per cent.

2. What is the simple interest on £250 for 4 yrs. at 3 per cent.; on £1720 for 3 yrs. at 5 per cent.; on £1832 for 20 yrs. at 10 per cent.; on £650, 10s. for 8 yrs. at 5 per cent.; on £1728 for 6 yrs. at 4 per cent?

3. Calculate the simple interest on—
£280 for 4 yrs. at 2½ per cent.
£760 for 7 yrs. at 3¾ per cent.
£1250 for 8½ yrs. at 3½ per cent.
£1605 for 7¼ yrs. at 6⅔ per cent.
£3325 for 17½ yrs. at 6¼ per cent.
£324, 6s. 8d. for 8 yrs. at 3¾ per cent.
£1923, 8s. 4d. for 6 yrs. at 7½ per cent.
£2170, 13s. 4d. for 2⅞ yrs. at 3⅓ per cent.

4. Find the amount of the following sums at simple interest—
£650 for 8 yrs. at 2½ per cent.
£1725 for 3⅛ yrs. at 3¾ per cent.
£533, 6s. 8d. for 2¾ yrs. at 7½ per cent.
£1717, 13s. 4d. at 3⅛ per cent. for 10½ years.
£2019, 6s. 3d. at 4⅜ per cent. for 1⅓ yrs.
£1011, 17s. 3d. for 3 yrs. 5 months, at 7 per cent.
£827, 14s. 6d. for 2½ yrs. at £8, 5s. per cent.
£3920, 16s. for 6 yrs. 7 months, at 2¼ per cent.
£3396, 18s. 8d. for 5 yrs. at £3, 1s. 3d. per cent.
£700, 10s. for 2 yrs. 9 months, at £3, 2s. 6d. per cent.
£20500, 13s. 6d. for 8 yrs. 11 months, at £9, 14s. 6d. per cent.

5. Find the simple interest on £385, 14s. 7d. for 5 yrs. 219 days, at 3¾ per cent. per annum.

6. What will be the simple interest of 729 francs, 75 centimes, for 5 yrs. 4 months, at 3 centimes' interest per annum for each franc lent?

7. What is the simple interest on £154, 16s. 8d. for 4 yrs. 219 days, at 3½ per cent. per annum?

8. Find the simple interest on £5208, 16s. at 7½ per cent. for 11 weeks and 3 days.

9. Find the simple interest on £742, 19s. 4d. at 3½ per cent. for 3 yrs. 5 months.

10. Find the simple interest for 3 yrs. 73 days, on £233, 9s. 3¼d. at 4 per cent. per annum.

11. What is the interest of £36, 13s. 4d. for 3 yrs. 13 wks., at 3⅞ per cent. per annum?

12. What is the simple interest of £2500 at 4½ per cent. per annum for 7 yrs. 21 wks. 3 days?

13. Find the simple interest of £215, 12s. for 3 yrs. 73 days, at 4½ per cent. per annum.

14. Required the amount of £450 in 3 yrs. and 35 days, at 3¾ per cent. per annum simple interest.

15. Find the interest on £189, 16s. 6d. for 341 days at 3¼ per cent. per annum.

16. Find the amount of £5433, 13s. 11½d. for 5 yrs. 5 months, at 2¾ per cent. per annum.

17. Find the simple interest on—

£516258 at 3½ per cent. for 2¼ yrs.
£735416 at 3½ per cent. for 4 yrs. 6 months.
£756419 at 2·2 per cent. for 3·5 yrs.

18. Find the simple interest on £8250, from March 24th to June 5th, at 5 per cent. per annum.

19. Find the amount of £460, 13s. 4d. when put out to simple interest from July 4th, 1825, to September 15th 1831, at 6⅔ per cent. per annum.

20. What is the amount of simple interest due on £800, 10s. 6d., if it be lent at 3⅓ per cent. per annum from January 25th to June 20th?

21. At what rate per cent. must £75 be put out for twelve months that at the end of that time it may amount to £81?

22. At what rate per cent. will £1912 amount to £2629 in 7½ years at simple interest?

23. What sum of money lent at 4½ per cent. per annum for 7 months will yield interest £17, 0s. 9¼d.?

24. What principal put out to simple interest will produce £157, 10s. of interest in 4 years at 5 per cent.?

25. What sum of money put out to simple interest will amount to £418, 14s. 11¾d. in 3 yrs., at 3½ per cent per annum?

26. If £417, 18s. 4d. amount to £454, 9s. 8¼d. in 3½ yrs. at simple interest, what is the rate per cent.?

27. What sum of money put out at simple interest will amount to £201, 0s. 7½d. in 3½ yrs., at 4¼ per cent.?

28. Find the principal, of which the interest for 3½ yrs. at 2½ per cent. simple interest will be £551, 18s. 6¾d.

29. In how many years will £2977, 18s. 4d. amount to £3446, 18s. 9¼d. at 3½ per cent. simple interest?

30. The interest on a sum of money at 3 per cent. per annum for 2½ yrs. is £17725; find the principal.

31. Find the sum of money which, put out at 4½ per cent. simple interest, produces an income of £168, 15s.

32. What sum of money put out for 3 yrs. at 3½ per cent. simple interest will amount to £384988, 18s. 1½d.

33. If the simple interest of £725, 10s. for a certain time at 3 per cent. per annum is £21, 15s. 3½d., at what rate per cent. will the same sum amount to £785, 8s. 0¾d. in the same time?

34. In what time will £229, 10s. amount to £258, 3s. 9d. at 5 per cent. per annum, simple interest?

35. In what time will £158, 6s. 8d. amount to £176, 14s. 8d. at 3 per cent. per annum simple interest?

36. For how many years must £418342 be lent at 2½ per cent. simple interest that it may gain £7320985?

37. In what time will £618053 gain £1004336125 if let out at 3¼ per cent. simple interest?

38. In what time will £956384 amount to £104564648, if let out to simple interest at 2⅓ per cent.?

39. If £735416 gain £11582802 in 4½ yrs. simple interest, at what rate was it lent?

40. If £756419 become £814663263 in 3 yrs. 6 months, what would have been the simple interest on £1400 in double the time, at the same rate?

## Exercise 37.

### Exercises in Compound Interest.

1. Find the compound interest on £800 for 2 yrs. at 5 per cent.

2. Find the amount of £1620 for 3 yrs. at 10 per cent.

3. What will be the value of £1830, 10s. in 4 years' time, compound interest being calculated at 5 per cent. per annum?

4. Find the compound interest on—

£759428 at 5 per cent. for 3 yrs.
£864235 at 5 per cent. for 2 yrs.
£716418 at 4 per cent. for 2 yrs.
£400 for 3½ yrs. at 6 per cent.
£675, 15s. for 3 yrs. 6 months, at 6 per cent. per annum.

5. What is the difference between the simple and the compound interest on £860, 8s. 4d. in 3 yrs. at 5 per cent.?

6. Find the difference between the amount of £850, 14s. 2d. for 2⅓ yrs., at 4 per cent. at simple interest and at compound interest.

7. If £960 be lent for 2 yrs. at 4 per cent., what is the difference in the compound interest when it is paid half-yearly as compared with that when it is paid yearly?

8. What principal must be put out to compound interest for 3 yrs. at 5 per cent. to gain £78, 16s. 3d.?

9. At what rate per cent. of compound interest must £750 be lent for 3 yrs., in order that it may amount to £843648?

10. Find the amount of £6500 in 2 yrs. at 10 per cent per annum, the interest (compound) being payable half-yearly.

11. To what sum will £850 amount in 4 yrs. 6 months, at 4¼ per cent. compound interest?

12. What is the compound interest on £8000 in 4 yrs. at 2½ per cent., interest being payable half-yearly?

## Exercise 38.

### Miscellaneous Examples.

1. What is the commission on £850, 10s. at ⅛ per cent.?

2. What is the brokerage on £200, 10s. at 6s. 4d. per cent.?

3. What is the premium of insurance on £1351, 3s. 4d. at £2, 16s. 10½d. per cent.?

4. What is the expense of insuring a vessel and cargo worth £5060, 10s. at 3½ per cent.?

5. A vessel was insured for £2056, 17s. 11d., which sum covered the value of the ship and cargo and the cost of insuring it at 5¾ per cent.; what was the value of the ship and cargo?

6. At £9, 8s. 4d. per cent., what is the cost of insuring goods which are worth £2450, so that in case of loss the owner may recover the value of the goods and also repayment of the premium?

7. To £1810, 10s. add the simple interest for 3 yrs. at 2½ per cent., and find the insurance on the "sum" at 3¾ per cent., and add it to the former "sum." Then find the commission on the total at 3s. 4d. per cent.

8. At 8¼ guineas per cent., how much must be insured on £1440, so that in case of loss the owner may receive the value of the goods and the premium?

9. If a man insures his life for £1000 at the yearly rate of £60, and dies immediately after his fifth annual payment, how much does the company lose by the transaction, reckoning money at 5 per cent. compound interest?

10. A servant deposits in the Post Office Savings Bank £1 each quarter out of her wages; simple interest at the rate of ½d. a month is allowed for each pound completed, and this interest is added to the principal at the end of each year. What sum will she possess at the end of 10 yrs.?

11. A labouring man deposits at the end of each month, out of his wages, 30s. 4d. in the Post Office Savings Bank; simple interest at the rate of ½d. in the month is allowed for each pound completed to the end of the year, when the interest is added to the principal; how much did he thus lay up in 11 yrs.?

12. A servant is able to put in the Government Savings Bank a sum of £2, 15s. the last day of each quarter; calculate her savings after 12 such payments, interest being calculated as in Exs. 10 and 11.

(N.B.—Each year is calculated from the date of the first deposit.)

# DISCOUNT.

**113. Discount** is the sum allowed by a creditor to his debtor for the payment of money before it is due.

If a person is under an obligation to pay a certain sum of money at some future time, he may arrange with his creditor to discharge the debt at once. And it is evident that he would be doing his creditor no injustice by paying him a smaller sum at once than he would if payment were deferred until it could be legally claimed. It is also evident that the deduction which the creditor will be willing to make on account of immediate payment will be mainly influenced by the current rate of interest; because, if interest be high, he will be able to lend the money on advantageous terms, but if interest be low (in other words, if money be plentiful) it will not be so much to his advantage to have ready money at his disposal, and therefore he will allow less Discount in the latter case than in the former.

We may therefore consider this point established that—

(1.) *The Rate of Discount varies directly with the Rate of Interest.*

Discount is most commonly allowed on the payment of *Bills* or *Notes of Hand*, which are written engagements to pay money at a future time. Thus, in purchasing a house, the buyer may either pay ready money or give a Promissory Note for an amount which will be payable in 3, 6, or 12 months (or any other specified time), but, for reasons given above, in every case the note will be drawn for an amount greater than that which the seller would accept as a present payment in cash. It may be, however, that the seller requires some ready money; he will therefore take the Promissory Note to a banker, who, if he considers the credit of the drawer good, will give him cash for it after deducting from the amount stated in the note the Simple Interest thereon at the current rate. The sum of money thus deducted is usually called "discount," but as we shall now see, it is but improperly so called. Thus, if the ready-money price of the house above-mentioned were £800, and the current rate of interest were 4 per cent., the seller would require a Promissory Note payable in, say 6 months, for (£800 + simple interest on £800 for 6 months at 4 per cent., *i.e.* £800 + £16, *i.e.*) £816, because this is the sum to which £800 would amount in the given time at the current rate of interest.

It is here plain that *the* **Present Worth** *of* £816 *due in* 6 *months at* 4 *per cent.* (*i.e.* the principal which, being out at that rate for that time, will produce £816) *is* £800.

But a banker would give £816—the simple interest on £816, *i.e.* £816 —£16, 6s. 4·8d., *i.e.* £799, 13s. 7·2d. for the note, *i.e.* 6s. 4·8d. less than the **real Present Worth.**

By carefully comparing the above remarks with this last given example, we are enabled to lay down the following additional points concerning Discount:—

(2.) **The (real) Present Worth** *of a Bill or Promissory Note is such a sum of money as, being put out to simple interest at a given rate per cent. and for the given time, will amount to the debt.*

(3.) There are two kinds of discount, viz.—

1. **Bankers', or Commercial Discount,** which is the simple interest on the future debt.
2. **True Discount,** which is the abatement made on the payment of a debt before it is due, and is the simple interest on the present worth * of the debt.

Hence—

The future debt − the present worth = true discount.

(4.) *Bankers' Discount exceeds True Discount by the Simple Interest on the True Discount.*

Thus, in the above example, we have—

Bankers' discount = simple interest on £816
= simple interest on £800 + simple interest on £16
= £16 + simple interest on £16
True discount = simple interest on £800
= £16
∴ Bankers' discount − true discount = £16 + simple interest on £16 − £16
= simple interest on £16
= simple interest on true discount.

---

* Unless otherwise specified, "the present worth" always means "the real present worth."

The same proposition may be demonstrated generally, as follows:—

Since—

*Present Worth* + *True Discount* = *Debt.*

∴ Simp. int. on (*Present Worth* + *True Discount*) = simp. int. on *Debt.*

$$\text{And } \therefore \left.\begin{array}{l}(\textit{Pres. Worth} + \text{simp. int. on } \textit{Pres. Worth}) + \\ (\textit{True Disc.} + \text{simp. int. on } \textit{True Discount})\end{array}\right\} = \left\{\begin{array}{l}\textit{Debt} + \text{simp. int.} \\ \text{on } \textit{Debt.}\end{array}\right.$$

But—*Pres. Worth* + simp. int. on *Pres. Worth* = *Debt.*

∴ *True Disc.* + simp. int. on *True Discount* = simp. int. on *Debt.*

∴ Simp. int. on *Debt* − *True Discount* = simp. int. on *True Discount.*

*i.e. Bankers' Discount* − *True Discount* = simp. int. on *True Discount.*

If £100 were put out to simple interest for 1 yr. at 5 per cent., it would amount to (£100 + £5 *i.e.*) £105. Now we see that the *present worth* of £105 due in 1 yr. at 5 per cent. is £100. Further, it is evident that £5, which is the simple interest on £100 for the given time at the given rate, is also the true discount on £100 increased by that simple interest; and we state generally, that—

(5.) *The Simple Interest on a sum of money for a given time, at a given rate, is equivalent to the True Discount on a sum of money which is made up of the former sum increased by the Simple Interest thereon.*

Or—

*The Simple Interest on the Present Worth of a Bill is equivalent to the True Discount on the Nominal Value* of the Bill.*

Let P. = principal.
S. I. = simple interest.
T. D. = true discount.
p. c. = per cent.

Then—

S. I. at 5 p. c. on P. = T. D. at 5 p. c. on (P. + S. I. at 5 p. c. on P.)

*i.e.* $\frac{5}{100}$ of P. = T. D. at 5 p. c. on (P. + $\frac{5}{100}$ of P.)

*i.e.* $\frac{1}{20}$ of P. = T. D. at 5 p. c. on (P. + $\frac{1}{20}$ of P.)

= T. D. at 5 p. c. on $1\frac{1}{20}$ of P.

= T. D. at 5 p. c. on $\frac{21}{20}$ of P.

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* *The nominal value* of a bill is the amount for which the bill is drawn.

Multiplying each side of the equation by 20, we get—

$$\text{P.} = \text{T. D. at 5 p. c. on 21 P.}$$

Dividing each side of the equation by 21, we get—

$$\frac{\text{P.}}{21} = \text{T. D. at 5 p. c. on P.}$$

Here we observe, that while we divide P. by 20 to find S. I. thereon at 5 p. c., we divide P. by (20 + 1, *i.e.* by) 21 to find T. D. thereon at the same rate; and, *generally*, we may observe that—

$$\text{(6.) } \textbf{T. D. on P.} = \textbf{P.} \times \frac{\textbf{Rate}}{\textbf{100 + Rate.}}$$

**Ex.** T. D. at 4 p. c. on P. $= \text{P.} \times \frac{4}{100+4} = \text{P.} \times \frac{4}{104} = \text{P.} \times \frac{1}{26}$.

The rule for finding both simple interest and true discount may be stated as follows—

Let P. = principal.

**RULE 1.—Take the rate per cent. as the numerator of a fraction having 100 for its denominator, then P. × this fraction gives the simple interest.**

**2. (*a.*) *Increase* the denominator of the fraction obtained in the former case *by* the rate per cent., then P. × this fraction gives the true discount. Or—**

**(*b.*) Reduce the fraction $\frac{\textbf{Rate P. C.}}{100}$ to an equivalent fraction having 1 for its numerator, and increase the new denominator by 1. Then P. × by this new fraction gives the true discount.**

**Ex.** *To find S. I. at 4 p. c.*

Multiply P. by $\frac{4}{100}$, *i.e.* by $\frac{1}{25}$;
*i.e.* Divide P. by 25.

*To find T. D. at 4 p. c.*

(*a.*) Multiply P. by $\frac{4}{100+4} = \frac{4}{104} = \frac{1}{26}$;
*i.e.* Divide P. by 26.

Or—

(*b.*) Multiply P. by $\frac{4 \div 4}{(100 \div 4) + 1} = \frac{1}{25+1} = \frac{1}{26}$.

**Questions involving the consideration of Time** are worked by this rule as in the following example—

$$\text{S. I. on P. for 8 yrs. at 4 p. c.} = \frac{8 \times 4}{100} \text{ of P.} = \frac{32}{100} \text{ of P.} = \frac{1}{3\frac{1}{8}} \text{ of P.}$$

$$\text{T. D. on P. for 8 yrs. at 4 p. c.} = \frac{32}{100 + 32} \text{of P.} = \frac{32}{132} \text{of P.} = \frac{1}{4\frac{1}{8}} \text{of P.}$$

$$= (\text{by Rule } 2b.) \frac{1}{3\frac{1}{8} + 1} \text{ of P.} = \frac{1}{4\frac{1}{8}} \text{ of P.}$$

It should be carefully noted that—

*The True Discount at* 4 *per cent. is not** ½ *the True Discount at* 8 *per cent., nor in any such case*—

$$\left.\begin{array}{l} \text{For T. D. at 4 p. c.} = \dfrac{1}{26} \text{ of P.} \\ \text{But T. D. at 8 p. c.} = \dfrac{1}{13\frac{1}{2}} \text{ of P.} \end{array}\right\} \text{where } 13\tfrac{1}{2} \text{ is NOT } \tfrac{1}{2} \text{ of } 26.$$

Also, that—

(7.) *The True Discount on a sum of money due in* 4 *years' time is not ** ½ *the True Discount on the same sum due in* 8 *years' time, nor in any such case*—

$$\text{For T. D. at 4 p. c. for 4 yrs.} = \frac{16}{116} \text{ of P.} = \frac{1}{7\frac{1}{4}} \text{ of P.}$$

$$\text{But T. D. at 4 p. c. for 8 yrs.} = \frac{32}{132} \text{ of P.} = \frac{1}{4\frac{1}{8}} \text{ of P.}$$

Where we notice that 4⅛ is NOT ½ of 7¼.

**114.** In Great Britain and Ireland there is an interval of three days, called "*Days of Grace*," allowed between the date on which the payment of a bill becomes *nominally* due, and that on which it becomes *legally* due.

Thus, although *a bill for £500, drawn on August 3rd at three months*, would become nominally due on November 3rd, it would not be legally due before November 6th, and a banker would include these three days in calculating the number of days for which he was deducting discount when finding the present worth of a bill.

A bill which falls nominally due on February 29th, 30th, or 31st, or on the 31st of any month having only 30 days, is in every case legally due on the 3rd of the following month.

In Great Britain all bills legally due on Sunday are paid on Saturday, but in Ireland on Monday.

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* *i.e.* "Does not exactly correspond with," "is not exactly equal to."

**Ex. 1.** *What is the true discount on £612, 17s. 6d. due 9 months hence at 5 per cent. per annum?*

We will do this in two ways—

(1.) **By the Rule 2*b*.—**

**Simple interest on £612, 17s. 6d. for 9 months at 5 p. c.**

$$= \frac{3}{4} \text{ of } \frac{\overset{1}{\cancel{5}}}{\underset{20}{\cancel{100}}} \text{ of } £612\tfrac{7}{8}$$

$$= \frac{3}{80} \text{ of } £612\tfrac{7}{8}$$

$$= \frac{1}{26\frac{2}{3}} \text{ of } £612\tfrac{7}{8}.$$

∴ **True discount on £612, 17s. 6d. for 9 months at 5 p. c. =**

$$\left(\frac{1}{26\frac{2}{3}+1} = \right) \frac{1}{27\frac{2}{3}} \text{ of } £612\tfrac{7}{8} = \frac{3}{83} \text{ of } £\frac{4903}{8} = \text{£22, 3s. } 0\tfrac{42}{83}\text{d.}$$

(2.) **By Case II. of Simple Interest—**

1st. *Find the present worth of £612, 17s. 6d. due in 9 months' time at 5 p. c. per annum; i.e.—*

*Find what sum of money put out to simple interest at 5 p. c. per annum will amount to £612, 17s. 6d. in 9 months.*

Here £100 in $\frac{3}{4}$ yr. at 5 p. c. will amount to £100 + £$3\frac{3}{4}$ = £$103\frac{3}{4}$.

$$\therefore £103\tfrac{3}{4} \ : \ £100 \ :: \ £612\tfrac{7}{8} \ : \ \textit{its present worth.}$$

$$\therefore \text{The present worth of } £612\tfrac{7}{8} = \frac{£612\frac{7}{8} \times 100}{103\frac{3}{4}} = \text{£590, 14s. } 5\tfrac{41}{83}\text{d.}$$

2nd. *From the nominal value (i.e. the amount of the future debt) subtract the present worth of the bill.*

£612, 17s. 6d. − £590, 14s. $5\frac{41}{83}$d. = **£22, 3s. $0\frac{42}{83}$d.**, *Answer.*

**Ex. 2.** *What does a banker gain by discounting a bill of £612, 17s. 6d., drawn July 12th, at 4 months, and discounted September 3rd, at 5 per cent.?*

The bill was legally due November 15th, and discounted on September 3rd, *i.e.* 73 days (or $\frac{1}{5}$ of a year) before it was legally due.

We will work this sum also by two different methods, viz.—

(1.) Since *bankers' discount exceeds true discount by the simple interest on the true discount*, we proceed thus—

S. I. on £612, 17s. 6d., for 73 days at 5 p. c. =
$\frac{1}{5}$ of $\frac{1}{20}$ of £612, 17s. 6d. = $\frac{1}{100}$ of £612, 17s. 6d.

∴ T. D. on £612, 17s. 6d. = $\left(\frac{1}{100+1}=\right)$ $\frac{1}{101}$ of £612, 17s. 6d.

Now S. I. at 5 per cent. for $\frac{1}{5}$ of a year on $\frac{1}{101}$ of £612, 17s. 6d.—

*i.e.* The difference between bankers' discount and true discount on that sum for that time and at that rate } = $\frac{1}{100}$ of $\frac{1}{101}$ of £612, 17s. 6d.
= $\frac{1}{10100}$ of £612, 17s. 6d.
= 1s. $2\frac{5690}{10100}$d., *Answer.*

(2.) S. I. on £$612\frac{7}{8}$ in $\frac{1}{5}$ yr. at 5 p. c. = $\frac{1}{100}$ of £$612\frac{7}{8}$ = £6, 2s. $6\frac{9}{10}$d.
T. D. on £$612\frac{7}{8}$ in $\frac{1}{5}$ yr. at 5 p. c. = $\frac{1}{101}$ of £$612\frac{7}{8}$ = £6, 1s. $4\frac{34}{101}$d.
Therefore the difference = 1s. $2\frac{5690}{10100}$d.

A consideration of the connection which we have shown to exist between the rules for finding Simple Interest and True Discount will be found useful in working such exercises as the one next following. (Page 217.)

Let P. = principal.

**By the rule—**

(1.) S. I. at 5 p. c. on P. = $\frac{P.}{20}$, and

T. D. at the same rate on P. = $\frac{P.}{20+1} = \frac{P.}{21}$.

(2.) S. I. at 4 p. c. on P. = $\frac{P.}{25}$, and

T. D. at the same rate on P. = $\frac{P.}{25+1} = \frac{P.}{26}$;

and a similar connection exists between the denominators of these fractions in all cases where simple interest and true discount are found.

We are therefore and thereby enabled to establish the following proportions—

T. D. at 5 p. c. on a sum of money : S. I. at 5 p. c. on that sum :: $\frac{P.}{21}$ : $\frac{P.}{20}$;

*i.e.* T. D. at 5 p. c. on a sum of money : S. I. at 5 p. c. on that sum :: 20 : 21;

and, generally, we may state that—

(8.) *T. D. at any given rate upon P. : S. I. at the same rate upon P. :: the number by which we divide P. to find S. I. thereon : the number by which we divide P. to find T. D. thereon.*

**Ex.** *Bankers' discount at 5 per cent. = £32, 14s. 6d.; find true discount on the same amount.*

True discount : bankers' discount :: 20 : 21;
*i.e.* True discount : £32, 14s. 6d. :: 20 : 21.
∴ 21 times true discount = 20 times £32, 14s. 6d.

$$\therefore \text{ True discount } = \frac{\text{£32, 14s. 6d.} \times 20}{21} = \text{£31, 3s. 4d., } \textit{Answer.}$$

**115.** In some branches of business it is usual to mark goods at a certain per-centage above the price which the seller is willing to take for his goods. Thus, a tradesman may wish to mark an article at such a price as will enable him to deduct 20 per cent. therefrom, and then get 25s. for it. Now 20 per cent. is $\frac{1}{5}$ of the principal; but if he adds to 25s. the fifth of itself, the *marked price* will be (25s. + $\frac{1}{5}$ of 25s. = 25s. + 5s. =) 30s. But when he comes to take off $\frac{1}{5}$ of this marked price, he will charge only (30s. − $\frac{1}{5}$ of 30s., *i.e.* 30s. − 6s. =) 24s. for it, *i.e.* just 1s. less than he intended to charge when marking it. But if, instead of adding one-*fifth*, he adds one-*fourth* of itself to the price he wishes to get, and so marks (25s. + $\frac{1}{4}$ of 25s. = 25s. + 6s. 3d. =) 31s. 3d. as the price, and then reduces this marked price by $\frac{1}{5}$ of itself, thus charging (31s. 3d. − $\frac{1}{5}$ of 31s. 3d., *i.e.* 31s. 3d. − 6s. 3d. *i.e.*) 25s. for the article, he will realise the price he anticipated.

The reason of the truth stated above, that in such cases, ***when we require to reduce the marked price by $\frac{1}{5}$ of itself, we must increase the real price by*** $\left(\frac{1}{5-1},\right.$ *i.e.*$\left.\right)$ $\frac{1}{4}$ ***of itself; to reduce the marked price by*** $\frac{1}{9}$, ***we must increase the real price by*** $\left(\frac{1}{9-1},\right.$ *i.e.*$\left.\right)$ $\frac{1}{8}$ &c., will be seen from considering that—

If R. = the real price to be obtained,
and M. = the marked price,
and the marked price is to be reduced 20 p. c.;
Then M. − $\frac{1}{5}$ of M. = R.,
*i.e.* $\frac{4}{5}$ of M. = R.,
*i.e.* 4 M. = 5 R.,
*i.e.* M. = $\frac{5\text{ R.}}{4}$ = $1\frac{1}{4}$ R.;
*i.e.* R. must be increased by $\frac{1}{4}$ of itself to make it equal to M.

## Exercise 39.

### Exercises in Discount.

1. Find the true discount upon the following bills :—

£350, 7s. 10½d., due in 1 yr., at 5 per cent.
£433, 14s. 3d., due in 1 yr., at 4 per cent.
£65, 3s. 9d., due in 1 yr., at 5 per cent.
£65, 6s. 6d., due in 1 yr., at 4 per cent.
£46, 1s. 6½d., due in 1 yr., at 6¼ per cent.
£113, 2s. 8d., due in 1 yr., at 6⅔ per cent.
£888, 1s. 6½d., due in 1 yr., at 2½ per cent.
£1476, 15s. 5¾d., due in 1 yr., at 16⅔ per cent.
£81, 5s. 4d., due in 8 months, at 6 per cent.

2. Bankers' discount on a sum of money, due in 1 yr., at 5 per cent., being £10, 8s. 3d., what was the nominal value of the bill?

3. Bankers' discount on a sum of money, due in 1 yr., at 6¼ per cent., being £18, 1s. 3d., for what sum was the bill drawn?

4. True discount on a sum of money, due in 4 months, at 5 per cent., being £1, 6s. 4d., what was the nominal value of the bill?

5. Find the present worth of £610, 4s. 2d., due in 73 days, discount being at 5 per cent.

6. Bankers' discount being at 5 per cent., a person receives £9, 5s. 6d. less than the nominal value of his bill. What should he receive for his bill if true discount only were deducted?

7. A person possesses a sum of money, the simple interest of which, at 4 per cent., is £134, 1s. 3d. With this money he purchases an estate, for which he pays by a note payable in 4 months' time, and which being discounted at 4 per cent., is worth at present exactly the money he possesses. For how much is the bill drawn?

8. The true discount on a bill due in 1 yr., and discounted at 5 per cent., being £8, 11s. 8d., what would have been the banker's discount thereon?

9. Find the present worth of £509, 4s. 2d., due in 73 days, discount being at 5 per cent.

10. Simple interest at 2½ per cent. being £9, 18s. 2d., find true discount on the same amount and at the same rate.

11. Find the present worth of £223, 9s. 3d., due in 1 yr., and discounted at 1 per cent.

12. True discount on a sum of money at 7⅐ per cent. being £8, find true discount on the same amount at 3⅛ per cent.

13. True discount, at 4 per cent., on a sum of money being £15, find simple interest on the same sum at 5 per cent.

14. True discount on a sum of money, due in 8 months, at 6 per cent. being £4, 3s. 3d., how much would a banker give for the bill 3 months before it is payable if the current rate of discount be 4½ per cent. ?

15. Find the difference between the present worth and the nominal value of a bill for £3152, 13s. 5½d. drawn Oct. 13th, at 4 months, and discounted Dec. 5th at 5 per cent.

16. Find the true discount on £900, 0s. 1½d. due in 2 yrs. and discounted at 5 per cent.

17. Find the present worth of two sums of £10 to be paid at the end of one and two years respectively, money making 5 per cent. per annum, and verify your result by showing that the value which you have obtained is just sufficient to pay the sums due at the end of each year.

18. Find true discount on £399, 1s. 6d. due in 13 months, at 5 per cent. per annum, and find the present worth of £396, 10s. 6d., due in 11 months, at 4 per cent. per annum.

19. If the discount on £226, 2s. 8d., due at the end of 1½ yrs., is £12, 16s., what is the rate of interest?

20. What is the discount on £756, one-half of which is payable in 6 months, the remainder in 12 months, at 7 per cent. per annum?

21. A person marks his goods so that he may allow a discount of 4 per cent. If after doing this he makes a profit of 15 per cent., what must be the marked price of an article which cost him £1, 17s. 6d.?

22. Owing £5, 2s. 6d., payable in eighteen months, £17, 18s. 6d. payable in 2½ years, and £10, 17s. payable in 5½ months, in what time ought I to pay down £33, 18s. for the whole debt?

# STOCKS.

**116.** When the Government of this or any other country is in immediate want of a greater sum of money than can be raised by taxation, it generally borrows money, and each lender receives from the Government a document called a "*Bond*," which is to him an acknowledgment of the debt, and by which he is entitled to receive a certain amount of interest, the rate of which is specified at the time the money is borrowed, and is determined by the circumstances then existing.

Thus, if A lent Government £100 at 3½ per cent., he would be supplied with a bond entitling him to receive £3, 10s. a year till Government chose to pay back the principal and so get rid of the debt.

As long as the debt remained unpaid, and A kept possession of his bond, he would be said to possess £100, 3½ per cent. Stock.

He might, however, sell his bond to some other person if he chose to do so, but the price which he would be able to get for it would depend upon the state of the money market at the time he sold it. If money were plentiful, interest would be low, and might fall below 3½ per cent., and A would then be able to get more than £100 for his bond, because it would require more than £100 in the then state of the market to produce an income of £3, 10s., which is the income derivable from A's bond. But if money were scarce, interest would be very high and might perhaps reach 7 per cent., in which case A would not be able to get more than £50 for his bond, because the interest upon £50 at that time would £3, 10s. a year, which is all that A's bond will produce.

The total amount owing to the bondholders, *i.e.*, the persons possessing Stock, is called "*The National Debt*," and the source from which the interest is paid is called "*The Public Funds*," which is, however, an imaginary property representing the credit of the borrowing country, which is pledged to the payment of the debts contracted by its Government; in other words, the interest is paid from the income of the Government, and this is mostly, and in most cases, derived from taxation.

Sometimes the per-centage offered by the borrowing Government is not so great as the current rate of interest, in which case it is evident that nobody will give £100 for a bond which represents £100 of such Stock. In such a case the price of Stock is said to be **below par**, *i.e.* £100 Stock may be purchased for less than £100 in ready money. If, however, the interest offered by the borrowing Government be higher than the current rate, and the credit of the borrowing country be considered good, the price of the Stock will be **above par,** *i.e.* more than £100 in ready money must be given to purchase £100 of this stock.

A distinction must be made between **£100 Stock,** which simply signifies £100 of the national debt, and **£100 in the Stocks,** which is the amount of stock purchased by £100 in ready money, and may be *more* or *less* than £100 Stock according as Stock is selling *below* or *above par.*

The above remarks may now be illustrated by three examples—

**Ex. 1.** If £100 Stock in the 3 per cents. could be purchased for £80 in ready money, the price of this Stock would be *below par,* and we should say that "*the* 3 *per cents. were selling at* 80," or, that "*the* 3 *per cents. were* **at a discount.**"

**Ex. 2.** If £100 Stock in the 3 per cents. could be purchased for £113 in ready money, the price of this Stock would be *above par,* and we should say that "*the* 3 *per cents. were selling at* £113," or, that "*the* 3 *per cents. were* **at a premium.**"

**Ex. 3.** If £100 Stock in the 3 per cents. could be purchased for £100 in ready money, we should say that "*the* 3 *per cents. were selling* **at par.**"

Stock is generally bought and sold by the agency of **Stockbrokers.** Thus, a person wishing to buy Stock does so through his broker, who buys it from the broker of another person desirous of selling. Each broker charges his employer a commission of ⅛ of a £ for the transfer of every £100 Stock, but in every exercise this consideration is to be omitted unless specially mentioned.

**117.** The following questions on "**Stocks**" may be easily worked by the help of the rules of **Interest** and the explanations here immediately preceding.

**Ex. 1.** *If the* 3 *per cents. are at* 84, *and a* 4 *per cent. Stock at* 98, *which is the best investment? And if a man has* £1000 *to invest, what will be the difference in his income according as he purchases the one or the other?*

(Examination for Certificates. Male students. 1866.)

1. *The* 3 *per cents.* being at 84, the principal required to produce an annual income of £1 therein will be (⅓ of £84, *i.e.*) £28.

*The* 4 *per cents.* being at 98, the principal required to produce an annual income of £1 therein will be ($\frac{1}{4}$ of £98, *i.e.*) £24, 10s.

Therefore *the* 4 *per cents. will be the best investment*, because a less principal invested therein will produce as good an income as a large principal invested in the 3 per cents.

2. *The annual income derived from investing* £1000 *in the* 3 *per cents. at* 84 will be found as follows—

The price of each "cent." (*i.e.* each £100) of Stock is £84,

$\therefore$ the number of "cents." purchased for £1000 will be £$\frac{1000}{84}$;

and the interest on each cent. being £3,

the interest on (£1000, *i.e.* on) $\frac{1000}{84}$ cents. will be £3 × $\frac{1000}{84}$ = £$35\frac{5}{7}$

But, *the annual income derived from investing* £1000 *in the* 4 *per cents. at* 98 will therefore be—

$$£4 \times \frac{1000}{98} = £40\tfrac{40}{49}.$$

Therefore *the difference of income resulting from investing* £1000 *in the* 3 *per cents. at* 84, *and in the* 4 *per cents. at* 98, will be—

£$40\frac{40}{49}$ − £$35\frac{5}{7}$ = £$5\frac{5}{49}$ = £5, 2s. $0\frac{1}{2}$d. nearly, *Answer.*

**Ex. 2.** *A person investing in the* 4 *per cents. receives* $4\frac{3}{8}$ *per cent. interest for his money; what was the price of the Stock?*

(Examination for Certificates. Male students. 1870.)

By First Principles—

The principal on which he receives £$4\frac{3}{8}$ interest is £100.

$\therefore$ ,, ,, ,, £1 ,, £$\frac{100}{4\frac{3}{8}}$

And ,, ,, ,, £4 ,, £$\frac{100 \times 4}{4\frac{3}{8}}$

Therefore the cost of the Stock must be $\left( £\frac{4 \times 100}{4\frac{3}{8}}, \textit{i.e.} \right)$ £$91\frac{3}{7}$, *Ans.*

**Ex. 3.** *When the* 3 *per cents. were at* 90, *I found that by selling out, and investing in Indian* 4 *per cents. at* 95, *I could improve my income by* £24, 6s. *What was the amount of my Stocks in the* 3 *per cents.?*

(Examination for Certificates. Male students. 1866.)

*The per-centage in the* 3 *per cents. was* $3\frac{1}{3}$, for—

As £90 : £100 :: £3 : *per-centage.*

$\therefore$ The per-centage $= £\frac{100 \times 3}{90} = £3\frac{1}{3}$.

*The per-centage in the Indian* 4 *per cents. was* $4\frac{4}{19}$, for—

As £95 : £100 :: 4 : *per-centage.*

$\therefore$ The per-centage $= £\frac{100 \times 4}{95} = £4\frac{4}{19}$

*So that by transferring* £100 from the 3 per cents. at 90 to the Indian 4 per cents. at 95, I improve my annual income by ($£4\frac{4}{19} - £3\frac{1}{3}$, *i.e.*) $£\frac{50}{57}$

$\therefore$ As $£\frac{50}{57}$ : $£24\frac{3}{10}$ :: £100 : *the money transferred.*

$\therefore$ The money transferred $= £100 \times £\frac{24\frac{3}{10}}{\frac{50}{57}} = £2770$, 4s.

We have now to find *how much Stock, sold out from the* 3 *per cents. at* 90, *would realise this money*—

By Proportion we have—

£90 : £100 :: $£2770\frac{1}{5}$ : *the Stock sold out to realise* $£2770\frac{1}{5}$.

$\therefore$ The Stock sold $= £2770\frac{1}{5} \times £\frac{100}{90} = £3078$, *Answer.*

## Exercise 40.

### Exercises on Stocks.

1. If the 3 per cents. are at 80, what income shall I obtain by investing £2287, 10s.?

2. What will be the annual income on £1496, 18s. 4d. invested in the $2\frac{1}{2}$ per cents. at 71?

3. £427, 8s. 6d. being invested in the 3 per cents. at 82, what will be the annual income therefrom?

4. A person having a legacy of £560, 8s. 2d. invests it in the 4 per cents. at 91; what is the present worth of his yearly dividend thereon, 3 months before it is payable, if the rate of interest be $3\frac{1}{2}$ per cent.?

5. What was the price of 3 per cent. Stock when £353, 9s. 2d. invested therein produced an income of £12, 9s. 6d.?

6. What is that Stock which, selling at 82, produces an annual income of £15, 16s. 3d. on an investment therein of £432, 4s. 2d.?

7. What sum must be invested in the 3 per cents. at 94 to produce a quarterly income of £8.?

8. What amount of Stock must be purchased in the 4 per cents. at 93 to produce a quarterly income of 9 guineas?

9. Sold Stock at 72⅜ for £461, 10s.; what was the Stock sold?

10. Sold £900¾ Stock for £1200, 16s.; required the price per cent.?

11. If the 2½ per cents. are at 96, what sum must be invested therein to produce an annual income which, one year before it is due, is worth £7, 6s. 6d. after being discounted by a banker at 2½ per cent.?

12. Sold £364 Stock at 92⅐ per cent. What was the sum received?

13. Sold Stock at 83⅝ for £203, 17s. What was the Stock sold?

14. If £650, 13s. be given for £550, 11s. Stock, what was the price per cent.?

15. If £225, 9s. Stock be sold for £125, 5s., what was the price per cent.?

16. A person sells out £350, 14s. Stock from the 3 per cents. at 128$\frac{4}{7}$; what does he receive, and what difference does he make in his income by investing the proceeds in 3½ per cents. at 144?

17. £800, 16s. Stock was sold out of the 2½ per cents. at 87½, and invested in railway 5 per cent. preference shares at 12 per cent. premium. What was the difference in the income derived?

18. I invest £1270 in 3 per cents. at 92½, and, selling out after allowing the interest to accumulate for 2 years, I find myself richer by £147, 10s.; at what price did I sell out?

19. If I invest £2800 in the 3 per cents. at 92, what is my income, and how much per cent. do I get for my money?

20. What is the price of the 3¼ per cents. when £3930 invested in them produces an income of £130 per annum?

21. Which is the better investment—the 3 per cents. at 90¼, or railway shares at 114, paying a half-yearly dividend of £2, 5s.?

22. I have £2400 Stock in the 3 per cents.; I sell out at 90½, invest the money in debentures, paying 5½ per cent., purchasing them at 111; what is the difference in annual income?

23. I lay out £1270 in the 3 per cents. at 92½; after allowing the simple interest to accumulate for 2 yrs., I sell out at 93, and invest the sum in debentures at 104, paying 4½ per cent. What is my income? (Fractions of a penny may be neglected.)

24. Is it better to invest in 3 per cent. Stock at 83½, or in shares at £233 each, on which a dividend of £7, 13s. 4d. is paid annually? If you have invested £1000 in the 3 per cents., and exchange it into the other security, what difference will it make in your income?

25. A person invests £4095 in the 3 per cents. at 91; he sells out £3000 stock when they have risen to 93½, and the remainder when they have fallen to 85? What does he gain or lose by the transaction? If he invests the produce in 4½ per cent. Stock at 102, what is the difference in his income?

26. A has stock in the 3 per cent. Consols which produce him £300 a year. He sells out one-half at 92, and invests the proceeds in the South Devon Railway when a £50 share is worth £23. What dividend per cent. per annum ought the South Devon Railway to pay so that he may increase his income £50 per annum by the transaction?

27. A buys 200 shares in the South Devon Railway, for which he gives £100 a share. When they are paying 2 per cent. he sells out at £46 a share, and invests the proceeds in 3 per cent. Consols at 92. Find the difference in his income in consequence of the operation.

28. A person invests £6825 in 3 per cents. at 91; he sells out £5000 Stock when they have risen to 93½, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction? If he invests the produce in 4½ per cent. Stock at par, what is the difference in his income?

29. What must a person have invested in 3 per cents. at 90⅝ if a transfer of ⅜ of his capital to the 4 per cents. at 115 would increase his income by £7?

30. A person invests £6200 in the 3 per cents. at 89⅛, and pays income-tax at 10d. in £1; when the Stock rises to 92 he sells out, and invests the proceeds in £50 Railway Shares, which yield an annual dividend, clear of income-tax, of 3½ per cent. What is the difference in his income?

31. A certain railway pays an annual dividend of £3, 10s. a share. A person bought 12 shares at such a price that they yielded 5⅗ per cent. on his investment, and when the price had risen £5, he sold out and invested the proceeds in 3¼ per cent. Stock at £85. Find the alteration in his income.

32. Having a legacy of £7823, I hesitate between investing it in 2 per cents. at 73, or 5 per cents. at 134, but finding that in the latter investment they deduct 13d. in the £1 from all yearly dividends, I decide on the former. How much do I gain or lose per annum by this choice?

33. I bought an estate, which let for £450 per annum at 28 yrs. purchase; how much Stock must I sell out of the 3 per cents. at 96 to pay for this estate, and how much more interest per cent. do I get by the estate than by the Funds?

## PROFIT AND LOSS.

**118. The Profit or the Loss** on the sale of articles is not generally expressed in commercial transactions by the difference between *the cost price* and *the selling price,* but by the ratio which this difference bears to the cost price, which ratio is generally expressed as *so much per cent.*

Thus, if a man purchased coals at 30s. a ton, and sold them at 36s. a ton, he would not express his gain as being "*6s. on the sale of a ton of coals,*" nor as being "*6s. on an outlay of 36s.,*" but as being ($\frac{1}{5}$ of, *i.e.*) "20 *per cent. on his outlay,*" meaning thereby that, selling his coals at 36s. a ton, *he gained £20 on every £100 he laid out.*

Whenever a gain or a loss is expressed as being *so much per cent.*, it must always be considered as being calculated upon *the original outlay,* and not on *the selling price,* unless so expressed.

In all questions of Profit and Loss there are three things to be considered, viz.—

1. **The Prime Cost,** or **Cost Price,** which is the sum originally given for the article.
2. **The Retail Price,** or **Selling Price,** which is the sum for which it is sold by the original purchaser.
3. **The Gain or Loss Per Cent.**, which is the difference between the cost price and the selling price.

**Ex.** If an article were purchased for 20 shillings and sold for 22 shillings, we should say that 20s. was *the prime cost,* 22s. was *the selling price,* and 2s. was *the gain.*

Here it is plain that if the gain on 20s. was 2s., the gain on 100s. at the same rate would be 10s., *i.e.* 5 times as much, because the principal is 5 times greater.

The same thing may be stated in a proportion, thus—

*As* 20s. : *the gain on* 20s. : : 5 *times* 20s. : *the gain on* 5 *times* 20s.,

*i.e.* 20s. : 2s. : : 100s. : *the gain on* 100s.

$$\therefore \text{The gain on 100s.} = 2\text{s.} \times \frac{100\text{s.}}{20\text{s.}} = 2\text{s.} \times 5\text{s.} = 10\text{s.};$$

and we may observe that generally—

$$\frac{\text{The gain on a given sum}}{\text{The given sum}} = \frac{\text{the gain on 100s.}}{\text{100s.}} = \frac{\text{the gain on £100}}{\text{£100}}.$$

Now, although "*per-centage*" signifies literally "*for a hundred*," it is usual to consider it as applying particularly to £100; so that, to take the above example, if an article be purchased for 20s. and sold for 22s., we say that *the gain was* 10 *per cent., i.e. at the rate of* £10 *gained for every* £100 *laid out in the original purchase;* therefore we see that—

Since, as above—

$$\frac{\text{The gain on a given sum}}{\text{The given sum}} = \frac{\text{the gain on £100}}{\text{£100}};$$

$$\therefore \frac{\text{The gain on a given sum}}{\text{The given sum}} = \frac{\text{the "per-centage"}}{100};$$

$$\textit{i.e.} \ \frac{\text{The gain}}{\text{The prime cost}} = \frac{\text{the "per-centage"}}{100}$$

$$\therefore \text{The per-centage} = \frac{\text{the gain}}{\text{the prime cost}} \times 100.$$

Hence the reason of the following rule—

**RULE.—To find the PER-CENTAGE of gain (or loss, for loss may be considered as a *minus* gain) on the sale of an article, *multiply* the fraction which expresses the ratio of the gain to the cost price *by* 100.**

**Ex.** *Bought an article for £3,* 10*s. and sold it for* 4 *guineas; what did I gain per cent. by the transaction?*

By the rule—

$$\text{The gain per cent.} = \frac{\text{4 guineas} - \text{£3, 10s.}}{\text{£3, 10s.}} \times 100$$

$$= \frac{\text{£4, 4s.} - \text{£3, 10s.}}{\text{£3, 10s.}} \times 100.$$

$$= \frac{14\text{s.}}{70\text{s.}} \times 100 = \tfrac{1}{5} \times 100 = 20, \text{ per cent., } \textit{Answer.}$$

The truth of the following proportion is also apparent—

$$\textit{Cost price} : \textit{selling price} :: \pounds 100 : \begin{array}{c} \pounds 100 + \textit{gain p. c. when a gain} \\ \textit{is effected}, \text{ or,} \\ \pounds 100 - \textit{loss p. c. when a loss is} \\ \textit{sustained.} \end{array}$$

Hence the reason of the following rules—

**RULE 1.—To find the SELLING PRICE of an article to *gain* a given per-centage, multiply the cost price by 100 increased by the rate per cent., and divide the product by 100.**

**Ex.** *Bought goods for £11, 7s. 1d.; find the selling price to gain 6 per cent.*

**By the rule—**

$$\text{The selling price} = \frac{\text{cost price} \times (100 + 6)}{100} = \frac{\pounds 11, \text{ 7s. 1d.} \times 106}{100} =$$

**£12, 0s. 8½d.,** *Answer.*

**RULE 2.—To find the SELLING PRICE of an article to *lose* a given per-centage, multiply the cost price by 100 diminished by the rate per cent., and divide the product by 100.**

**Ex.** *Bought goods for £120, 17s. 6d.; find the selling price to lose 13⅓ per cent.*

**By the rule—**

$$\text{The selling price} = \frac{\text{cost price} \times (100 - 13\tfrac{1}{3})}{100} = \frac{\pounds 120, \text{ 17s. 6d.} \times 86\tfrac{2}{3}}{100}$$

$$= \frac{\pounds 120, \text{ 17s. 6d.} \times 260}{300} \ \pounds 104, \text{ 15s. 2d.}, \ \textit{Answer.}$$

**RULE 3.—To find the COST PRICE of an article when a *gain* of a certain per-centage has been made, multiply the selling price by 100, and divide the product by 100 increased by the rate per cent.**

**Ex.** *Sold goods for £5, 10s. 3d., and gained 8 per cent. on my outlay; what was the cost?*

**By the rule—**

$$\text{The cost price} = \frac{\text{the selling price} \times 100}{100 + 8} = \frac{\pounds 5, \text{ 10s. 3d.} \times 100}{108} = \pounds 5, \text{ 2s. 1d.}$$

**RULE 4.—To find the COST PRICE of an article when a *loss* of a certain per-centage has been sustained, multiply the selling price by 100, and divide the product by 100 diminished by the rate per cent.**

**Ex.** *Sold goods for* £19, 11*s.* 8*d.*, *and lost* 6 *per cent. on my outlay; what was the cost?*

**By the rule—**

$$\text{The cost price} = \frac{\text{the selling price} \times 100}{100 - 6} = \frac{£19,\ 11\text{s. } 8\text{d.} \times 100}{94} =$$

**£20, 16s. 8d.,** *Answer.*

The above rules are useful in many cases, especially for Mental Arithmetic; but most questions of Profit and Loss may more profitably, though perhaps less expeditiously, be worked by First Principles, as in the following examples—

**Ex. 1.** *A man bought* 296 *sheep at* £1, 7*s.* 6*d. each, and after paying* $\frac{2}{11}$ *of his outlay for their keep, exchanged them for* 37 *cows, which he sold at* £15 *per head; what did he gain per cent. on his expenditure?*

(Examination for Certificates. Male students. 1871.)

The prime cost of the sheep = £1, 7s. 6d. × 296;

Their keep = $\frac{2}{11}$ of their prime cost = $\frac{2}{11}$ of £1, 7s. 6d. × 296;

∴ *His total expenditure* by them = $1\frac{2}{11}$ of £1, 7s. 6d. × 296

$$= \frac{13}{11} \times £\frac{11}{8} \times 296 = £481.$$

But *his total receipts* by them = the price of 37 cows at £15 a head

= £15 × 37 = £555;

∴ His **gain** on them = £555 − £481

= £74.

This gain must now be expressed as a per-centage, which is done by the following proportion—

*The gain on* £481 : *the gain on* £100 :: £481 : £100.

$$\text{Therefore (the gain on £100, } i.e.\text{) the gain p. c.} = \frac{\text{the gain on £481} \times £100}{£481}$$

$$= \frac{£74 \times 100}{481} = £15\tfrac{5}{13}\ \textit{Answer.}$$

**Ex. 2.** *If a postmaster is allowed* $1\frac{1}{4}$ *p. c. profit on selling postage stamps, what is his gain on a sheet of* 120 *stamps?*

(Examination for Certificates. Male students. 1867.)

The selling price of 120 penny stamps would be 10s.

In this case the per-centage would be reckoned on the selling price, so that his profit will be 1¼ p. c. on 10s., *i.e.* £½.

$$\therefore \textit{As } \pounds 100 \ : \ \pounds\tfrac{1}{2} \ :: \ \pounds 1\tfrac{1}{4} \ : \ \textit{his profit.}$$

$$\therefore \textit{His profit} = \pounds 1\tfrac{1}{4} \times \pounds\frac{\frac{1}{2}}{100} = \pounds\frac{\cancel{5}}{4} \times \tfrac{1}{2} \times \frac{1}{\underset{20}{\cancel{100}}} = \pounds\frac{1}{160} = 1\tfrac{1}{2}\text{d. } \textit{Answer.}$$

**Ex. 3.** *A lb. of tea and 6 lb. of sugar together cost 7s. 6d.; if tea were to rise 50 p. c., and sugar fall 12½ p. c., they would cost 8s. 9d. What is the price of each?*

(Examination for Certificates. Male students. 1867.)

(1.) By the question we have—

*Cost of* 1 *lb. of tea* + *cost of* 6 *lb. of sugar* = 7s. 6d. (at the original price).

Or, multiplying each side of the equation by 6, for a reason which will be apparent presently, we get—

*Cost of* 6 *lb. of tea* + *cost of* 36 *lb. of sugar* = 45s. (I.)

But, by the question, we have also—

(*a.*) Cost of 1½ lb. of tea at the *old* price = cost of 1 lb. of tea at the *new* price.

(*b.*) Cost of ⅞ lb. of sugar at the *old* price = cost of 1 lb. of sugar at the *new* price.

∴ Cost of (⅞ × 6) lb. of sugar at the *old* price = cost of 6 lb. of sugar at the *new* price.

*i.e.* Cost of $\frac{21}{4}$ lb. of sugar at the *old* price = cost of 6 lb. of sugar at the *new* price.

Therefore—

$$\left.\begin{array}{l}\text{Cost of } 1\tfrac{1}{2} \text{ lb. of tea at the } \textit{old}\\ \quad\text{price + cost of } \tfrac{21}{4} \text{ lb. of}\\ \quad\text{sugar at the } \textit{old} \text{ price.}\end{array}\right\} = \left\{\begin{array}{l}\text{Cost of 1 lb. of tea at } \textit{new}\\ \quad\text{price + cost of 6 lb. of}\\ \quad\text{sugar at } \textit{new} \text{ price.}\end{array}\right.$$
$$= \text{8s. 9d. (by the question.)}$$

Therefore, we have—

*Cost of* 1½ *lb. of tea* + *cost of* $\frac{21}{4}$ *lb. of sugar* = 8s. 9d. (at the original price).

Or, multiplying each side of this equation by 4, to eliminate the fractions, and make the number of pounds of tea the same as in Equation I., we have—

*Cost of* 6 *lb. of tea* + *cost of* 21 *lb. of sugar* = 35s. (II.)

By now subtracting Equation II. from Equation I. we shall be able to find the price of 1 lb. of sugar, and, from that, the price of 1 lb. of tea; thus—

*The cost of* 6 *lb. of tea* + *the cost of* 36 *lb. of sugar* = 45s. (I.)
*The cost of* 6 *lb. of tea* + *the cost of* 21 *lb. of sugar* = 35s. (II.)

By subtracting, we have—

The cost of 15 lb. of sugar = 10s.
$\therefore$ The cost of 1 lb. of sugar = $\frac{10}{15}$s. = 8d.

Therefore, since—

*The cost of* 1 *lb. of tea* + *the cost of* 6 *lb. of sugar* = 7s. 6d. (by Equation I.)
*The cost of* 1 *lb. of tea* + 4s. = 7s. 6d.
$\therefore$ *The cost of* 1 *lb. of tea* = 7s. 6d. − 4s. = 3s. 6d.

*Answer*, Sugar, 8d. per lb.; Tea, 3s. 6d. per lb.

**Ex. 4.** *A quantity of tea is sold for* 4s. 2d. *per lb., the gain is* 10 *per cent., and the total gain is* £12. *What is the quantity of the tea?*

(Examination for Certificates. Male students. 1870.)

Here, *The prime cost* : *the selling price* :: 100 : 110;
*i.e. The prime cost per lb.* : 4s. 2d. :: 100 : 110;

$\therefore$ The prime cost per lb. $= \dfrac{\text{4s. 2d.} \times 100}{110} = \frac{10}{11}$ of 4s. 2d.;

$\therefore$ The gain on 1 lb. = $\frac{1}{11}$ of 4s. 2d.

But $\dfrac{\text{The total gain}}{\text{The gain per lb.}}$ = the number of lb. sold;

*i.e.* $\left(\dfrac{\text{£}12}{\frac{1}{11}\text{ of 4s. 2d.}} = \dfrac{\text{240d.} \times 12}{\frac{1}{11}\text{ of 50d.}} = \dfrac{24\cancel{0}\text{d.} \times 12 \times 11}{5\cancel{0}} = \dfrac{3168}{5} = \right)$

633$\frac{3}{5}$ lb. = the number of lb. sold.

*Answer*, 633$\frac{3}{5}$ lb.

## Exercise 41.

### Exercises in Profit and Loss.

1. What is the selling price of an article which cost £1, 13s. 5d., and is sold so as to gain 20 per cent. on its prime cost?

2. What was the prime cost of an article which, being sold for £1·75, produced a gain of 25 per cent.?

3. An article is sold for £3·6 at a loss of 20 per cent.; for what must it be sold to gain 10 per cent.?

4. How much sugar at £·025 per lb. must I give in exchange for 36 lb. of tea at £·25 per lb. so as to gain 20 per cent. by the exchange?

5. A stationer buys 45 reams of paper at 13s. 9d. per ream, and 73 reams at 12s. 10d. per ream; at what average price per quire must he retail it to make a gain of 30 per cent.?

6. A man has £500 of stock in trade, and borrows £200 at 6¼ per cent. to increase it; at what price must he retail his goods to make 8⅓ per cent. per annum on the whole, deducting £150 for expenses?

7. A man has 30 cattle, by which he hopes to make a certain profit per cent., but he loses 10 suddenly by the cattle-plague, and his loss per cent. is now as great as the profit would have been. Find the per-centage of loss.

8. A newsagent purchases 23⅓ dozen of penny papers daily; he gives 9d. a dozen for them, and sells them at 1d. each; what does he gain per cent.?

9. A man buys land at £42, and having built upon it, sells the land and houses so as to gain 20 per cent. on his whole outlay, and receives £9004, 16s. How much did he spend in building?

10. What is a person's income who pays £86, 16s. 4d. when the income-tax is 6d. in the £1? and what will he pay when the tax is reduced 25 per cent.?

11. A house built for £664 is sold for £830; what is the gain per cent.? If it had been built for £830 and sold for £664, what would have been the loss per cent.? Why do the rates differ?

12. A woman buys a certain number of eggs at 21 a shilling, and the same number at 19 a shilling; she mixes them together, and sells them at 20 a shilling; how much does she gain or lose per cent. by the transaction?

13. A woman buys a certain number of apples at 3 a penny, and the same number at 2 a penny, and sells them at 5 for 2 pence. How much does she gain or lose per cent. by the transaction?

14. A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses 20 per cent. The price of flour rises 50 per cent., and other trade expenses are thereby increased 25 per cent. What advance must he make in the price of a fivepenny loaf that he may still realise the same amount of profit by it?

15. A person sells 20 lb. of tea at a profit of 5 per cent., and 30 lb. at a profit of 8 per cent.; if he had sold it all at a profit of 6 per cent., he would have received one shilling less; what was the prime cost?

16. A dealer bought 56 sheep at a sale at so much per head, and afterwards sold $\frac{3}{8}$ of them at a profit of 15 per cent., $\frac{2}{7}$ at a profit of 10 per cent., and the remainder at a loss of 5 per cent., thereby gaining on the transaction £5, 14s. What did he pay for the sheep?

17. A grocer buys 10 cwt. 3 qrs. 21 lb. of sugar for £30, and pays 12s. 6d. for expenses; at what rate must he sell it per lb. to gain not less than 25 per cent.?

18. If I gain 5 per cent. by selling an article for £5, 5s., how much shall I gain or lose per cent. by selling it for $4\frac{1}{2}$ guineas?

19. If wheat, having been at £2, 10s. 4d. per quarter, rises 6.25 per cent., and afterwards falls 5 per cent., what is the price at which it finally sells per quarter?

20. A grocer bought 200 lb. of tea and 1000 lb. of sugar, the price of the sugar being $\frac{1}{6}$ of that of the tea. He sold the tea at a profit of 40 per cent., and the sugar at a loss of $2\frac{1}{2}$ per cent., gaining on the whole £9, 9s. 7d. What were his buying and selling prices?

21. If eggs be bought at 5 for 3d., at what price must they be sold to gain 20 per cent.?

22. If 17 lb. of tea worth 4s. per lb. is mixed with 25 lb. worth 4s. 8d. per lb., and the whole is sold at 5s. 4d. per lb., what is the total gain, and what the profit per cent?

23. Goods are purchased at £28, 10s. 6d. per cwt.; the trade profits are 15 per cent. on invested capital; the income-tax due thereon at 9d. in the pound amounts to £24, 3s. 6d.; how many cwt. were purchased?

24. How much per cent. is paid to the booksellers when 11s. 4d. is charged to them for a 16s. book; 25 copies are given for 24; 10 per cent. is deducted from the amount for commission, and the money is paid 5 months after the expiration of the year, the interest of the money due being reckoned at 5 per cent. for the last 5 months?

25. I buy a set of watches at 50s. each; I sell them again at a profit of $\frac{1}{10}$ the prime cost, but for ready money I deduct 5 per cent.; for every 8 that are sold for ready money 5 are sold without discount; what will be the gain per cent. on the money invested?

26. A merchant has teas worth 4s. 6d. and 3s. 6d. per lb. respectively, which he mixes in the proportion of 2 lb. of the former to 1 lb. of the latter, and sells the mixture at 4s. 4d. per lb. What does he gain or lose per cent.?

27. A person who began business $5\frac{3}{4}$ yrs. ago increased his capital at the rate of 15 per cent. per annum simple interest, and it now amounts to £5960. What had he at first?

28. With what capital did a tradesman commence business if at the end of twelve months his net gain amount to £210, 14s., a certain portion only of this being accounted trade profit, the remainder, viz. 5s. out of every 9s., being 7 per cent. interest on capital?

29. A quantity of wine is sold to A at a certain loss per cent. Then A sells it to B, losing at the same rate; but B sells it to C for the original cost price, and gains thereby 44 per cent. on his outlay. What was the loss per cent. at which A and B sold the wine?

30. Sold a quantity of logwood at £4, 4s. 6d. per ton, by which I cleared 17 per cent., but I afterwards raised the price to £4, 9s. per ton. What rate per cent. did I gain at this latter price?

31. A merchant sent goods to France, on the sale of which he was a gainer of 40 per cent. The value of the goods was returned to him in French goods, on the sale of which he lost 15 per cent. What percentage of his original gain remained?

32. A corndealer bought 800 qrs. at 25s. per qr., which, being kept for twelve months, lost 8 per cent. in measure, and was then sold at 30s. per qr. Find his gain or loss per cent., allowing £32, 10s. 6d. for granary expenses, and reckoning interest at 5 per cent. per annum.

## PER-CENTAGES, PROPORTIONAL PARTS, AND MISCELLANEOUS RULES AND PROBLEMS.

**119.** In Profit and Loss the term "*per cent.*" was applied with especial reference to £100, but the term "*cent.*" more widely, and yet more strictly, signifies *a hundred units of any kind, and of any denomination.*

Thus, if a man sells 100 bushels of wheat so as to receive as much for them as would equal the value of 105 bushels if bought at the price he gave for them, it is evident that—

$$\frac{\text{His gain}}{\text{His outlay}} = \frac{5}{100};$$

*i.e.* He gains 5 bushels on every 100 bushels.

*i.e.* He gains 5 p. c.

The whole meaning and manner of working of exercises in "**Per-Centages**" will be evident from the solution of the following examples—

**Ex. 1.** *If* 8 *per cent. of the persons employed in a mine in which* 625 *persons were engaged were women, what was the total number of the women employed?*

$$\textit{As}\ 8 : 100 :: \textit{the women in the mine} : \textit{the total number of persons engaged there;}$$

$$\textit{i.e. As}\ 8 : 100 :: \textit{the Answer} : 625\ \textit{persons;}$$

$$\textit{i.e. The Answer} = \frac{625\ \textit{persons} \times 8}{100} = 50\ \textit{women.}$$

**Ex. 2.** *The Board of Trade Returns for* 1861 *show that, with regard to shipwrecks, there were saved by lifeboats,* 13½ *per cent.; by rockets,* 8 *per cent.; by ships' boats,* 62 *per cent.; by individuals,* ½ *per cent.; and* 16 *per cent. were lost. Determine the number of lives saved by the several means, corresponding to an excess of* 2619 *rescued by ships' boats over those saved by lifeboats.*

| | |
|---|---|
| The per-centage saved by ships' boats | = 62 |
| " " " lifeboats | = 13½ |
| The excess rescued by ships' boats | = 48½ p. c. |

But by the question—

$$\frac{48\frac{1}{2}}{100} = \frac{2619}{\text{the total number wrecked.}}$$

$$\therefore \text{The total number wrecked} = \frac{2619 \times 100}{48\frac{1}{2}} = \frac{2619 \times 100 \times 2}{97} = 5400.$$

And by the question—

$$\text{(1.)}\ \frac{13\frac{1}{2}}{100} = \left(\frac{\text{No. saved by lifeboats}}{\text{The total No. wrecked}} = \right)\frac{\text{No. saved by lifeboats}}{5400};$$

$$\therefore \text{The No. saved by lifeboats} = \frac{5400 \times 13\frac{1}{2}}{100} = 729.$$

$$\text{(2.)}\ \frac{8}{100} = \left(\frac{\text{No. saved by rockets}}{\text{The total No. wrecked}} = \right)\frac{\text{No. saved by rockets}}{5400};$$

$$\therefore \text{The No. saved by rockets} = \frac{5400 \times 8}{100} = 432.$$

$$(3.)\ \frac{62}{100} = \left(\frac{\text{No. saved by ships' boats}}{\text{The total No. wrecked}} = \right)\frac{\text{No. saved by ships' boats}}{5400};$$

$$\therefore \text{ The No. saved by ships' boats} = \frac{5400 \times 62}{100} = \qquad 3348.$$

$$(4.)\ \frac{\frac{1}{2}}{100} = \left(\frac{\text{No. saved by individuals}}{\text{The total No. wrecked}} = \right)\frac{\text{No. saved by individuals}}{5400};$$

$$\therefore \text{ The No. saved by individuals} = \frac{5400 \times \frac{1}{2}}{100} = \qquad 27$$

$$(5.)\ \frac{16}{100} = \left(\frac{\text{The No. of lives lost}}{\text{The total No. wrecked}} = \right)\frac{\text{No. of lives lost}}{5400};$$

$$\therefore \text{ The No. of lives lost} = \frac{5400 \times 16}{100} = \qquad 864.$$

*Answer*, 720; 432; 3348; and 27.

**120.** Questions in **Proportional Parts** mostly require for their solution the use of the following proposition :—

**Proposition.—To divide a number or (quantity) into parts which shall be connected together by given ratios.**

**Ex. 1.** *Two persons, A and B, gained in trade £375, one having put in £500, and the other £850, what part of the profits ought each to receive?*

(Examination for Certificates. Male students. 1870.)

Let $x$ = the profit per £1 of invested capital.

Now the whole amount of invested capital was (£500 + £850=) £1350;

$\therefore$ The total amount of profit = 1350 $x$ = £375 by question;

$$\therefore x = £\frac{375}{1350}.$$

And since A invested £500—

$$\text{A's profit} = £\frac{375}{1350} \times 500.$$

Also, since B invested £850,

$$\text{B's profit} = £\frac{375}{1350} \times 850.$$

Hence the reason of the following rule—

**RULE.—To divide a given number** (*or quantity*) **into parts which shall be to each as given numbers** (*i.e. shall be connected together by*

***given ratios*), take the sum of the given numbers as the common denominator of fractions which shall have the separate numbers themselves as numerators. The given number (or quantity), being multiplied by each of these fractions in succession, will give the numbers (or quantities) required for the Answer.**

**Ex. 1.** may be worked by the rule as follows—

$$£500 + £850 = £1350;$$

$$\therefore \text{The share of A} = £\frac{500}{1350} \text{ of } £375 = £138\tfrac{8}{9};$$

$$\text{And the share of B} = £\frac{850}{1350} \text{ of } £375 = £236\tfrac{1}{9}.$$

*Answer*, £138$\frac{8}{9}$; £236$\frac{1}{9}$.

**Ex. 2.** *Divide* 64 *into two parts having the ratio of* 3 : 5.

By the rule—

$$3 + 5 = 8;$$

Therefore the fractions are $\frac{3}{8}$ and $\frac{5}{8}$;

And consequently, the parts required by the answer are—

(1) $\frac{3}{8}$ of 64, and (2) $\frac{5}{8}$ of 64;

*i.e.* (1) 24 and (2) 40, *Answers.*

**Ex. 3.** *Divide* £639 *among three persons in the ratio of* 13, 17, *and* 41.

By the rule—

$$13 + 17 + 41 = 71;$$

Therefore the fractions are $\frac{13}{71}$, $\frac{17}{71}$, and $\frac{41}{71}$;

$$\left.\begin{aligned} \text{And } \tfrac{13}{71} \text{ of } £639 &= £117 \\ \tfrac{17}{71} \text{ of } £639 &= £153 \\ \tfrac{41}{71} \text{ of } £639 &= £369 \end{aligned}\right\} \textit{Answer.}$$

**Ex. 4.** *Sugar being composed of* 49·856 *per cent. of oxygen,* 43·265 *per cent. of carbon, and the remainder hydrogen, find how many pounds of each of these materials there will be in a ton of sugar.*

(Examination for Certificates. Male students. 1867.)

Here 49·856 + 43·265 = 93·121;

Therefore (100 − 93·121 = ) 6·879 = the per-centage of hydrogen.

**Now, by the rule—**

**The sum of all these parts = 100 parts;**
**Therefore the fractions are—**

$$\frac{49\cdot856,\ 43\cdot265, \text{ and } 6\cdot879}{100};$$

**i.e. multiplying each numerator and the common denominator by 1000—**

$$\frac{49856,\ 43265, \text{ and } 6879}{100000}.$$

$$\left.\begin{array}{l}\text{Now } \frac{49856}{100000} \text{ of 2240 lb.} = 1116\cdot7744 \text{ lb. (oxygen)}\\ \text{And } \frac{43265}{100000} \text{ of 2240 lb.} = 969\cdot136 \text{ lb. (carbon)}\\ \text{And } \frac{6879}{100000} \text{ of 2240 lb.} = 154\cdot0896 \text{ lb. (hydrogen)}\end{array}\right\}\textit{Answer.}$$

**121.** In the old text-books a rule called "**Alligation**" was employed for the solution of a certain class of questions which are of very little practical utility, and which we prefer to work by Algebraical processes, as in the following examples—

**Ex. 1.** *In what ratio must two kinds of flour, worth 2½d. and 3¾d. per lb. respectively, be taken to make a mixture worth 3¼d. per lb.?*

Let $x$ = the number of lb. at 2½d. per lb.,
and $y$ = " " 3¾d. "
then $x + y$ = " " 3¼d. "

Now ($x \times 2\frac{1}{2}$, i.e. $x \times \frac{5}{2}$, i.e.) $\frac{5x}{2}$ = the cost, *in pence*, of that portion of the flour which is worth 2½d. per lb.

And ($y \times 3\frac{3}{4}$, i.e. $y \times \frac{15}{4}$, i.e.) $\frac{15y}{4}$ = the cost, *in pence*, of that portion of the flour which is worth 3¾d. per lb.

Also $(x + y) \times 3\frac{1}{4}$, i.e. $(x + y) \times \frac{13}{4}$ = the cost, *in pence*, of all the flour sold.

Therefore, by the question—

$$\frac{5x}{2} + \frac{15y}{4} = (x + y) \times \frac{13}{4};$$

Or, multiplying each side of the equation by 4, we have—

$$\begin{aligned} 10x + 15y &= (x + y) \times 13; \\ \text{i.e. } 10x + 15y &= 13x + 13y; \\ \text{i.e. } 15y - 13y &= 13x - 10x. \\ \text{i.e. } 2y &= 3x; \\ \text{i.e. } \frac{y}{x} &= \frac{3}{2}. \end{aligned}$$

From which we see that for every 3 lb. of the flour at 3¾d., we must take 2 lb. of that at 2½d.

$\therefore$ *Answer*, 2 : 3.

**Ex. 2.** *In what proportions must wines, worth* 10*s.*, 14*s.*, 17*s.*, *and* 18*s. per gall. respectively, be mixed so that the compound may be worth* 16*s. per gall.?*

Let $a$ = the number of gallons at 10s. per gall.
$b$ = " " 14s. "
$c$ = " " 17s. "
$d$ = " " 18s. "

Then $a + b + c + d$ = the number of gallons at 16s. per gall.

$\therefore$ $10a$ = the cost of the gallons at 10s. per gall.
$14b$ = " " 14s. "
$17c$ = " " 17s. "
$18d$ = " " 18s. "

$16(a + b + c + d)$ = the cost of the whole quantity of wine.

$$\begin{aligned} &\therefore 10a + 14b + 17c + 18d = 16(a + b + c + d); \\ &\text{i.e. } 10a + 14b + 17c + 18d = 16a + 16b + 16c + 16d; \\ &\therefore 10a - 16a + 14b - 16b + 17c - 16c + 18d - 16d = 0; \\ &\therefore -6a - 2b + c + 2d = 0; \\ &\therefore c + 2d = 6a + 2b. \end{aligned}$$

If in this last equation we assume any values for any three of the letters $a$, $b$, $c$, $d$, we can find the value of the remaining letter, and unless this value prove a *minus* quantity, the four values of the four letters will be the answer.

For example, if we suppose that—

$$a = 4, \quad c = 8, \quad d = 9,$$

and we require to know the value of $b$—

Since $c + 2d = 6a + 2b$;
Therefore $8 + 18 = 24 + 2b$;
i.e. $26 = 24 + 2b$;
i.e. $2b = 2$;
$\therefore\ b = 1$.

Or, to put the same thing in words—

*If 4 galls. of wine at 10s. per gall., 8 galls. at 17s. per gall., and 9 galls. at 18s. per gall., be mixed together with a certain number of gallons at 14s. per gall., and the whole mixture be sold at 16s. per gall., at cost price, the number of gallons at 14s. per gall. must be (by the above equation)* 1.

It is evident that if other values be given to $a$, $c$, $d$ (i.e. if other quantities of wine be taken at 10s., 17s. and 18s. per gall. respectively), the value of $b$ will also vary, thus—

If $a = 3$, $c = 10$, and $d = 7$,
Then, since $c + 2d = 6a + 2b$,
$10 + 14 = 18 + 2b$,
$10 + 14 - 18 = 2b$;
$\therefore\ 6 = 2b$;
$\therefore\ b = 3$;

and it is at once evident that there is no limit to the answers which may be given to this question.

**Ex. 3.** *How much linen, at 2s. per yard, must be taken with the same quantity at 2s. 5d. per yard and 216 yds. at 3s. 4d. per yard, that the whole may be worth 2s. 6d. per yard at an average?*

Let $a$ = the number of yards at (2s., i.e.) 24d. per yard;
then $a$ = ,, ,, (2s. 5d., i.e.) 29d. ,,

Therefore by the question—

$24a + 29a + (216 \times 40) = (a + a + 216) \times 30$;
i.e. $24a + 29a + 8640 = 30a + 30a + 6480$;
i.e. $53a + 8640 = 60a + 6480$;
$\therefore\ 7a = 2160$;
$\therefore\ a = 308\frac{4}{7}$, *Answer.*

**Ex. 4.** *What quantities of tea, worth 8s., 7s. 6d., and 6s. 6d. per lb. respectively, must be mixed together to form a parcel containing 112 lb. worth 7s. 4d. per lb.?*

Let $a =$ the number of lbs. at (8s., *i.e.*) 96 pence per lb.
and $b =$ ,, ,, (7s. 6d., *i.e.*) 90 ,, ,,
$\therefore 112-(a+b) =$ ,, ,, (6s. 6d., *i.e.*) 78 ,, ,,

Therefore by the question—

$$96a + 90b + 78\ (112 - a - b) = (112 \times 88 =)\ 9856;$$
$$i.e.\ 96a + 90b + 8736 - 78a - 78b = 9856;$$
$$i.e.\ 18a + 12b = (9856 - 8736 =)\ 1120;$$
$$i.e.\ 3a + 2b = 186\tfrac{2}{3}.$$

Assume that $a = 26\frac{2}{3}$ (at 8s. per lb.)
Then since $3a + 2b = 186\frac{2}{3}$,
we have $80 + 2b = 186\frac{2}{3}$;
*i.e.* $2b = 186\frac{2}{3} - 80 = 106\frac{2}{3}$;
$\therefore\ b = 53\frac{1}{3}$ (at 7s. 6d. per lb.)
And since $112 - a - b =$ the number of lbs. at 6s. 6d. per lb.,
we have $112 - 26\frac{2}{3} - 53\frac{1}{3} = 112 - 80 = 32$ (lb. at 6s. 6d. per lb.)

***Answer*, 26⅔, 53⅓, and 32.**

**122.** In conducting commercial transactions between different nations, it is sometimes required to know how much money in the coinage of one country will liquidate a debt in the coinage of another country. The rule by which this is found is called the **Rule of Exchange.**

**Def.** THE PAR OF EXCHANGE between two countries is the *intrinsic* value of the money of one country as compared with the money of another country.

This *par* will only vary when the coins in either country vary in fineness, weight, and nominal value, except when the standard coins are of different metals, *e.g.*—

*Originally,* $\dfrac{\text{The English (gold) sovereign}}{\text{The Dutch (silver) florin}} = \dfrac{11}{1} = 11;$

*Afterwards,* gold having become scarcer, and silver having become more plentiful,

$$\frac{\text{The English (gold) sovereign}}{\text{The Dutch (silver) florin}} = \frac{11\frac{1}{2}}{1} = 11\tfrac{1}{2}.$$

This variation of the Par of Exchange was owing to the fact that, while England has adopted gold for its standard coin, Holland has chosen silver as the metal of which to make its standard coins.

To provide against the difficulty occasioned by this and other causes of variation in the *Par of Exchange*, another term has been invented and adopted; this is—

THE COURSE OF EXCHANGE, which, at any particular time, is the sum of money of one country which, at that time, is given for a fixed sum of the money of another.

Thus, although the *Par of Exchange* fixes the value of £1 sterling in English money at 25·22 francs in French money, an English merchant would regulate his payments by the *Course of Exchange* which would perhaps fix the exchange at 26·32 francs for £1 English.

The causes which chiefly influence the *Course of Exchange* will be more easily understood after the use of

**Bills of Exchange** has been explained.

For various reasons it has become a universal practice with merchants seldom to send coined money in payment for goods, but to employ these written instruments, called *Bills of Exchange* instead. A bill of exchange may be defined as "*a written instrument drawn up by one person and directing a second person to pay a third person a certain sum of money, at a certain date, charging the same to the drawer of the bill.*"

The following illustration will explain this definition—

A merchant in England, whom we will call A, buys goods of B in America to the value of £1200, while C in America buys goods of D in England to the value of £1000. If these debts are paid in coin, £1200 must pass from England to America, and £1000 from America to England, and a moment's consideration will show that if all commercial transactions were conducted in the same manner, there would be such a great demand for coined money that many merchants would have to restrict their business for want of cash. Besides this great inconvenience, there would be the smaller ones, such as *loss of interest* on the money during its transit from place to place, the *cost of insuring* it against robbery or shipwreck, and *the reduction in the amount of current coin* whenever any was lost in the sea.

All these difficulties are obviated in the following manner—

A will send to B a Bill of Exchange for £1200 payable in London, and

C will give D a Bill of Exchange for £1000 payable in America, and wherever the credit of A is good (*i.e.* wherever he is considered sure to pay the money when due), B can get the value of £1200 for the bill he holds from A. He may take this bill to C, and buy goods from him with it; C will send it to D to pay his debt of £1000, receiving a similar bill for £200 from D to put the difference straight. D will present the bill for £1000 to A, perhaps for goods received from him; and thus, by means of this one bill, goods have been bought and sold to the extent of £4000 without the passing of a single coin.

Since there are, in every country having foreign commercial dealings, thousands of merchants wanting to buy bills, and others desirous of selling them, and those anxious to buy bills may not have time to seek those who have them to sell, the business of buying and selling bills is usually conducted by *bill-brokers.*

Now, it is evident that whenever the imports from France into England exceed the exports from England into France, the total amount of money owing by England to France will be in excess of that owing by France to England; in other words, the total amount of the Bills of Exchange drawn in England and payable in France will be in excess of the bills of France payable in England. As a natural consequence, the bill-brokers in England would be called upon for more bills on France than their agents in France would have corresponding bills on England. However, the English brokers would still issue bills of their own payable in France, and, to make up the difference to their agents in France, they would send over to them either coin or bullion. Now, as we said above, the passage of this coin would be accompanied by risk and loss of interest, therefore the brokers would charge a little above the *Par of Exchange* for these bills, and indeed for all their bills on France, and the varying price they thus charge is the great cause of the variations in the *Course of Exchange.*

It should be noticed that the Course of Exchange never differs very much from the Par of Exchange, because, if it was so, merchants would send coined money to pay their foreign debts instead of sending bills.*

In foreign exchanges,† one place always gives another a *fixed sum* or piece of money for a *variable sum.* The former is called the CERTAIN RATE, the latter the VARIABLE RATE.

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* The student is referred to "*Principles of Political Economy,*" by John Stuart Mill, book iii. chap. xx., for further information on Exchanges.

† "*Arithmetic in Theory and Practice,*" by James Thomson, LL.D., to which the student is referred for further particulars concerning the details of exchange between commercial countries.

Thus, in exchange with Portugal, London gives for one milree sometimes 65 pence, sometimes 68 pence. Here 1 milree is the *fixed price*, and 65 pence, 68 pence, &c., the *variable price*.

When England has more to pay to France than to receive from her, bills on France are said to be *above par*, or *at a premium*; and whenever bills on France are at a premium in England, bills on England are *at a discount* in France. In such a case the exchange *is said to be* against England, and *vice versâ*. If, however, bills are *at a par* in either country, they are at a par also in the other.

There is one other term of exchange which requires to be explained. This is—

ARBITRATION OF EXCHANGE, which may be defined as the estimation of the Rate of Exchange, found by comparing the price of bills, bullion, &c., in several different countries at the same time, or in all the countries of the world.

In explanation of this term it will be sufficient to quote the following passage from Stuart Mill's "*Political Economy*:"—

"It remains to observe, that the exchanges do not depend on the balance of the debts and credits with each country separately, but with all countries taken together. England may owe a balance of payment to France, but it does not follow that the exchange with France will be against England, and that bills on France will be at a premium; because a balance may be due to England from Holland or Hamburg, and she may pay her debts to France with bills on those places, which is technically called *Arbitration of Exchange*. There is some little additional expense—partly Commission and partly loss of Interest—in settling debts in this circuitous manner, and to the extent of that small difference the exchange with one country may vary apart from that with others; but in the main, the exchanges with all foreign countries vary together, according as the country has a balance to receive or to pay on the general result of its foreign transactions."

*The Par of Exchange* is sometimes called the "*Arbitration Price*," being evidently the Rate of Exchange found by comparing the price of bullion in different countries.

*Arbitration* is called *Simple* or *Compound*, according as three or more places are concerned in the estimation.

*The Rule called Arbitration of Exchange* may be explained to be "the process of finding the Rate of Exchange between the first and last of a given number of places when the Course of Exchange between the first and second, second and third, &c., &c., is given.

**123.** All **Questions in Exchange** depend for their solution upon the principles of Practice and Proportion. The following will serve as examples—

**Ex. 1.** *Reduce £84, 19s. 2d. English money into French money at 25·2 francs to the £1 sterling.*

**By Proportion we have—**

As £1 : 25·2 francs : : £84, 19s. 2d. : its equivalent in French francs;

$$\therefore\ 25{\cdot}2 \text{ francs} \times \frac{\text{£84, 19s. 2d.}}{\text{£1}} = \textit{Answer}.$$

$$\textit{i.e.} \left( 25{\cdot}2 \text{ francs} \times \frac{20390\text{d.}}{240\text{d.}} = \right) 2140{\cdot}95 \text{ francs} = \textit{Answer}.$$

Or—

**By Practice we have—**

| | |
|---|---|
| 10s. = ½ of £1. | 25·2 francs = the equivalent of £1. |
| | 84 |
| | 1008 |
| | 2016 |
| | 2116·8 francs = the equivalent of £84. |
| 5s. = ½ of 10s. | 12·6 „ = „ „ 10s. |
| 3s. 4d. = ⅓ of 10s. | 6·3 „ = „ „ 5s. |
| 10d. = ¼ of 3s. 4d. | 4·2 „ = „ „ 3s. 4d. |
| | 1·05 „ = „ „ 10d. |

*Answer*, ∴ 2140·95 francs = the equivalent of £84, 19s. 2d.

**Ex. 2.** *A bill upon Hamburg is bought in London at* 13 *marks* 7½ *schillings per £*1 *sterling. It is afterwards sold in*

*Paris at* 101 *francs for* 54 *marks. What is the Rate of Exchange between London and Paris?*

This question may be thus stated—

"*If* £1 *in English money be worth* 13 *marks* $7\frac{1}{2}$ *schillings in money of Hamburg, and* 54 *marks in money of Hamburg be worth* 101 *francs, how many francs are worth* £1 *in English money?*

Since 16 schillings = 1 mark, we have

$$13 \text{ marks } 7\tfrac{1}{2} \text{ schillings} = 13\left(\tfrac{7\frac{1}{2}}{16} = 13\tfrac{15}{32}\right) \text{ marks.}$$

(1.) **By First Principles**, we proceed thus—

$$54 \text{ marks} = 101 \text{ francs};$$

$$\therefore 1 \text{ mark} = \frac{101}{54} \text{ francs};$$

$$\therefore 13\tfrac{15}{32} \text{ marks} = \frac{101}{54} \text{ francs} \times 13\tfrac{15}{32} = \left(\tfrac{101}{54} \times \tfrac{43531}{1728}\right) \text{ francs} = 25\tfrac{331}{1728} \text{ francs.}$$

But $13\frac{15}{32}$ marks = £1;

$$\therefore £1 = 25\tfrac{331}{1728} \text{ francs, } \textit{Answer.}$$

(2.) **By the Rule of Chain** (§ 103, page 179) we proceed thus—

£1 = $13\frac{15}{32}$ marks,
and
54 marks = 101 francs.

*Dividing* the product of the numbers on the right *by* the product of those on the left, we have—

$$£\,(1 \times 54) = (13\tfrac{15}{32} \times 101) \text{ francs};$$

$$\therefore £1 = \frac{13\frac{15}{32} \times 101}{54} \text{ francs} = 25\tfrac{331}{1728} \text{ francs, } \textit{Answer.}$$

**Ex. 3.** *The exchange between London and Frankfort is* 11 *florins* 52 *kreutzers for the £ sterling; the exchange between Paris and Frankfort is* 20 *francs for* 9 *florins* 20 *kreutzers: what is the exchange between London and Paris?* (1 *florin* = 60 *kreutzers*).

(Examination for Certificates. Male students. 1863).

**By the Rule of Chain**—

£1 = (11 florins 52 kreutzers =) 712 *kreutzers*,
(9 florins 20 kreutzers = ) 560 *kreutzers* = 20 *francs;*

$$\therefore £\,(1 \times 560) = (712 \times 20) \text{ francs,}$$

$$\therefore £1 = \frac{712 \times 20}{560} \text{ francs} = 25\tfrac{3}{7} \text{ francs, } \textit{Answer.}$$

N.B.—In using this *Rule of Chain* it should be noticed that the *first* term on the *left* gives its name to the product of all the numbers on the left, and the *last* or *bottom* term on the *right* gives its name to the product of all the numbers on the right.

**Ex. 4.** *If for* 1 *sovereign I can buy* 11 *gulden* 12 *kreutzers, or* 25·5 *francs, and for one* 20-*franc piece I can buy* 9 *gulden* 20 *kreutzers, how much do I gain per cent. by buying French gold with my English gold before buying German money?*

Here 1 gulden = 60 kreutzers;
Therefore—
(1.) Buying German money in London, I get $11\frac{1}{5}$ guld. for £1 sterling.
(2.) Buying German money through Paris, I get as below—

$$\begin{aligned} \pounds 1 &= 25\cdot 5 \text{ francs,} \\ 20 \text{ francs} &= 9\tfrac{1}{3} \text{ guld.} \\ \hline \therefore \pounds 1 &= \frac{25\cdot 5 \times 9\frac{1}{3}}{20} \text{ guld.} = 11\tfrac{9}{10} \text{ guld.} \end{aligned}$$

But this is an advance of $(11\frac{9}{10} - 11\frac{1}{5} =)$ $\frac{7}{10}$ guld. upon $11\frac{1}{5}$ guld., the price received in England.

And a gain of $\frac{7}{10}$ guld. upon $11\frac{1}{5}$ guld. $= \left(\frac{\frac{7}{10}}{11\frac{1}{5}} \times 100 = \right) 6\frac{1}{4}$ p. c. *Ans.*

## Exercise 42.

### Exercises in Per-centages, Alligation, &c.

(Collected from Examination Papers.)

1. If £600 is to be divided among 24 men, 36 women, and 72 children, in such a manner that 2 men receive as much as 3 women, and 1 woman as much as 2 children, how much does each child receive?

2. The number of boys in a school at Christmas 1871 was 960, but the next year at Christmas it was 1020; what was the increase per cent.?

3. The sum of £26 was divided amongst four men, A, B, C, D, in the following manner:—B's share was three times A's, C's was three times B's, D's was as much as A's, B's, and C's, together; what was the share of each?

4. In a town of 18600 electors ·625 voted for one candidate, and ·08 did not vote at all. How many votes were polled by the other candidate, and by what per-centage of the total number of votes was the contest won, supposing that each elector used his vote for only one candidate?

5. If an engine and tender cost £4000, 12s. 4d., and the engine costs one-half more than twice as much as the tender; what was the separate cost of each?

6. Tea at 3s. 6d. per lb. is mixed with tea at 2s. 4d. per lb.; what is the proportion of the mixture when the mixed tea is worth 2s. 10d. per lb.?

7. If 15 men, 12 women, and 9 boys can complete a piece of work in 50 days; how long would 9 men, 15 women, and 18 boys be in doing the same work, the parts done by each man, woman, and boy respectively being as the numbers 3, 2, 1?

8. A man, his wife, and boy, worked in a garden: the man for 2 days, his wife for $3\frac{1}{2}$ days, the boy for 4 days. Their daily wages were—man's to woman's in the ratio of 7 to 4, and woman's to boy's in the ratio of 7 to 3. Their total earnings amounted to £1. What were the daily wages of each?

9. A, B, and C, respectively, invested £6413, £5247, and £4081 in a business in which the net profits in one year were £1123, 1s. $2\frac{1}{4}$d. What was the share of each?

10. A, B, C are sent to empty a cistern by means of two pumps of the same bore. A and B go to work first, making 37 and 40 strokes, respectively, per minute; but, after 5 minutes, they make each 5 strokes less a minute, and after 10 minutes more A gives way to C, who works at the rate of 30 strokes a minute. The cistern is emptied in 22 minutes altogether, and the men are paid 12s. 7d. for their labour. How shall the money be divided?

11. A, B, C join in an enterprise, to which they are to contribute in the proportion of 3, $3\frac{1}{3}$, $3\frac{2}{3}$, respectively. A pays down £220, 10s., B £205, 4s., and C £213, 15s. What must each one pay to the others, or receive from them, to make the proportion of capital accurate?

12. Three merchants make a stock of £700; their profits are respectively, £23, 1s. $5\frac{1}{7}$d., £64, 3s. $3\frac{27}{35}$d., and £39, 8s. $7\frac{3}{35}$d. How much did each contribute?

13. A ship's company take a prize of £1001, 19s. 2d., which is divided according to their pay and time on board. The officers and midshipmen have been on board six months, the sailors three months; the officers receive 40s., the midshipmen 30s., the sailors 22s. a month. There are 4 officers, 12 midshipmen, 110 sailors; what will be each man's share?

14. A bankrupt owes £900 to three creditors, and his whole property amounts to £675. The claims of two of the creditors are £125 and £375 respectively. What sum will the remaining creditor receive for his dividend?

15. Divide £2850 between A, B, and C, giving $\frac{6}{11}$ of B's share to A, and to C £300 more than what is given to A and B together.

16. A merchant has teas worth 4s. 6d. and 3s. 6d. per lb. respectively, which he mixes in proportion of 2 lb. of the former to 1 lb. of the latter, and sells the mixture at 4s. 4d. per lb. What does he gain or lose per cent.?

17. The exchange between Russia and England is 6·28 roubles for £1, and between Russia and America 6·28 roubles for 4·55 dollars, also between America and England 4·85 dollars for £1. Find the gain in transmitting 6000 roubles through England to America.

18. Suppose that from an official return, the arrivals of oxen, calves, sheep, pigs, and horses, in the port of London for a certain week, it appears that there were 3 times as many sheep as oxen, the number of pigs was 13⅓ per cent. of the number of sheep, that for every 28 pigs there were 25 calves, that the horses were $\frac{3}{10}$ per cent. of the whole, and that the horses and oxen together were 3587; required, the number of oxen.

19. F and G are partners in trade. F contributes ⅜ of the joint capital for 10½ months, and G receives ⅘ of the gain; what was his period of investment?

20. In what ratio must beer of two kinds, worth 2½d. and 3¾d. per pint respectively, be mixed, so that the mixture may sell at 3¼d. per pint.

21. Divide £7000 among three persons in such a way that if the shares of each be multiplied respectively by 2½, 3, and 3½, the products shall be equal.

22. How much coffee at 2s. 6d., 3s. 4d., 3s. 8d. per lb. must be mixed with 45 lb. at 4s. 6d. to sell at 4s. per lb.?

23. If 20 bushels of wheat at 5s. per bush. be mixed with rye at 3s. per bush., barley at 2s. per bush., and oats at 1s. 6d. per bush., how much must be taken of rye, barley, and oats, respectively, to make the mixture worth 2s. 8d. per bush.?

24. A wine merchant is desirous of mixing 18 galls. of Canary at 6s. 9d. per gall., with Malaga at 7s. 6d. per gall., sherry at 5s. per gall., and white wine at 4s. 3d. per gall.; how much of each sort must he take that the mixture may be sold for 7s. 6d. per gall. at a profit of 25 per cent. upon the cost price?

25. In what quantities must teas at 5s., 6s., 8s., and 9s. per lb. respectively, be mixed, to obtain a composition of 87 lb., worth 7s. per lb.?

26. Desiring to remit £1430 to Portugal, will it be better for my Portuguese correspondent to have a direct remittance from London to Lisbon at 68d. per milree, or to have the money transmitted through Amsterdam and Paris, the exchange between London and Amsterdam being at 37s. 3d. Flemish per pound sterling; and between Amsterdam and Paris at 56 pence Flemish for 3 francs; while between Paris and Lisbon the exchange is 460 rees for 3 francs, an expense of $1\frac{1}{2}$ per cent. being incurred in the circuitous course? (*N.B.* 1000 rees = 1 milree.)

## POWERS AND ROOTS.

**124. Def. 1.** When any number is multiplied by itself once or more than once (as the case may be), the product is said to be a **Power** of that number; and, conversely—

**Def. 2.** When a given number, being multiplied by itself once or more than once (as the case may be), will produce a second given number, the first number is said to be a **Root** of the second number.

**125.** *The Integral Powers of any given number are greater or less than that given number according as it is integral or fractional, i.e. according as the given number is greater or less than unity, e.g.—*

**Ex. 1.** $2 \times 2 = 4$, *i.e.* the given number + 2.
**Ex. 2.** $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, *i.e.* the given number − $\frac{1}{4}$.
**Ex. 3.** $1\frac{1}{3} \times 1\frac{1}{3} \times 1\frac{1}{3} = \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = \frac{64}{27} = 2\frac{10}{27}$.
**Ex. 4.** $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$.

**126. Def. 3.** When any given number is multiplied by itself once, the product is called the "**Square**" of that number.

Thus, since $6 \times 6 = 36$, we say that 36 *is the Square of* 6.

To signify that 36 is the product obtained by multiplying together *two* sixes, a small figure 2 is used as below—

$$36 = 6^2.$$

Similarly, since $216 = 6 \times 6 \times 6$, we have—

$$216 = 6^3.$$

And, generally, if $x$ be any given number, we express—

$x \times x$ by $x^2$.
$x \times x \times x$ by $x^3$
$x \times x \times x \times x$ by $x^4$.
$x \times x \times x \times x$ ...... n times $= x^n$.

These small figures used for this particular purpose are called **Indices**, and their use, together with that of the word POWER, will be apparent from the following table—

$x^2 = x \times x =$ the *second* power (or SQUARE) of $x$.
$x^3 = x \times x \times x =$ the *third* power (or CUBE) of $x$.
$x^4 = x \times x \times x \times x =$ the *fourth* power of $x$.
$x^5 = x \times x \times x \times x \times x =$ the *fifth* power of $x$.
$x^n =$ the nth power of $x$.

**Def. 4.** The **Square Root** of any given number is a number which, when multiplied by itself once, will produce that number.

Thus, since $36 = 6 \times 6$, it follows that *the Square Root of* 36 *is* 6.

To signify this we introduce a new sign, $\surd$, which is an altered form of the first letter of the Latin word *radix*, signifying *a root.*

The use of this sign, *together with a new use of the indices*, will be apparent from the following table—

$\sqrt[2]{x} =$ the *Square* Root of $x$.
$\sqrt[3]{x} =$ the *Cube* Root of $x$.
$\sqrt[4]{x} =$ the *fourth* Root of $x$.
$\sqrt[n]{x} =$ the nth Root of $x$.

N.B.—The index is usually omitted from the sign for the Square Root, *i.e.* instead of $\sqrt[2]{\phantom{x}}$ we generally use $\surd$ alone.

## I. Square Root.

**127.** It is evident that since the square of 10 is 100, the square of any number less than 10 will be less than 100.

Also, that since $100^2 = 10000$, the square of any number less than 100 will be less than 10000.

Again, since $1000^2 = 1000000$, the square of any number less than 1000 will be less than 1000000.

These and similar considerations enable us to determine the following points—

1. The **Square** of any number less than 10 (*i.e.* any number expressed by one figure,) is expressed by not more than two figures.

2. The **Square** of any number between 10 and 99, both inclusive, is expressed by more than two figures and not more than four.

3. The **Square** of any number between 100 and 999, both inclusive, is expressed by more than four figures and not more than six.

And, generally—

4. The **Square** of 10, or any power of 10, is expressed by 1 and twice as many ciphers as there are ciphers in the given number.

And, conversely—

1. The **Square Root** of any number expressed by 1 and an even number of ciphers, is expressed by 1 and half that number of ciphers.

2. The **Square Root** of any number expressed by more than four figures and not more than six, is some number not more than 999 and not less than 100.

3. The **Square Root** of any number expressed by more than two figures and not more than four, is some number not more than 99 and not less than 10.

4. The **Square Root** of any number expressed by less than three figures, is some number less than 10.

**128.** By Algebra we find that—

$$(a + b)^2 = a^2 + 2ab + b^2.$$

The truth contained in this equation may be explained and illustrated by the following Arithmetical proposition—

**Prop.** *The Square of a number is the sum of—*

1st. *The Square of the tens.*
2nd. *The product of double the tens* "**into**"* *the units.*
3rd. *The Square of the units.*

**Ex. 1.** *Let it be required to square* 12.

$$\begin{array}{rl} \text{Now } 12 = & 10 + 2; \\ \text{Therefore} & (10 + 2) \\ \times & \underline{(10 + 2)} \\ & 100 + 20 \\ & \qquad + 20 + 4 \\ \hline & 100 + 2\cdot20 + 4 = \mathbf{144}, \textit{ Answer.} \end{array}$$

**Ex. 2.** $43^2 = 40^2 + 2\cdot40\cdot3 + 3^2$

$$= 1600 + 240 + 9$$
$$= 1849;$$

Therefore $\sqrt{1849} = 43.$

Let us now suppose that, being totally ignorant of the answer, we are required to find the value of $\sqrt{1849}$.

$$\text{Since } 40^2 = 1600,$$
$$\text{and } 50^2 = 2500,$$

it is evident that the answer is a number greater than 40 and less than 50, *i.e.* it is some number having 4 in its *tens'* place.

Therefore we put a 4 in the quotient, and take $40^2$ from 1849, *i.e.* from 1849 we take 1600, and find 249 remainder.

$$\begin{array}{r|l} 1849 & \mathbf{43}, \textit{ Answer.} \\ \underline{1600} & \\ 80 + 3 = 83)\ 249 & \\ 249 & \end{array}$$

Now we know that this 249 which remains is the sum of—

(1.) The product of double the tens "**into**" the units; and
(2.) The square of the units.

Therefore since double the tens = (2 × 4 tens =) 8 tens = 80, we divide 249 by 80 as a trial divisor; and since the quotient is a little over 3, we place 3 in the units' place of the answer, and then we find that 249 is the exact sum of—

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* See page 8.

(1.) The product of 8 tens "into" 3, and
(2.) The square of 3.
Therefore 3 is the units' figure.
∴ *Answer* = 43.

From this brief explanation the student will be able to see the reason of the following rule—

**RULE.—To find the Square Root of any given number, separate it first into groups of two figures each. This grouping must be commenced on the right, and proceed towards the left.**

**Find now the greatest square in the group on the extreme left, subtract it from this group, and beside the remainder bring down the next group towards the right, thus forming a new dividend, at the same time placing in the quotient the number whose square was subtracted from the group on the left.**

**Double this root, and find how many times the product is contained in the figures preceding that on the extreme right in the new dividend. Write the quotient on the right of the root found before, and also on the right of the last divisor, and then multiply this new divisor by the last figure added to the quotient. Subtract the product from the new dividend, and, if any number remains, treat it as the last remainder was treated.**

**Ex. 1.** *Find the Square Root of* 2704.

Here the given number contains two groups—viz., 27 and 04.

$$\begin{array}{rl} & 2\dot{7}0\dot{4}\,(52 \\ & \underline{25} \\ 102) & \overline{204} \\ & 204 \end{array}$$

*Answer*, 52.

**Ex. 2.** *Find the Square Root of* 105625.

Here the given number contains three groups—viz., 10, 56, and 25.
The 62 = 3 × 2, with 2 annexed.
The 645 = 32 × 2, with 5 annexed.

$$\begin{array}{rl} & 1\dot{0}5\dot{6}2\dot{5}\,(325 \\ & 9 \\ 62) & \overline{156} \\ & \underline{124} \\ 645) & 3225 \\ & 3225 \end{array}$$

*Answer*, 325.

**Ex. 3.** *Find the Square Root of* 4558225.

The 41 = 2 × 2, with 1 annexed.
The 423 = 21 × 2, with 3 annexed.
The 4265 = 213 × 2, with 5 annexed.

$$\begin{array}{rl} & \dot{4}5\dot{5}8\dot{2}2\dot{5}\,(2135 \\ & 4 \\ 41) & \overline{55} \\ & \underline{41} \\ 423) & \overline{1482} \\ & \underline{1269} \\ 4265) & \overline{21325} \\ & 21325 \end{array}$$

*Answer*, 2135.

**Ex. 4.** *Find the Square Root of* 20·25.

$$\begin{array}{r l} & 2\dot{0}\cdot2\dot{5}\,(4\cdot5,\ \textit{Answer.} \\ & 16 \\ \hline 85) & 425 \\ & 425 \end{array}$$

The square root of a number which contains decimals is found exactly as the square root of a number composed wholly of integers; but *in grouping the figures* we always commence with the *units'* figure, and then count off twos to the right and the left; and as soon as the integers are exhausted by the process for finding the square root, *a decimal point* is placed in the quotient, and the remainder of the quotient is a decimal fraction.

**Ex. 5.** *Find the Square Root of* 4·124961.

$$\begin{array}{r l} & \dot{4}\cdot1\dot{2}4\dot{9}6\dot{1}\,(2\cdot031,\ \textit{Answer.} \\ & 4 \\ \hline 403) & \cdot1249 \\ & \phantom{\cdot}1209 \\ \hline 4061) & \phantom{\cdot}4061 \\ & \phantom{\cdot}4061 \end{array}$$

Here, after bringing down the ·12, we find that the 1 of this number is not divisible by 4. We therefore place one cipher in the quotient, and another on the right of the 4, and bring down the next group of figures—viz., 49, and place them on the right of the other group, 12.

**Ex. 6.** *Extract the Square Root of* 72 *correct to three places of decimals.*

$$\begin{array}{r l} & 72\cdot(8\cdot485,\ \textit{Answer.} \\ & 64 \\ \hline 164) & 800 \\ & 656 \\ \hline 1688) & 14400 \\ & 13504 \\ \hline 16965) & 89600 \\ & 84825 \\ \hline & \phantom{8}4775 \end{array}$$

Since 72 = 72·000000, &c., in this case we bring down ciphers in groups of two each until the answer has been carried out to the required number of decimal places.

## II. Cube Root.

**129.** Reasoning in a manner similar to that pursued in the consideration of the Square Root, we shall be able to establish a rule by which the Cube Root of a given number may be found, but advantage is taken of this opportunity to refer the student to works on Algebra for the scientific explanations of the rules for finding both the Square Roots and the Cube Roots of numbers.

**Ex. 1.** $30^3 = 27000$;

Hence, $\sqrt[3]{27000} = 30$; *i.e.* $\sqrt[3]{27}$ with *one* cipher annexed.

**Ex. 2.** $400^3 = 64000000$;

Hence, $\sqrt[3]{64000000} = 400$; *i.e.* $\sqrt[3]{64}$ with *two* ciphers annexed.

**Ex. 3.** $5000^3 = 125000000000$;

Hence, $\sqrt[3]{125000000000} = 5000$; *i.e.* $\sqrt[3]{125}$ with *three* ciphers annexed.

And, generally—

$$\text{The number of ciphers in the CUBE ROOT} = \frac{\text{the number of ciphers in the GIVEN NUMBER.}}{3}$$

**130.** By Algebra we find that

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

The truth contained in this equation may be explained and illustrated by the following arithmetical proposition:—

**Prop.** *The Cube of a number is the sum of*—

1st. *The Cube of the tens.*

2nd. *The product of three times the Square of the tens* "**into**" *the units.*

3rd. *The product of three times the tens* "**into**" *the Square of the units.*

4th. *The Cube of the units.*

**Ex.** *Let it be required to find the Cube of* 45.

By the foregoing proposition we have—

$$\begin{aligned} 46^3 &= 40^3 + 3\cdot40^2 6 + 3\cdot40\cdot6^2 + 6^3 \\ &= 64000 + 28800 + 4320 + 216 \\ &= 97336; \end{aligned}$$

Therefore, $\sqrt[3]{97336} = 46$.

Let us now suppose that, being totally ignorant of the answer, we are required to find the value of $\sqrt[3]{97336}$.

Now, since $40^3 = 64000$, and $50^3 = 125000$, it is evident that the answer is greater than 40 and less than 50; therefore it is some number having a 4 in its place of *tens*.

We therefore, as below, put a 4 in the tens' place of the quotient, and subtract 64000 (*i.e.* $40^3$) from 97336, and find 33336 remainder.

Now, we know that this remainder is the sum of—

(1.) The product of three times the square of *the tens* "**into**" *the units.*

(2.) The product of three times *the tens* "**into**" the square of *the units.*

(3.) The cube of *the units.*

Therefore, since three times the square of *the tens* "**into**" *the units* =

$$3 \cdot 40^2 \cdot \textit{the units} =$$
$$3 \cdot 1600 \cdot \textit{the units} =$$
$$4800 \cdot \textit{the units},$$

we shall be able to find *the units' figure* of the answer by dividing 33336 by 4800, for (2) and (3) of the component parts of 33336 are of so small a value compared with (1) that we may neglect them from the present consideration.

$$\text{Now } \frac{3336}{4800} = \frac{333 \cdot 36}{48}, \text{ and we find that } \frac{333 \cdot 36}{48} = 6 + ;$$

we therefore put 6 in the place of units, and we then find that 3336 is the exact sum of—

(1.) The product of three times the *square of* 4 *tens* "into" 6 *units.*

(2.) The product of three times 4 tens into *the square of* 6 *units.*

(3.) *The cube of* 6 *units.*

Therefore 6 is the units' figure.

$$\therefore \textit{Answer} = 46.$$

The preceding explanation will assist the student in comprehending the reason of the following rule—

**RULE.—To find the Cube Root of any given number:—If the given number is expressed by not more than three figures, its root is in the units (for 10, the smallest number of two figures, requires *four* figures to express its cube); if the number contains more than three figures, divide it into groups of three figures each, commencing on the right and proceeding towards the left; (the last group may contain less than three figures).**

**Find the greatest perfect cube contained in the group on the extreme left, place its cube root on the right of the given number (*i.e.* in the place where the answer is written), subtract the cube itself from the group operated upon, and beside the remainder place the group next on the right. From this new**

**group cut off the two figures on the right, divide the remainder by** ***three times the square of that figure of the answer found above,*** **and place the quotient on the right of that figure, as the second figure of the answer. If, now, the new group formed above be exactly equal to** ***three times the square of the figure on the left of the answer × the figure on the right, + three times the figure on the left × the square of the figure on the right,*** **+ the cube of the figure on the** ***right,*** **the answer is expressed by the two figures found above,** ***i.e.*** **if there be no more groups of figures in the given number.**

Ex. 1. *Find the Cube Root of* 175616.

Dividing the figures of the given number into groups of three, we have

$$17\dot{5}61\dot{6},$$

where the right-hand figure of each group is indicated by a dot placed over it.

Now it is evident, from reasons given above, that the cube root of this number is a number of two figures, so that the answer will have one figure in the place of tens and one in the place of units.

$$\begin{array}{rl} & 17\dot{5}61\dot{6}\left(\begin{array}{l} 50+6 \\ =\mathbf{56} \end{array}\right. \\ & \underline{125} \\ & \mathbf{50616} \\ 3\cdot50^2\cdot6 = 45000 & \\ 3\cdot50\cdot6^2 = \phantom{0}5400 & \\ \underline{6^3 = \phantom{00}216} & 50616 \end{array}$$

*Answer*, **56.**

It will also be evident that the given number is composed of four components, of which *the cube of the tens* is by far the greatest; and (since $10^3 = 1000$, and $100^3 = 1000000$) *the cube of the tens* must be in *the thousands;* therefore we separate three figures on the right, as above, and from the remainder, 175, subtract the greatest perfect cube contained therein: this is 125, the cube of 5, therefore 5 is the *tens'* figure of the answer. The figure 5 is therefore placed on the right of the given number (*i.e.* in the place of the answer), and its cube, 125, is placed beneath the 175, and subtracted therefrom: the remainder is 50.

To the right of this 50 the next group, 616, is placed, thus making 50616. This number, 50616, contains three components, of which the greatest is equal to *three times the square of the tens' figure of the answer × the units' figure of the answer.* But this product cannot be less than a hundred (because $10 \times 10 \times x = 100x$, where $x$ represents the units' figure of the answer), and for this reason the two figures, 16, on the right of 5016, are separated, and the remainder, 506, is divided by *three times the square of* 5, and the quotient is put in units' place of the answer.

$$\text{Now, } \frac{506}{5^2 \times 3} = \frac{506}{25 \times 3} = \frac{506}{75} = 6 + ;$$

therefore 6 is placed on the right of the 5 in the answer, thus making 56, and that 6 is the real units' figure of the answer is proved by showing that—

$$\begin{aligned} 50616 &= 3 \cdot 50^2 \cdot 6 + 3 \cdot 50 \cdot 6^2 + 6^3 \\ &= 3 \cdot 2500 \cdot 6 + 3 \cdot 50 \cdot 36 + 216 \\ &= 45000 + 5400 + 216 \\ &= 50616. \end{aligned}$$

## Ex. 2. *Find the Cube Root of* 177504328.

Dividing this number also into groups of three figures each, we have

$$17\dot{7}50\dot{4}32\dot{8}.$$

We proceed, as before, to find the cube root of 177504; then we continue the process to find the cube root of the whole given number 177504328, thus—

$$\begin{array}{lcr|ll} & & & \mathbf{17}\dot{\mathbf{7}}50\dot{4}32\dot{8} & (50 + 6) \text{ tens} + \mathbf{2} \\ & & & 125 & \quad = 56 \text{ tens} + \mathbf{2} \\ & & & \overline{52\mathbf{504}} & \quad = 562 \\ 3 \cdot 50^2 \cdot 6 = 3 \cdot 2500 \cdot 6 & = & 45000 & & \\ 3 \cdot 50 \cdot 6^2 = 3 \cdot 50 \cdot 36 & = & 5400 & & \\ 6^3 = 6 \cdot 6 \cdot 6 & = & \underline{216} & 50616 & \\ & & & \overline{1888\mathbf{328}} & \\ 3 \cdot 560^2 \cdot 2 = 3 \cdot 313600 \cdot 2 & = & 1881600 & & \\ 3 \cdot 560 \cdot 2^2 = 3 \cdot 560 \cdot 4 & = & 6720 & & \\ 2^3 = 2 \cdot 2 \cdot 2 & = & \underline{8} & 1888328 & \end{array}$$

*Answer*, **562**.

Of the three figures of the answer, the 56 was obtained as in Ex. 1; then, considering these two figures as 56 *tens* (§ 10, N.B. 2), we separate two figures from the right of 1888328, and divide 18883, the remainder, by ($56^2 \times 3$, *i.e.* by $3136 \times 3$, *i.e.* by) 9408.

$$\text{Now } \frac{18883}{9408} = 2+ ;$$

therefore we put **2** in the units' place of the answer, and, as above, we verify this result by considering the 56 as being 56 *tens*.

## Ex. 3. *Find the Cube Root of* 178263433152.

Dividing the given number into groups of three figures each, we have—

$$17\dot{8}\ 26\dot{3}\ 43\dot{3}\ 15\dot{2}.$$

We proceed, as before, to find the cube root of 178263433; then we continue the process to find the whole given number, 178263433152—

$$17\dot{8}26\dot{3}43\dot{3}15\dot{2}\left(\begin{array}{r}(500+60+2)\,\text{tens}+\\ 8=\mathbf{5628},\ \textit{Ans.}\end{array}\right.$$

$$\begin{array}{rrr|l}
 & & 125 & \\ \hline
 & & & 53\mathbf{263}\\
3\cdot50^2\cdot6= & 3\cdot2500\cdot6= & 45000 & \\
3\cdot50\cdot6^2= & 3\cdot50\cdot36= & 5400 & \\
6^3= & 6\cdot6\cdot6= & 216 & 50616\\ \hline
 & & & 2647\mathbf{433}\\
3\cdot560^2\cdot2= & 3\cdot313600\cdot2= & 1881600 & \\
3\cdot560\cdot2^2= & 3\cdot560\cdot4= & 6720 & \\
2^3= & 2\cdot2\cdot2= & 8 & 1888328\\ \hline
 & & & 759105\mathbf{152}\\
3\cdot5620^2\cdot8= & 3\cdot31584400\cdot8= & 758025600 & \\
3\cdot5620\cdot8^2= & 3\cdot5620\cdot64= & 1079040 & \\
8^3= & 8\cdot8\cdot8= & 512 & 759105152\\ \hline
\end{array}$$

*Answer*, **5628.**

Of the four figures of the answer, the 562 was obtained as in Ex. 2; then, considering these three figures as 562 *tens* (§ 10, N.B. 2), we separate two figures from the right of 759105152, and divide 7591051, the remainder, by ($562^2 \times 3$, *i.e.* by $315844 \times 3$, *i.e.* by) 947532.

$$\text{Now } \frac{759105152}{947532} = 8+\,;$$

therefore we put 8 in the units' place of the answer, and, as above, we verify this result by considering the 562 as being 562 *tens.*

**Ex. 4.** *If the internal content of a cubical box be* 18·191447 *cub. ft., what is the length of one of its sides?*

By § 36 we see that—

$$\text{The Answer} = \sqrt[3]{18{\cdot}191447}\ \text{cub. ft.}$$

Therefore—

$$1\dot{8}{\cdot}19\dot{1}44\dot{7}\ (2{\cdot}8$$

$$\begin{array}{rcrcr|l}
 & & & & 8 & \\ \hline
 & & & & & 10{\cdot}\mathbf{191}\\
3\cdot20^2\cdot8 & = & 3\cdot400\cdot8 & = & 9600 & \\
3\cdot20\cdot8^2 & = & 3\cdot20\cdot64 & = & 3840 & \\
8^3 & = & 8\cdot8\cdot8 & = & 512 & 13952
\end{array}$$

Proceeding thus far, we find that 8 is too great a number for the second figure of the answer, because 13952 exceeds 10191. Similarly, we shall find that 7 is also too great.

$$\begin{array}{r|l}
 & \mathbf{1\dot{8}}\cdot19\dot{1}44\dot{7}\ (2\cdot63 \\
 & 8 \\ \cline{2-2}
 & 10\mathbf{191} \\
3\cdot20^2\cdot6 = 3\cdot400\cdot6 = 7200 & \\
3\cdot20\cdot6^2 = 3\cdot20\cdot36 = 2160 & \\
6^3 = 6\cdot6\cdot6 \quad = \ \ 216 & 9576 \\ \hline
 & 615\mathbf{447} \\
3\cdot260^2\cdot3 = 3\cdot67600\cdot3 = 608400 & \\
3\cdot260\cdot3^2 = 3\cdot260\cdot9 \quad = \quad 7020 & \\
3^3 = 3\cdot3\cdot3 \quad = \quad 27 & 615447 \\ \hline
\end{array}$$

*Answer*, **2·63 ft.**

In this case we have proceeded in a manner precisely similar to that which we should have followed if the given number had been 18191447 instead of 18·191447, except that, after using the group of figures containing *whole numbers*, we place a decimal point after that figure of the answer already found.

**Ex.** 5. *If the solid content of a cube be* 8615·125 *cub. inches, what is the area required for* 100 *such cubes to stand upon?*

Here (1) the length of one side of the cube = $\sqrt[3]{8615\cdot125.}$

$$\begin{array}{r|l}
 & \dot{8}61\dot{5}\cdot125\ (20\cdot5 \\
 & 8 \\ \cline{2-2}
 & \mathbf{615125} \\
3\cdot200^2\cdot5 = 3\cdot40000\cdot5 = 600000 & \\
3\cdot200\cdot5^2 = 3\cdot200\cdot25 \quad = \quad 15000 & \\
5^3 = 5\cdot5\cdot5 \quad = \quad 125 & 615125 \\ \hline
\end{array}$$

Therefore (2) the area of one side of the cube = 20·5 in. × 20·5 in.
= 420·25 sq. in.

And (3) one hundred such cubes would require (420·25 sq. in. × 100 =) 42025 sq. in. to stand upon.

*Answer*, **42025 sq. in.**

# SOLUTIONS OF PROBLEMS.

## I. Questions selected from Government Examinations of Training Colleges.

1. *What will the paving of a court cost at $4\frac{3}{4}$d. per square yard, the length being* 58 *ft.* 6 *in., and the breadth* 54 *ft.* 9 *in.?*

$$\begin{aligned}\text{The area of the court} &= 58\text{ ft. }6\text{ in.} \times 54\text{ ft. }9\text{ in.}\\ &= 58\tfrac{1}{2}\text{ ft.} \times 54\tfrac{3}{4}\text{ ft.}\\ &= \left(\frac{117}{2} \times \frac{219}{4}\right)\text{sq. ft.}\end{aligned}$$

Therefore, since the paving of 9 sq. ft. costs $4\frac{3}{4}$d. (by question), we have the following proportion—

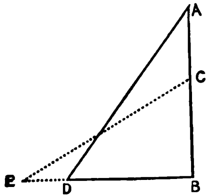
$$9\text{ sq. ft.} \ : \ \left(\frac{117}{2} \times \frac{219}{4}\right)\text{sq. ft.} \ :: \ 4\tfrac{3}{4}\text{d.} \ : \ \text{Answer.}$$

$$\therefore \text{ Answer} = 4\tfrac{3}{4}\text{d.} \times \frac{\frac{117}{2} \times \frac{219}{4}}{9} = \frac{19}{4}\text{d.} \times \frac{117 \times 219}{2 \times 4 \times 9} = \frac{117 \times 219 \times 19}{2 \times 4 \times 9 \times 4}\text{d.}$$

$$= \frac{54093}{32}\text{d.} = \text{£}7,\ 0\text{s. } 10\tfrac{13}{32}\text{d., } \textit{Answer.}$$

2. *The foot of a ladder,* 30 *ft. long, is* 14 *ft. from a house, and its top reaches the upper part of a circular window; when the foot is drawn away to a distance of* 17 *ft. from the house, the top reaches the lower edge of the window; what is the diameter of the window?*

In the accompanying diagram, let AC represent the diameter of the window, let DA represent the position of the ladder in the first case, and EC its position in the second case.



By Euclid I. 47, we have—

1st. $DA^2 = DB^2 + BA^2$;

*i.e.* $30^2 = 14^2 + BA^2$;

$\therefore$ $BA^2 = (30^2 - 14^2)$ ft. $= (900 - 196)$ ft.

$= 704$ ft.;

$\therefore$ $BA = \sqrt{704}$ ft.

2nd. $EC^2 = EB^2 + BC^2$;
*i.e.* $30^2 = 17^2 + BC^2$;
$\therefore$ $BC^2 = (30^2 - 17^2)$ ft. $= (900 - 289)$ ft. $= 611$ ft.
$\therefore$ $BC^2 = \sqrt{611}$ ft.

Therefore—
$CA = (\sqrt{704} - \sqrt{611})$ ft. $= (26{\cdot}53 - 24{\cdot}71)$ ft. $=$ **1·82 ft.**, *Answer.*

3. *One-fourth of an estate belongs to A, $\frac{2}{9}$ of it to B, and the remainder to C, which is worth £2500 more than A's share; what is the value of the whole?*

A's share $= \frac{1}{4}$, and B's share $= \frac{2}{9}$;
$\therefore$ The combined shares of A and B $= \frac{1}{4} + \frac{2}{9} = \frac{17}{36}$;
$\therefore$ The share of C $= 1 - \frac{17}{36} = \frac{36}{36} - \frac{17}{36} = \frac{19}{36}$.

But by the question—
A's share + £2500 = C's share;
*i.e.* $\frac{1}{4}$ of the estate + £2500 = $\frac{19}{36}$ of the estate;
$\therefore$ $(\frac{19}{36} - \frac{1}{4})$ of the estate = £2500;
*i.e.* $(\frac{10}{36} =)\ \frac{5}{18}$ of the estate = £2500;
$\therefore$ $\frac{1}{18}$ of the estate $= £\frac{2500}{5}$;
$\therefore$ $\frac{18}{18}$ of the estate $= £\frac{2500 \times 18}{5} =$ **£9000**, *Answer.*

4. *A starts on a journey, and walks at the rate of 3 miles an hour; B starts an hour and a quarter after A, and walks 4 miles an hour. How far on the road will they be when B overtakes A?*

When B starts, A has been travelling $1\frac{1}{4}$ hrs. at 3 miles an hour;
*i.e.* A is $(3 \times 1\frac{1}{4})$ miles ahead; therefore B has to gain $(3 \times 1\frac{1}{4})$ miles to overtake A.
Now, in 1 hour B travels 1 mile farther than A;
$\therefore$ in $(3 \times 1\frac{1}{4})$ hrs. B would gain $(3 \times 1\frac{1}{4})$ miles upon A.
But in $(3 \times 1\frac{1}{4})$ hrs. B would travel $(3 \times 1\frac{1}{4} \times 4)$ miles.
$\therefore$ *Answer* $= (3 \times 1\frac{1}{4} \times 4)$ miles = **15 miles.**

5. *A lump of iron, containing* 11 *cubic feet, is drawn out into a cylindrical rod* 14 *yards long; what will be the thickness of the rod?*

(I.) *The solid content of a cylinder = height × area of base;*

$\therefore$ 11 cubic ft. = 14 yds. × area of base
= 42 ft. × area of base;

$\therefore$ area of base $= \frac{11}{42}$ sq. ft.

(II.) *The area of a circle* $= \pi r^2 = \frac{22}{7} r^2$;

$\therefore \frac{11}{42}$ sq. ft. $= \frac{22 r^2}{7}$;

*i.e.* Multiplying each side of the equation by 42, we have—

11 sq. ft. $= 132 r^2$;

$\therefore r^2 = \frac{11}{132}$ sq. ft. $= \frac{1}{12}$ sq. ft. = 12 sq. in.;

$\therefore r = \sqrt{12}$ in.

$\therefore 2r$ (*i.e.* the diameter of the circle of the base, *i.e.* the thickness of the cylindrical rod) $= 2\sqrt{12}$ in.
= 3·464 in. × 2 = 6·928 in., *Answer.*

6. *If* 2 *metres* 7 *decimetres* 3 *centimetres of ribbon be worth* 8 *francs* 19 *centimes, how much ribbon may be bought, at the same rate, for* 11 *francs* 43 *centimes?*

Here 2 metres 7 decimetres 3 centimetres = 273 centimetres.
8 francs 19 centimes = 819 centimes.
11 francs 43 centimes = 1143 centimes.

$\therefore$ As 819 *centimes* : 1143 *centimes* :: *the ribbon bought for* 819 *centimes* : *the ribbon bought for* 1143 *centimes*;

*i.e.* $\frac{\text{819 centimes}}{\text{1143 centimes}} = \frac{\text{273 centimetres}}{\text{Answer}}$;

$\therefore$ *Answer* = 273 centimetres $\times \frac{1143}{819}$ = 381 centimetres =
3 metres 8 decimetres 1 centimetre.

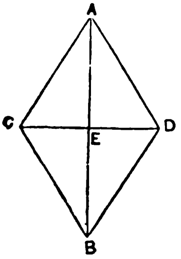
7. *Find the difference between $\frac{13}{24}$ of 3 kilograms 1 hectogram 6 dekagrams 1 decigram 5 centigrams 2 milligrams, and $\frac{16}{19}$ of 2 kilograms 2 dekagrams 7 decigrams 7 milligrams, and express the result in kilograms alone, and in centigrams alone.*

Here 3 kilograms 1 hectogram 6 dekagrams 1 decigram 5 centigrams 2 milligrams = 3160152 *milligrams.*

And 2 kilograms 2 dekagrams 7 decigrams 7 milligrams = 2020707 *milligrams.*

Therefore, the *Answer* = $\frac{13}{24}$ of 3160152 millig. − $\frac{16}{19}$ of 2020707 millig.
= 1711749 millig. − 1701648 millig.
= 10101 millig.
= **1010·1 centigrams**
= **·010101 kilograms.**

8. *A diamond-shaped kite, 2 feet broad, and 4 feet long, is covered on one face with gold leaf, which is sold in pieces 4 inches square. How many leaves does it take?*



Let the figure ACBD represent the kite.
Then AB is 4 feet and CD is 2 feet.

It is evident, therefore, that CE is a line, 1 foot long, drawn from the apex of the triangle ACB, perpendicular to its base.

Then the area of the triangle ACB $= \frac{AB \times CE}{2}$

$$= \frac{4 \text{ ft.} \times 1 \text{ ft.}}{2} = 2 \text{ sq. ft.}$$

Similarly, the area of the triangle ADB = 2 sq. ft.

∴ The area of the figure ACDB = 4 sq. ft. = (144 × 4) sq. in.

But the gold leaf is in pieces 4 inches square, *i.e.* each piece of gold leaf contains (4 × 4, *i.e.*) 16 sq. in.

$$\therefore \textit{Answer} = \frac{144 \times 4}{16} = \textbf{36 pieces.}$$

9. *A and B exchange goods: A gives B 15 cwt. of hops, the retail price of which is 58s. per cwt., but he reckons them at £3, 3s. per cwt.; B gives A 12 barrels of beer, of the retail*

*value of* 1*s.* 2*d. a gallon, but the value of which he raises in proportion to the increased value of the hops; how much must be paid in money?*

$$\text{Here } \frac{\text{the real value}}{\text{the exchange value}} = \frac{58\text{s.}}{63\text{s.}} = \frac{58}{63}.$$

***A** gives **B*** goods whose exchange value is 63s. × 15 = 945s.
***B** gives **A*** goods whose real value is 1s. 2d. × 12 times 36, and
whose exchange value is therefore 1s. 2d. × 12 × 36 × $\frac{63}{58}$

$$= \frac{\overset{7}{\cancel{14}}}{\cancel{12}}\text{s.} \times \frac{\cancel{12}}{1} \times \frac{36}{1} \times \frac{63}{\underset{29}{\cancel{58}}} = \frac{15876\text{s.}}{29} = 547\tfrac{13}{29}\text{s.}$$

Therefore *A owes B* 945s. − 547$\frac{13}{29}$s. = 447$\frac{16}{29}$s. = £22, 7s. 6$\frac{18}{29}$d., *Ans.*

10. *What must be the depth of a cylindrical tank to hold* 50,000 *gallons, its diameter being* 12 *feet?*

I. *The imperial standard gallon contains* 2772·74 *cubic inches;*
∴ 50000 gallons must contain 277·274 cubic inches × 50000,
= 13863700 cubic inches.

II. *The area of a circle* = $\pi r^2$;
∴ The area of the base of the cylindrical tank = $(\frac{22}{7} \times 12 \times 12)$ sq. ft.
= $(\frac{22}{7} \times 144 \times 144)$ sq. in.

III. *The cubic content of a cylinder = height × base;*
∴ The cubic content of the cylindrical tank = height × $(\frac{22}{7}\cdot 144\cdot 144)$;
*i.e.* 13863700 inches = height × $(\frac{22}{7}\cdot 144\cdot 144)$;

$$\therefore \text{Height} = \frac{13863700 \text{ inches}}{\frac{22}{7}\cdot 144\cdot 144} = \frac{(13863700 \times 7) \text{ inches}}{22 \times 144 \times 144}$$

$$= \frac{24261475}{114048}\text{inches} = 212\tfrac{83299}{114048} \text{ inches} = 17 \text{ feet } 8\tfrac{83299}{114048} \text{ inches, } \textit{Ans.}$$

11. *The length of a piece of four-sided timber is* 18 *ft.* 6 *in., the breadth at the greater and less ends* 1 *ft.* 6 *in. and* 1 *ft.* 3 *in. respectively, the thickness at the greater and less ends* 1 *ft.* 3 *in. and* 1 *ft. respectively; find the solid content.*

The average *breadth* is $\frac{1\,ft.\ 6\ in.\ +\ 1\,ft.\ 3\ in.}{2} = 1$ ft. $4\frac{1}{2}$ in.

The average *thickness* is $\frac{1\,ft.\ 3\ in.\ +\ 1\,ft.}{2} = 1$ ft. $1\frac{1}{2}$ in.

$\therefore$ *The solid content* = 1 ft. $4\frac{1}{2}$ in. × 1 ft. $1\frac{1}{2}$ in. × 18 ft. 6 in.

= $1\frac{3}{8}$ ft. × $1\frac{1}{8}$ ft. by $18\frac{1}{2}$ ft.

$= \left(\frac{11}{8} \times \frac{9}{8} \times \frac{37}{2}\right)$ cubic ft. $= \frac{3663}{128}$ cubic ft.

= **28 cubic ft. $1066\frac{1}{2}$ cubic in.**, *Answer.*

12. *A man allows his agent* 1*s. in the pound for collecting his rents; he spends* $\frac{1}{7}$*th of his net income in insuring his life, and this part is exempt from Income-tax; his Income-tax* (*at* 10*d. in the pound*) *is* £38, 19*s.; what is his gross income?*

I. His Income-tax, at 10d. in the pound, was £38, 19s.;

$\therefore$ 10d. : £38, 19s. : : £1 : *the amount upon which he paid Income-tax;*

$\therefore$ The amount upon which he paid Income-tax = £1 × $\frac{£38,\ 19s.}{10d.}$

$= \frac{£9348}{10} =$ £934, 16s.

II. The cost of insuring his life = $\frac{1}{7}$ of his net income;

$\therefore$ The amount upon which he paid Income-tax = $\frac{6}{7}$ of his net income;

*i.e.* £934, 16s. = $\frac{6}{7}$ of his net income;

$\therefore$ $\frac{£934,\ 16s.}{6} = \frac{1}{7}$ of his net income;

$\therefore$ $\frac{£934,\ 16s.}{6} \times 7 = \frac{7}{7}$ of his net income;

*i.e.* £1090, 12s. = his net income.

III. Of his gross income 1s. in the pound is expended in collecting;

$\therefore$ his net income = $\frac{19}{20}$ of his gross income;

*i.e.* £1090, 12s. = $\frac{19}{20}$ of his gross income;

$\therefore$ $\frac{£1090,\ 12s.}{19} = \frac{1}{20}$ of his gross income;

$\therefore$ $\frac{£1090,\ 12s.}{19} \times 20 = \frac{20}{20}$ of his gross income;

*i.e.* **£1148** = his gross income, *Answer.*

13. *A piece of cloth, when measured with a yard measure which is two-thirds of an inch too short, appears to be* $10\frac{1}{2}$ *yards long; what is its true length?*

$$\frac{\text{The apparent length}}{\text{The true length}} = \frac{1\text{ yd.}}{1\text{ yd.} - \frac{2}{3}\text{ in.}} = \frac{36\text{ in.}}{35\frac{1}{3}\text{ in.}} = \frac{108}{106};$$

$$i.e.\ \frac{10\frac{1}{2}\text{ yds.}}{\text{The true length}} = \frac{108}{106};$$

$\therefore$ 108 times the true length = 106 times $10\frac{1}{2}$ yds. = 1113 yds.

$$\therefore\ \textit{The true length} = \frac{1113\text{ yds.}}{108} = 10\tfrac{11}{36}\text{ yds., } \textit{Answer.}$$

14. *A circular plate of gold,* 3 *inches in diameter, and* $\frac{1}{8}$ *of an inch thick, is extended by hammering so as to cover* 5 *square yards; find its present thickness.*

(I.) The area of a circle 3 inches in diameter = $(\frac{22}{7}\cdot3\cdot3)$ sq. in.;
$\therefore$ *The solid content of the plate* = $(\frac{22}{7}\cdot3\cdot3\cdot\frac{1}{8})$ cubic in.

(II.) Similarly, *surface* × *thickness* = *cubic content;*
$\therefore$ 5 square yds. × *thickness* = $(\frac{22}{7}\cdot3\cdot3\cdot\frac{1}{8})$ cubic in.;
*i.e.* 6480 square in. × *thickness* = $(\frac{22}{7}\cdot3\cdot3\cdot\frac{1}{8})$ cubic in.;

$$\therefore\ \textit{Thickness} = \frac{\frac{22}{7}\cdot3\cdot3\cdot\frac{1}{8}}{6480} = \frac{\overset{11}{\cancel{22}}\cdot\cancel{3}\cdot\cancel{3}\cdot1}{7\cdot\underset{\underset{360}{\cancel{720}}}{\cancel{6480}}\cdot8}\text{ in.} = \frac{11}{20160}\text{ in., } \textit{Answer.}$$

15. *It is half-past three o'clock; at what hour will the two hands of the watch meet for the first time?*

At half-past three o'clock *the hour hand* is at (15 + $2\frac{1}{2}$, *i.e.*) $17\frac{1}{2}$ mins. past 12,
*the minute hand* is at 30 mins. past 12, *i.e.* it is (30 + $17\frac{1}{2}$, *i.e.*) $47\frac{1}{2}$ mins. behind the hour hand in going round the dial-plate.

Now the minute hand gains 55 minutes on the dial-plate in an hour, from which it follows that—

55 mins. : $47\frac{1}{2}$ mins. :: 1 hr. : time required for the minute hand to gain $47\frac{1}{2}$ mins.

$\therefore$ The time required for the minute hand to gain $47\frac{1}{2}$ mins. =

$$\frac{47\frac{1}{2}}{55}\text{ hr.} = \frac{95}{110}\text{ hr.} = \frac{19}{22}\text{ hr.} = 51\tfrac{9}{11}\text{ mins.}$$

The time at starting was 3 hr. 30 mins.

$$\begin{array}{r} 51\tfrac{9}{11}\text{ mins.} \\ \hline 4\text{ hr. }21\tfrac{9}{11}\text{ mins.} \end{array}$$ **, *Answer*.**

16. *What number is that from which if you deduct* $\frac{7}{9} - \frac{3}{7}$, *and to the remainder add the quotient of* $\frac{3}{17}$ *divided by* $5\frac{1}{4}$, *the sum will be* $\frac{122}{357} + 10\frac{1}{21}$ ?

Let N be the number required.

$$\text{Then } N - \left(\frac{7}{9} - \frac{3}{7}\right) + \frac{\frac{3}{17}}{5\frac{1}{4}} = \frac{122}{357} + 10\tfrac{1}{21};$$

$$i.e.\ N - \frac{22}{63} + \frac{4}{119} = 10\tfrac{139}{357};$$

$$\therefore N = 10\tfrac{139}{357} + \frac{22}{63} - \frac{4}{119}$$

$$= 10\frac{417 + 374 - 36}{1071} = 10\tfrac{755}{1071}, \textit{ Answer.}$$

17. *Find the Square Root of* $\dfrac{5{\cdot}04}{{\cdot}021}$.

$$\frac{5{\cdot}04}{{\cdot}021} = \frac{5{\cdot}04 \times 1000}{{\cdot}021 \times 1000} = \frac{5040}{21} = 240;$$

$$\therefore \sqrt{\frac{5{\cdot}04}{{\cdot}021}} = \sqrt{240} = \text{(by the Rule) } 15{\cdot}5 \text{ nearly.}$$

*Answer*, **15·5 nearly.**

18. *In a cricket match a certain number of the players make* 147 *runs among them; in another match the same players, increased by* 2, *make* 234 *runs, the average of each being* 5 *more in the second case than in the first. Find the averages and the number of the players.*

$$\left.\begin{array}{l}\text{Let } N = \text{number of players} \\ \text{and } A = \text{average}\end{array}\right\}\text{ in the first match.}$$

$$\left.\begin{array}{l}\text{Then } N + 2 = \text{number of players} \\ \text{and } \quad A + 5 = \text{average}\end{array}\right\}\text{ in the second match.}$$

Then by the question we have—

$$\text{(1.) } N{\cdot}A = 147;\ \therefore N = \frac{147}{A}.$$

$$
\begin{aligned}
&(2.)\ (N+2)\cdot(A+5)=234;\\
&i.e.\ N\cdot A+2A+5N+10=234;\\
&i.e.\ 147+2A+5\cdot\frac{147}{A}+10=234;\\
&\therefore\ 2A+\frac{5\times147}{A}=234-147-10=77;\\
&\therefore\ 2A^2+735=77A;\\
&\therefore\ 2A^2-77A=-735;\\
&\therefore\ A^2-\frac{77A}{2}=-367\tfrac{1}{2};\\
&\therefore\ A^2-\frac{77A}{2}+\left(\frac{77}{4}\right)^2=-367\tfrac{1}{2}+\left(\frac{77}{4}\right)^2=-367\tfrac{1}{2}+370\tfrac{9}{16}\\
&\qquad\qquad=3\tfrac{1}{16};\\
&\therefore\ A-\frac{77}{4}=\sqrt{3\tfrac{1}{16}}=\sqrt{\tfrac{49}{16}}=\tfrac{7}{4}=1\tfrac{3}{4};\\
&i.e.\ A-19\tfrac{1}{4}=1\tfrac{3}{4};
\end{aligned}
$$

$$
\left.\begin{aligned}
&\therefore\ A=1\tfrac{3}{4}+19\tfrac{1}{4}=21\\
&\therefore\ A+5=21+5=26\\
&\text{And } N=\frac{147}{A}=\frac{147}{21}=7\\
&\therefore\ N+2=7+2=9
\end{aligned}\right\}\textit{Answer.}
$$

19. *Multiply* 3·456789 *by* 107·34163. *The result is required correct to 5 places of decimals only, and no more figures are to be used than are necessary to secure this result.*

$$
\begin{array}{r}
3{\cdot}456789\\
107{\cdot}34163\\
\hline
345{\cdot}6789\phantom{00}\\
24{\cdot}19752\cancel{3}\\
1{\cdot}03703\\
{\cdot}13827\\
{\cdot}00346\\
{\cdot}00207\\
{\cdot}00010\\
\hline
371{\cdot}05735,\ \textit{Answer.}
\end{array}
$$

(1.) Multiply by the 1 *hundred.*

(2.) „ „ 7 *tens.*

(3.) „ 3·4567 by ·3, because this multiplicand containing *four* decimal places, and the multiplier *one*, the product will contain *five* decimal places.

But, to obtain these decimal places correct, say,

*three eights* = 24,

and then, rejecting the 4, carry the 2 to the product of *three sevens.*

N.B.—If the product of the *three eights* had been 25, or any number greater than 24 and less than 35, we should have carried 3 instead of 2. Similarly, 4 would be carried for all numbers greater than 34 and less than 45.

(4.) Multiply 3·456 by ·04.

(5.) „ 3·45 by ·001.

(6.) „ 3·4 by ·0006.

(7.) „ 3 by ·00003.

The answer is now found by adding up these results.

20. *In an election where the number of voters was* 2424, *the majority of the winning candidate was* 159, *the votes polled by the candidates were in the ratio of* 7 *to* 6 ; *how many voters did not go to the poll ?*

Let $L =$ the number of votes polled by the *losing* candidate.
Then $L + 159 =$ ,, ,, ,, *winning* ,,

Then, by the question—

$$L : L + 159 :: 6 : 7.$$

$$\therefore 7L = 6(L + 159) = 6L + 954;$$
$$\therefore 7L - 6L = 954;$$
$$\left.\begin{array}{l} i.e.\ L = 954 \\ \therefore L + 159 = 1113 \end{array}\right\} \therefore L + L + 159 = 954 + 1113 = 2067.$$

Therefore $2424 - 2067 =$ **357**, *Answer.*

Or—

The winning candidate gains 1 vote out of every 7 polled by him;

*i.e. The total number of votes polled by him* = 7 times the number he gained over his opponent.

*i.e. The total number of votes polled by him* $= 159 \times 7 = 1113$.

Also, *The total number of votes polled by the losing candidate* $= 159 \times 6 = 954$;

$\therefore$ *The number of voters who did not go to the poll* $= 2424 - (1113 + 954) = 2424 - 2067 =$ **357**, *Answer.*

21. £1000 *was divided between A, B, and C in such a way that B had* $\frac{7}{8}$ *of A's share, and C had* $\frac{5}{6}$ *of B's share; what was the share of each ?*

If A's share be represented by 1,
Then B's share $= \frac{7}{8}$;
And therefore C's share $= \frac{5}{6}$ of $\frac{7}{8} = \frac{35}{48}$;
$\therefore$ The united shares of A, B, and $C = 1 + \frac{7}{8} + \frac{35}{48} = 2\frac{29}{48}$, *i.e.* £$1000 = 2\frac{29}{48}$.

$$\therefore \text{As } 2\tfrac{29}{48} : 1 :: £1000 : \text{the share of A.}$$

Therefore, *the share of A* $= £\frac{1000}{2\frac{29}{48}} = £\frac{\frac{\cancel{1000}\,^{8}}{1}}{\frac{\cancel{125}\,_{1}}{48}} = £48 \times 8 = £384;$

*The share of B* $= \frac{7}{8}$ of £384 = £336;
*The share of C* $= \frac{5}{6}$ of £336 = £280.

***Answer*, £384, £336, £280.**

22. *How many gallons of spirits, at 31s. per gallon, must be added to 16 gallons, at 17s. per gallon, and 19 gallons, at 27s. per gallon, to make the mixture worth 26s. per gallon?*

Let $x$ = the gallons at 31s. per gallon.

Then by the question we have—

$$(17\text{s.} \times 16) + (27\text{s.} \times 19) + (31\text{s.} \times x) = 26\text{s.} \times (16 + 19 + x);$$

$$\text{or } (17 \times 16) + (27 \times 19) + (31x) = 26(16 + 19 + x)$$

$$272 + 513 + 31x = 416 + 494 + 26x;$$

$$\therefore 31x - 26x = 416 + 494 - 272 - 513;$$

*i.e.* $5x = 910 - 785$

$\therefore x = 25$, *Answer.*

## Examination Paper. Male Students of the Second Year. Christmas, 1872.

1. *A man bought a horse and carriage for £100, and sold the horse at a gain of 50 per cent. on its prime cost, and the carriage at a loss of 25 per cent., thus gaining 5 per cent. on his whole outlay; what were the buying and the selling prices of the horse and carriage respectively?*

Let H = prime cost of the *horse* in pounds.
$\therefore$ £100 − H = „ *carriage* „

Then by the question—

$$\frac{150}{100}\text{H} + \frac{75}{100}(£100 - \text{H}) = \frac{105}{100} \text{ of } £100;$$

$$\textit{i.e. } \frac{3\text{H}}{2} + \tfrac{3}{4}(£100 - \text{H}) = £105;$$

$$\therefore 6\text{H} + 3(£100 - \text{H}) = £105 \times 4$$

$$\textit{i.e. } 6\text{H} + £300 - 3\text{H} = £420;$$

$$\therefore 3\text{H} = £420 - £300 = £120;$$

$$\left.\begin{array}{ll} & \therefore \text{H} = £40, \text{ the prime cost of the horse.} \\ \text{And} & \therefore £100 - \text{H} = £60 \quad ,, \quad ,, \quad \text{carriage.} \\ & \therefore \tfrac{3}{2} \text{ of H} = £60, \text{ the selling price of the horse;} \\ \text{and} & \tfrac{3}{4} \text{ of } £60 = £45, \quad ,, \quad ,, \quad \text{carriage.} \end{array}\right\} \textit{Answer.}$$

2. *A butcher sold a haunch of mutton at 9½d. per lb., and a shoulder at 8½d. per lb., thus receiving 2½d. more than if he had sold them together at 9d. per lb. Their total weight was 13 lb.; what was the weight of each?*

**It is evident that if the haunch and shoulder had been of equal weights the average price would have been 9d. per lb., and in this case the butcher would neither gain nor lose anything by selling them together at 9d. per lb.**

**It is also clear that, for every pound by which the weight of the haunch exceeds that of the shoulder, the butcher will gain ½d. by selling them separately, and since his total gain is 5 times ½d., it follows that—**

***The weight of the haunch > the weight of the shoulder* by 5 lb.**

**Therefore—**

$$\left.\begin{array}{l}\left(\frac{13-5}{2}\text{ lb., } \textit{i.e. } \frac{8}{2}\text{ lb., } \textit{i.e. }\right) 4\text{ lb.} = \text{weight of the shoulder.} \\ \text{And } (13-4,\ \textit{i.e.})\ 9\text{ lb.} = \text{weight of the haunch.}\end{array}\right\} \textit{Answer.}$$

3. *Show that, if the numerator and denominator of a fraction are both divided by the same number, the result is an equivalent fraction; and reduce $\frac{134397}{310323}$ to its lowest terms.*

**See § 59, and proceed thus—**

**The G.C.M. of 134397 and 310323 is 327 (§ 33.)**

$$\therefore \frac{134397}{310322} = \frac{134397 \div 327}{310323 \div 327} = \frac{411}{949}, \textit{ Answer.}$$

4. *On the 23rd of July, 1872, a meteoric stone, weighing 47 kilograms, fell in the south of France; on being analysed it was found to contain 17·2 per cent. of silica, 13·84 per cent. of magnesia, 11·33 per cent. of iron protoxide, 0·12 per cent. of sodium chloride, 1·24 per cent. of water; find the weight of each of these materials in the whole substance.*

By the question—

$$(1.)\ \frac{17{\cdot}2}{100} = \frac{\text{Silica}}{47\text{ kilog.}};$$

$$\therefore\ \text{Silica} = \frac{47\text{ kilog.} \times 17{\cdot}2}{100} = 8{\cdot}084\text{ kilog., } \textit{Answer.}$$

$$(2.)\ \frac{13{\cdot}84}{100} = \frac{\text{Magnesia}}{47\text{ kilog.}};$$

$$\therefore\ \text{Magnesia} = \frac{47\text{ kilog.} \times 13{\cdot}84}{100} = 6{\cdot}5048\text{ kilog., } \textit{Answer.}$$

$$(3.)\ \frac{11{\cdot}33}{100} = \frac{\text{Iron protoxide}}{47\text{ kilog.}};$$

$$\therefore\ \text{Iron protoxide} = \frac{47\text{ kilog.} \times 11{\cdot}33}{100} = 5{\cdot}3251\text{ kilog., } \textit{Answer.}$$

$$(4.)\ \frac{0{\cdot}12}{100} = \frac{\text{Sodium chloride}}{47\text{ kilog.}};$$

$$\therefore\ \text{Sodium chloride} = \frac{47\text{ kilog.} \times {\cdot}12}{100} = {\cdot}0564\text{ kilog., } \textit{Answer.}$$

$$(5.)\ \frac{1{\cdot}24}{100} = \frac{\text{Water}}{47\text{ kilog.}};$$

$$\therefore\ \text{Water} = \frac{47\text{ kilog.} \times 1{\cdot}24}{100} = {\cdot}5828\text{ kilog., } \textit{Answer.}$$

5. *Simplify* $\dfrac{1\frac{8}{15} + 2\frac{7}{11}}{2\frac{15}{19} - 1\frac{11}{13}} \times \dfrac{1\frac{3}{7} + 2\frac{1}{13}}{1\frac{11}{21} + 1\frac{9}{10}} \div \dfrac{1\frac{10}{17} + 2\frac{1}{11}}{1\frac{18}{19} - 1\frac{1}{14}}.$

Here—

$$1\tfrac{8}{15} + 2\tfrac{7}{11} = (1 + 2) + (\tfrac{8}{15} + \tfrac{7}{11}) = 3 + \frac{88 + 105}{165} = 3 + \tfrac{193}{165} = 4\tfrac{28}{165}.$$

And—

$$2\tfrac{15}{19} - 1\tfrac{11}{13} = (2 - 1) + (\tfrac{15}{19} - \tfrac{11}{13}) = 1 + \frac{195 - 209}{247} = 1 - \tfrac{14}{247} = \tfrac{233}{247}.$$

$$\therefore\ \frac{1\frac{8}{15} + 2\frac{7}{11}}{2\frac{15}{19} - 1\frac{11}{13}} = \frac{4\frac{28}{165}}{\frac{233}{247}} = \frac{\frac{688}{165}}{\frac{233}{247}} = \frac{688 \times 247}{165 \times 233}.$$

Also—

$$1\tfrac{3}{7} + 2\tfrac{1}{13} = (1 + 2) + (\tfrac{3}{7} + \tfrac{1}{13}) = 3 + \frac{39 + 7}{91} = 3 + \tfrac{46}{91} = 3\tfrac{46}{91}.$$

And—

$$1\tfrac{11}{21} + 1\tfrac{9}{10} = (1 + 1) + (\tfrac{11}{21} + \tfrac{9}{10}) = 2 + \frac{110 + 189}{210} = 2 + \tfrac{299}{210} = 3\tfrac{89}{210}.$$

$$\therefore\ \frac{1\frac{3}{7} + 2\frac{1}{13}}{1\frac{11}{21} + 1\frac{9}{10}} = \frac{3\frac{46}{91}}{3\frac{89}{210}} = \frac{\dfrac{319}{\cancel{91}\ 13}}{\dfrac{719}{\cancel{210}\ 30}} = \frac{319 \times 30}{719 \times 13}$$

Again, $1\frac{10}{17} + 2\frac{1}{11} = (1 + 2) + (\frac{10}{17} + \frac{1}{11}) = 3 + \frac{110 + 17}{187} = 3\frac{127}{187}$.

And $1\frac{18}{19} - 1\frac{1}{14} = (1 - 1) + (\frac{18}{19} - \frac{1}{14}) = \frac{252 - 19}{266} = \frac{233}{266}$.

$$\therefore \frac{1\frac{10}{17} + 2\frac{1}{11}}{1\frac{18}{19} - 1\frac{1}{14}} = \frac{3\frac{127}{187}}{\frac{233}{266}} = \frac{\frac{688}{187}}{\frac{233}{266}} = \frac{688 \times 266}{187 \times 233}.$$

Therefore the question may now stand thus—

$$\textit{Simplify}\ \frac{688 \times 247}{165 \times 233} \times \frac{319 \times 30}{719 \times 13} \div \frac{688 \times 266}{187 \times 233}.$$

$$\therefore \textit{Answer} = \frac{\cancel{688} \times \overset{\cancel{19}}{\cancel{247}}}{\underset{\cancel{11}}{\cancel{165}} \times \cancel{233}} \times \frac{\overset{29}{\cancel{319}} \times \overset{\cancel{2}}{\cancel{30}}}{719 \times \cancel{13}} \times \frac{187 \times \cancel{233}}{\cancel{688} \times \underset{\underset{7}{\cancel{14}}}{\cancel{266}}} =$$

$$\frac{29 \times 187}{719 \times 7} = \frac{5423}{5033} = 1\tfrac{390}{5033},\ \textit{Answer.}$$

6. *A person travelled by coach, rail, and sea; the distance by coach was $\frac{1}{4}$ that by rail, and the distance by rail was $\frac{1}{2}$ that by sea; the cost of the journey by coach was $\frac{1}{8}$ that by rail, and the cost by rail was $\frac{9}{16}$ that by sea. The length of the whole journey was 2304 miles; what was its cost, the railway journey costing 2d. per mile?*

(1.) *Find the distances travelled by sea, rail, and coach respectively.*

Let S = miles travelled by sea;

then $\frac{S}{2}$ = „ „ rail;

and $\left(\frac{S}{2} \div 4,\ \textit{i.e.}\right)\frac{S}{8}$ = „ „ coach.

$\therefore\ S + \frac{S}{2} + \frac{S}{8}$ = total distance = by question, 2304 miles.

Multiplying each side of this equation by 8, we have—

$8S + 4S + S = 18432$ miles;

*i.e.* $13S = 18432$ miles.

$\therefore\ S = 18432$ miles $\div\ 13 = 1417\frac{11}{13}$ miles travelled by *sea;*

and $\frac{S}{2} = 1417\frac{11}{13}$ miles $\div\ 2 = 708\frac{12}{13}$ „ „ *rail;*

and $\frac{S}{8} = 1417\frac{11}{13}$ miles $\div\ 8 = 177\frac{3}{13}$ „ „ *coach.*

(2.) ***Find the cost by rail, and, from that, the cost of the whole journey.***

**By the question—**

**The railway journey cost (2d. × $708\frac{11}{13}$ =) $1417\frac{9}{13}$ pence.**
**The journey by coach cost ($\frac{1}{3}$ of $1417\frac{9}{13}$d. =) $472\frac{8}{13}$ pence.**
**The sea journey cost ($\frac{16}{9}$ of $1417\frac{9}{13}$d. =) $2520\frac{8}{13}$ pence.**

**∴ The cost of the whole journey = $4411\frac{1}{13}$ pence = £18, 7s. $7\frac{1}{13}$d., *Answer.***

7. *A grocer mixed* 45 *lb. of tea at* 1*s.* 8*d. per lb.*, 95 *lb. at* 2*s. per lb., and* 160 *lb. at* 3*s. per lb.; at what price must he sell the mixture so as to make a profit of* 20 *per cent.?*

**(1.) The cost of 45 lb. at 1s. 8d. per lb. = £3, 15s. 0d.**
**,, 95 lb. at 2s. per lb. = £9, 10s. 0d.**
**,, 160 lb. at 3s. per lb. = £24, 0s. 0d.**

**Therefore the whole 300 lb. cost £37, 5s. 0d.**

**(2.) As 120 : 100 :: the selling price : the cost price**
***i.e.* 6 : 5 :: the selling price : the cost price.**

**∴ 6 times the cost price = 5 times the selling price (§ 99.)**

**∴ The selling price = $\frac{£37,\ 5s.\ \times 6}{5} = \frac{£223,\ 10s.}{5}$ = £44, 14s.**

**(3.) Therefore *the total selling price* = £44, 14s.;**
**And *the selling price per lb.* = £44, 14s. ÷ 300 = 2s. $11\frac{19}{25}$d., *Answer.***

8. *Divide* 1705·7 *by* 36·881, *and multiply the quotient by* 21·6216, *giving your reason for the position of the decimal point in the result of each process.*

**The position of the decimal point may be explained by Arts. 79 and 78, and the final answer may be found as below—**

**Since $1705{\cdot}7 \div 36{\cdot}881 = \frac{1705{\cdot}7}{36{\cdot}881}$, it follows that $(1705{\cdot}7 \div 36{\cdot}881) \times 21{\cdot}6216$**

**$= \frac{1705{\cdot}7 \times 21{\cdot}6216}{36{\cdot}881} = \frac{36879{\cdot}96312}{36{\cdot}881}$ = 999·97, &c., *Answer.***

9. *Add together* £3, 1*s.* $1\frac{1}{2}$*d.* × $5{\cdot}\dot{4}$, £2, 3*s.* 1*d.* × $27{\cdot}\dot{4}\dot{5}$, *and* £5, 15*s.* $7\frac{1}{2}$*d.* × $4{\cdot}\dot{1}8\dot{9}$.

$$
\begin{aligned}
\text{Here } £3,\ 1\text{s. } 1\tfrac{1}{2}\text{d.} \times 5{\cdot}\dot{4} &= £3,\ 1\text{s. } 1\tfrac{1}{2}\text{d.} \times \left(\frac{54-5}{9},\ \textit{i.e.}\right)\frac{49}{9} \\
&= \frac{£3,\ 1\text{s. } 1\tfrac{1}{2}\text{d.} \times 49}{9} \\
&= 6\text{s. } 9\tfrac{1}{2}\text{d.} \times 49 \\
&= £16,\ 12\text{s. } 9\tfrac{1}{2}\text{d.} \\
\text{Also } £2,\ 3\text{s. } 1\text{d.} \times 27{\cdot}\dot{4}\dot{5} &= £2,\ 3\text{s. } 1\text{d.} \times \left(\frac{2745-27}{99},\ \textit{i.e. } \frac{2718}{99},\ \textit{i.e.}\right)\frac{302}{11} \\
&= \frac{£2,\ 3\text{s. } 1\text{d.} \times 302}{11} \\
&= 3\text{s. } 11\text{d.} \times 302 \\
&= £59,\ 2\text{s. } 10\text{d.} \\
\text{And } £5,\ 15\text{s. } 7\tfrac{1}{2}\text{d.} \times 4{\cdot}\dot{1}8\dot{9} &= £5, 15\text{s. } 7\tfrac{1}{2}\text{d.} \times \left(\frac{4189-4}{999} = \frac{4185}{999} = \frac{465}{111} = \right)\frac{155}{37} \\
&= \frac{£5,\ 15\text{s. } 7\tfrac{1}{2}\text{d.} \times 155}{37} \\
&= 3\text{s. } 1\tfrac{1}{2}\text{d.} \times 155 \\
&= £24,\ 4\text{s. } 4\tfrac{1}{2}\text{d.}
\end{aligned}
$$

$$
\begin{aligned}
\text{Therefore, } \textit{Answer} &= £16,\ 12\text{s. } 9\tfrac{1}{2}\text{d.} + £59,\ 2\text{s. } 10\text{d.} + £24,\ 4\text{s. } 4\tfrac{1}{2}\text{d.} \\
&= £100.
\end{aligned}
$$

10. *If* 25 *francs be worth* 20*s., what will a person lose who receives payment of a debt of* £16, 5*s. in francs valued at* 9¾*d. each.*

(1.) ***Find the number of francs he receives in payment of the debt.***

By question we have—

$$\frac{£16,\ 5\text{s.}}{9\tfrac{3}{4}\text{d.}} = \text{number of francs received;}$$

$$\textit{i.e. } 400 = \text{number of francs received.}$$

(2.) ***Find the loss on each franc, and from that the total loss.***

By question we have—

$$\frac{20\text{s.}}{25} = \text{real value of 1 franc, in shillings;}$$

$$\textit{i.e. } \frac{4}{5}\text{s.} = \text{real value of 1 franc.}$$

$$\text{But } \left(9\tfrac{3}{4}\text{d.},\ \textit{i.e. } \frac{39}{48}\text{s.},\ \textit{i.e.}\right)\frac{13}{16}\text{s. is the value of 1 franc as it was paid;}$$

$$\text{therefore } \left(\frac{13}{16} - \frac{4}{5}\right)\text{s.} = \text{the loss on each franc;}$$

$$\text{And } \therefore \left(\frac{13}{16} - \frac{4}{5}\right)\text{s.} \times 400 = \text{the loss on 400 francs} = \text{the total loss.}$$

$$\textit{i.e. } \left(\frac{1}{80}\text{s.} \times 400,\ \textit{i.e.}\right) 5\text{s.} = \text{the total loss, } \textit{Answer}.$$

11. *If* 11 *men, working* 10 *hours a day, earned* 643 *francs* 50 *centimes in* 13 *days, how many days would* 25 *men, working* 9 *hours a day, be in earning* 2673 *francs when the pay for an hour's work had increased* 20 *per cent.?*

**By First Principles—**

**11 men working 10 hrs. a day earned 643·5 francs in 13 days;**
**∴ 1 man ,, 10 ,, ,, would earn 643·5 ,, (13 × 11) days;**
**∴ 1 ,, ,, 1 hr. ,, ,, 643·5 ,, (13 × 11 × 10) days;**
**∴ 1 ,, ,, 1 ,, ,, ,, 1 ,,** $\frac{13\times11\times10}{643\cdot5}$ **days;**
**∴ 1 ,, ,, 1 ,, ,, ,, 2673 ,,** $\frac{13\times11\times10\times2673}{643\cdot5}$ **dys.;**
**∴ 1 ,, ,, 9 hrs. ,, ,, 2673 ,,** $\frac{13\times11\times10\times2673}{643\cdot5\times9}$ **dys.;**
**∴ 25 men ,, 9 ,, ,, ,, 2673 ,,** $\frac{13\times11\times10\times2673}{643\cdot5\times9\times25}$ **dys.**

**And, if the wages were increased, as by the question, at the rate of 20 per cent., the days occupied by these 25 men in earning the 2673 francs would be . . . . . .** $\frac{13\times11\times10\times2673}{643\cdot5\times9\times25}\times\frac{100}{120}$
**= 22 days, *Answer.***

12. *Find the value of the castings of the great steam hammer, weighing* 225 *tons, at the rate of* 1¾ *lb. for* 3*d.*

**Here 225 tons = 504000 lb.**

**Therefore—**

**As 1¾ lb. : the value of 1¾ lb. : : 504000 lb. : the value of 504000 lb.;**
***i.e.* 1¾ lb. : 3d. : : 504000 lb. : Answer.**

$$\therefore \textit{Answer} = 3\text{d.} \times \frac{504000 \text{ lb.}}{1\frac{3}{4} \text{ lb.}} = £\,\frac{1}{80} \times \frac{504000 \times 4}{7} = £3600.$$

13. *A invested* £5600, *B invested* £4200, *in a joint business, on condition that each should receive for management* 5 *per cent. on the profits, and that the remainder should be divided between them according to their capital; after* 9 *months A retired from active participation in the management, and B invested* £2800

*more in the business, and thenceforth received* 10 *per cent. on the profits for management. What share ought each to receive of the gross profits, £*2565, 8*s.* 4*d., made at the end of* 12 *months?*

(1.) *Find the profits for management due to each.*

The uniform profit for management was 10 per cent. on the gross profits;
*i.e.* 10 per cent. on £2565, 8s. 4d.;
*i.e.* £256, 10s. 10d.

Now A managed jointly with B for 9 months,
B ,, ,, A for 9 months, and alone for 3 months; but during that 3 months his profit was proportioned to that of 6 months while managing jointly with A; so that—

$$\frac{\text{A's share}}{\text{B's share}} = \frac{9 \text{ months}}{(9+6) \text{ months}} = \frac{9}{15} = \frac{3}{5} \quad \text{(I.)}$$

$$\left.\begin{aligned} &\text{Therefore } \frac{3}{3+5} = \frac{\text{A's share}}{\text{Total receipts}}, \textit{ i.e. } \text{A's share} = \tfrac{3}{8} \text{ of total receipts} \\ &\text{and } \frac{5}{3+5} = \frac{\text{B's share}}{\text{Total receipts}}, \textit{ i.e. } \text{B's share} = \tfrac{5}{8} \text{ of total receipts} \end{aligned}\right\} \text{(II.)}$$

Now $\frac{3}{8}$ of £256, 10s. 10d. = £96, 4s. 0¾d.
and $\frac{5}{8}$ of £256, 10s. 10d. = £160, 6s. 9¼d.

(2.) *Find the profits on investment.*

A invested £5600 for 12 months, *i.e.* £(5600 × 12) for 1 month.
B invested £4200 for 12 months, and £2800 for 3 months, *i.e.* £(4200 × 12 + 2800 × 3) for 1 month.

Therefore—

$$\frac{\text{A's share}}{\text{B's share}} = \frac{£5600 \times 12}{£4200 \times 12 + £2800 \times 3} = \frac{67200}{50400 + 8400} = \frac{67200}{58800} = \frac{8}{7} \quad \text{(I.)}$$

$$\left.\begin{aligned} &\therefore \frac{8}{8+7} = \frac{\text{A's share}}{\text{Gross profits} - \text{profits for management}} \\ &\text{and } \frac{7}{8+7} = \frac{\text{B's share}}{\text{Gross profits} - \text{profits for management}} \end{aligned}\right\} \text{(II.)}$$

$$\textit{i.e. } \frac{8}{15} = \frac{\text{A's share}}{£2565,\ 8\text{s. } 4\text{d.} - £256,\ 10\text{s. } 10\text{d.}} = \frac{\text{A's share}}{£2308,\ 17\text{s. } 6\text{d.}}$$

Therefore—

A's share = $\frac{8}{15}$ of £2308, 17s. 6d. = £1231, 8s.
And, similarly—
B's share = $\frac{7}{15}$ of £2308, 17s. 6d. = £1077, 9s. 6d.

(3.) ***Find the total profits of each.***

$$\left.\begin{array}{lll} \text{A's share for} \left\{\begin{array}{l}\text{management} \\ \text{investment}\end{array}\right. \begin{array}{l} = \text{£96, 4s. } 0\tfrac{3}{4}\text{d.} \\ = \underline{\text{£1231, 8s. 0d.}} \end{array} & & \\ \qquad\qquad\text{Total,} \quad = & & \text{£1327, 12s. } 0\tfrac{3}{4}\text{d.} \\ \text{B's share for} \left\{\begin{array}{l}\text{management} \\ \text{investment}\end{array}\right. \begin{array}{l} = \text{£160, 6s. } 9\tfrac{1}{4}\text{d.} \\ = \underline{\text{£1077, 9s. 6d.}} \end{array} & & \\ \qquad\qquad\text{Total,} \quad = & & \text{£1237, 16s. } 3\tfrac{1}{4}\text{d.} \end{array}\right\}$$

*Answer*, **£1327, 12s. 0$\frac{3}{4}$d.; £1237, 16s. 3$\frac{1}{4}$d.**

## Examination Paper. Male Students of the Second Year. Christmas, 1869.

1. *What is the price of lead per cwt., if the sale of* 48 *cwt. at* £45, 11*s.* 3*d. give a profit of* £12, 10*s. per cent. on the first outlay?*

By the question we have—

£112$\frac{1}{2}$ : £100 :: £45, 11s. 3d. : *Prime cost of* 48 *cwt. of lead;*

*i.e.* 9 : 8 :: £45, 11s. 3d. : *Prime cost of* 48 *cwt. of lead.*

∴ Prime cost of 48 cwt. of lead = $\frac{8}{9}$ of £45, 11s. 3d. = £40, 10s.

∴ Prime cost of 1 cwt. of lead $\frac{\text{£40, 10s.}}{48}$ = **16s. 10$\frac{1}{2}$d.**, *Answer.*

2. *Bought a quantity of goods for* £47, 9*s.* 8*d., payable twelve months hence; sold them again for* £57, 8*s.* 2*d., payable nine months hence. What is the gain in ready money, allowing true discount at* 4$\frac{1}{2}$ *per cent.?*

(1.) ***Find Present Worth of £47, 9s. 8d., due in 1 year, at 4$\frac{1}{2}$ per cent.***

By Art. 113 we have—

100 : 104$\frac{1}{2}$ :: *Present worth of* £47, 9s. 8d. : *its nominal value;*

*i.e.* 100 : 104$\frac{1}{2}$ :: *Present Worth of* £47, 9s. 8d. : £47, 9s. 8d.

$$\therefore \textit{Present Worth of } \text{£47, 9s. 8d.} = \frac{100}{104\frac{1}{2}} \text{ of £47, 9s. 8d.}$$
$$= \text{£45, 8s. } 9\tfrac{5}{19}\text{d.}$$

(2.) *Find Present Worth of £57, 8s. 2d., due in ¾ year, at 4½ per cent.*

By Art. 106 we have—

S. I. on £100 for ¾ yr. at 4½ per cent. = ¾ of £4½ = £3⅜.

Therefore by Arts. 109 and 113 we have—

100 : 103⅜ :: *Present Worth of* £57, 8s. 2d. : *its nominal value;*

*i.e.* 100 : 103⅜ :: *Present Worth of* £57, 8s. 2d. : £57, 8s. 2d.

$$\therefore \text{ Present Worth of £57, 8s. 2d.} = \frac{100}{103\frac{3}{8}} \text{ of £57, 8s. 2d.}$$

$$= \text{£55, 10s. 8d.} +$$

(3.) *Find the difference between the two results found in* (1) *and* (2).

*The Answer* = £55, 10s. 8d. − £45, 8s. 9d.

= **£10, 1s. 11d.**

3. *Find, to two places of decimals, the value of—*

$$\frac{15 + \sqrt{10}}{15 - \sqrt{10}} + \frac{30 - \sqrt{10}}{30 + \sqrt{10}}.$$

(1.) $$\frac{15 + \sqrt{10}}{15 - \sqrt{10}} = \frac{(15 + \sqrt{10}) \times (15 + \sqrt{10})}{(15 - \sqrt{10}) \times (15 + \sqrt{10})} \text{ (§ 59)} = \frac{(15 + \sqrt{10})^2}{15^2 - 10} =$$

$$\frac{225 + 30\sqrt{10} + 10}{225 - 10} = \frac{235 + 30\sqrt{10}}{215} = \frac{235 + 30 \text{ times } 3{\cdot}1622}{215} =$$

$$\frac{235 + 94{\cdot}866}{215} = \frac{329{\cdot}866}{215} = \mathbf{1{\cdot}534.}$$

(2.) $$\frac{30 - \sqrt{10}}{30 + \sqrt{10}} = \frac{(30 - \sqrt{10}) \times (30 - \sqrt{10})}{(30 + \sqrt{10}) \times (30 - \sqrt{10})} \text{ (§ 59)} = \frac{(30 - \sqrt{10})^2}{30^2 - 10} =$$

$$\frac{900 - 60\sqrt{10} + 10}{900 - 10} = \frac{910 - 60\sqrt{10}}{890} = \frac{91 - 6\sqrt{10}}{89} =$$

$$\frac{91 - 6 \text{ times } 3{\cdot}1622}{89} = \frac{91 - 18{\cdot}9732}{89} = \frac{72{\cdot}0268}{89} = \mathbf{{\cdot}809.}$$

(3.) Therefore—

*Answer* = 1·534 + ·809 = **2·343, &c.**

4. *Show that—*

$$1 + \frac{1}{3 \times 5} + \frac{1}{2 \times 3 \times 5^2} + \frac{1}{2 \times 3 \times 4 \times 5^3} = 1{\cdot}073\dot{6}.$$

$$1 = \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1\cdot$$

$$\frac{1}{3 \times 5} = \frac{1}{15} = \quad . \quad . \quad . \quad . \quad . \quad . \quad \cdot066\dot{6}$$

$$\frac{1}{2 \times 3 \times 5^2} = \frac{1}{(3 \times 5)\,(2 \times 5)} = \frac{\cdot0\dot{6}}{10} = \quad . \quad . \quad . \quad \cdot006\dot{6}$$

$$\frac{1}{2 \times 3 \times 4 \times 5^3} = \frac{1}{(2 \times 3 \times 5^2)\,(4 \times 5)} = \frac{\cdot00\dot{6}}{20} = \frac{\cdot000\dot{6}}{2} = \quad \cdot000\dot{3}$$

$$\text{Total} = \quad . \quad . \quad . \quad . \quad 1\cdot073\dot{6}$$

5. *If the third satellite of Jupiter be* ·000085 *of the mass of the planet, and* ·02947 *of the mass of the earth, find the mass of the earth in terms of the mass of the planet, to three places of decimals.*

Let P = mass of the planet,
and E = mass of the earth.

Then by the question we have—

$$\cdot02947\,E = \cdot000085\,P.$$

∴ Multiplying each side of this equation by 1000000 we have—

$$29470\,E = 85\,P.$$

$$\therefore\ E = \frac{85}{29470}P = \frac{17}{5894}P = \cdot002\tfrac{2606}{2947}\,P.$$

*Answer*, ·002.

6. *If I buy* 15 *chests of tea, each containing* 36 *kilogrammes* 37 *grammes, at* 315 *francs* 4 *centimes a chest, and retail the tea at* 20 *grammes for* 25 *centimes, do I gain or lose? How much?*

(1.) The prime cost of the tea = 315 fr. 4 c. × 15 = 4725 fr. 60 c. = 472560 c.

(2.) The weight of the tea = 36 kilog. 37 gr. × 15 = 540555 gr.

Therefore *the cost price* of the tea $= \frac{472560\text{ c.}}{540555}$ per gr.

But *the selling price* of the tea $= \frac{25\text{ c.}}{20}$ per gr.

Therefore the **gain** on the sale of 1 gr. $= \frac{25 \text{ c.}}{20} - \frac{472560 \text{ c.}}{540555}$

$$= \frac{5 \text{ c.}}{4} - \frac{31504 \text{ c.}}{36037}$$

$$= \frac{(5 \times 36037) \text{ c.} - (4 \times 31504) \text{ c.}}{4 \times 36037}$$

$$= \frac{180185 \text{ c.} - 126016 \text{ c.}}{4 \times 36037}$$

$$= \frac{54169 \text{ c.}}{4 \times 36037}.$$

∴ The **total gain** (*i.e.* the gain on the sale of 540555 gr.) $= \frac{54169 \text{ c.}}{4 \times 36037} \times 540555$

$$= \frac{54169 \times \overset{15}{\cancel{540555}}}{4 \times \cancel{36037}} \text{ c.}$$

$$= \frac{812535 \text{ c.}}{4}$$

$$= 203133{\cdot}75 \text{ c.}$$

= **2031 fr. 33·75 c.**, *Answer.*

7. *Find the value of* 17·875 *ells* – 4·65625 *yds.* – $1{\cdot}26714\dot{6}$ *of* 7 *yds.* 3 *nls., at* 4*d. a yd.*

| | | |
|---|---|---|
| (1.) 17·875 ells = $(17{\cdot}875 \times \frac{5}{4})$ yds. = | | 22·34375 yds. |
| (2.) 4·65625 yds. = | 4·65625 yds. | |
| (3.) $1{\cdot}26714\dot{6}$ of 7 yds. 3 nls. = <br> $1{\cdot}26714\dot{6}$ of $7\frac{3}{16}$ yds. = | 9·107616 yds. | 13·76386 yds. |
| | | 8·57989 yds. |
| But the price per yd. = . . . | | 4d. |
| Hence the value of 8·57989 yds. = . . | | 34·31956d. |

*Answer* = 34·3196d. = **2s. 10·3196d.**

8. *A person has stock in the* 3 *per cents. which brings him in* £240 *a year. He sells out one-fourth of the stock at* $87\frac{1}{4}$, *and invests the proceeds in railway stock, when a* £100 *share is worth* £$174\frac{1}{2}$. *What dividend per cent. per annum ought the railway stock to pay so that he may increase his income* £40 *a year by the transaction?*

**Here $\frac{£240}{£3}$, *i.e.* 80 = the number of "cents." in the 3 p. c. Stock.**

**$\therefore$ $80 \div 4$, *i.e.* 20 = the number of "cents." sold out at $87\frac{1}{4}$.**
**Therefore $£87\frac{1}{4} \times 20$ = amount invested in Railway Stock;**

**And $\therefore$ $\frac{87\frac{1}{4} \times 20}{174\frac{1}{2}}$ = the number of "cents." of Railway Stock purchased.**

**And, if R = the rate per cent. per annum paid in Railway Stock,**
**$\therefore$ $£\frac{87\frac{1}{4} \times 20}{174\frac{1}{2}} \times R$ = income received from Railway Stock.**

**Now the total income in the 3 p. c. before selling out was £240;**
**$\therefore$ The income on one-fourth of the original 3 p. c. Stock was $£\frac{240}{4} = £60$;**
**$\therefore$ The income on the Railway Stock was £60 + £40, *i.e.* £100.**

**Therefore we have—**

$$\frac{£87\frac{1}{4} \times 20 \times R}{174\frac{1}{2}} = £100;$$

$$\textit{i.e. } £87\tfrac{1}{4} \times 20R = £100 \times 174\tfrac{1}{2} = £17450;$$

$$\textit{i.e. } 1745R = £17450;$$

$$\therefore R = 10.$$

***Answer*, £10 per cent.**

9. *Assuming that an express train runs* 40 *miles an hour, and an ordinary train* 30 *miles an hour, and that the express fare is* $\frac{1}{4}$*d. per mile more than the ordinary fare—find how much a man's time is worth if it costs him the same to travel by one or by the other.*

***The extra expense* incurred by travelling 40 miles by the express train $= \frac{1}{4}\text{d.} \times 40 = 10\text{d.}$**

***The extra time* required to travel 40 miles by the ordinary train $= \left(\frac{40}{30} - 1\right)$ hr. $= \frac{1}{3}$ hr.**

**$\therefore$ The value of $\frac{1}{3}$ hr. = 10d.**
**$\therefore$ The value of 1 hr. = 10d. × 3 = 2s. 6d.**

***Answer*, 2s. 6d. per hour.**

10. *A man embarks his property in four successive ventures. In the first he clears* 100 *per cent., and in each of the others he loses* 20 *per cent. Show that there remains to him* 2·4 *per cent. profit on his original outlay.*

Let P = his *first* investment;
$\therefore$ P = also his first profit;
and 2P = his *second* investment;
$\therefore$ 2P − 20 per cent. on $2P = \left(\frac{80}{100}, \text{ i.e.}\right) \frac{4}{5}$ of $2P = \frac{8P}{5}$ = his *third* investment;
$\therefore$ also $\left(\frac{4}{5} \text{ of } \frac{8P}{5}, \text{ i.e.}\right) \frac{32P}{25}$ = his *fourth* investment;
$\therefore$ Similarly $\left(\frac{4}{5} \text{ of } \frac{32P}{25}, \text{ i.e.}\right) \frac{128P}{125}$ = final remainder;
And therefore $\left(\frac{128P}{125} - P, \text{ i.e.}\right) \frac{3P}{125}$ = the final gain.

But this is the gain on P;

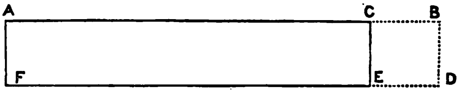
$$\text{Therefore } \frac{3P}{125} : P :: \textit{the gain per cent.} : 100.$$

$$\therefore \frac{3P}{125} \times 100 = \textit{the gain per cent.} \times P;$$

$$\therefore \frac{3}{125} \times 100 = \textit{the gain per cent.}$$

$$\text{i.e.} \left(\frac{300}{125}, \text{ i.e.}\right) 2{\cdot}4 = \textit{the gain per cent.}$$

11. *The area of a rectangle, one of whose sides is 22 feet longer than the other, exceeds six times the square of the shorter side by 8 sq. ft.; find the dimensions of the rectangle.*



(1.) Let ABDF be the rectangle, and CBDE the square on BD, the shorter side.

Then by the question AC is 22 ft. long.

(2.) Let $x$ = length of CB, or CE, in ft.

Then $(22 + x)x$ = area, in sq. ft., of ABDF;
and $x^2$ = area, in sq. ft., of CBDE.
$\therefore (22 + x)x = 6x^2 + 8$;
i.e. $22x + x^2 = 6x^2 + 8$;
i.e. $5x^2 - 22x = -8$.

And, solving this quadratic equation, we have—

$$x - \frac{11}{5} = \frac{\pm 9}{5}$$

$$\therefore x = \frac{11}{5} \pm \frac{9}{5} = 4 \text{ or } \tfrac{2}{5}.$$

So that AB = (22 + 4, *i.e.*) 26 ft., in which case BD = 4 ft.;
or (22 + $\frac{4}{5}$, *i.e.*) 22$\frac{4}{5}$ ft., in which case BD = $\frac{4}{5}$ ft.
*Answer* = **26 ft. and 4 ft.; or, 22$\frac{4}{5}$ ft. and $\frac{4}{5}$ ft.**

12. *The square of the arithmetic mean between two numbers exceeds that of the geometric mean by* 400; *and the square of the geometric mean exceeds that of the harmonic mean by* 144; *find the two numbers.*

Let $a$ and $b$ be the two numbers required.

Then $\frac{a+b}{2}$ = their *Arithmetic* mean;

$\sqrt{ab}$ = their *Geometric* mean;

and $\frac{2ab}{a+b}$ = their *Harmonic* mean.

But by the question we have—

$$\text{(I.) } \frac{(a+b)^2}{4} - ab = 400;$$
$$\therefore\ (a+b)^2 - 4ab = 1600;$$
$$i.e.\ a^2 + 2ab + b^2 - 4ab = 1600;$$
$$i.e.\ a^2 - 2ab + b^2 = 1600;$$
$$\therefore\ a - b = 40;$$
$$\therefore\ a = b + 40 \text{ (I.)}$$
$$\text{(II.) } ab - \frac{4a^2b^2}{(a+b)^2} = 144;$$
$$i.e.\ \frac{ab(a+b)^2 - 4a^2b^2}{(a+b)^2} = 144;$$
$$i.e.\ ab\,\frac{a^2 + 2ab + b^2 - 4ab}{(a+b)^2} = 144;$$
$$i.e.\ ab\,\frac{a^2 - 2ab + b^2}{(a+b)^2} = 144;$$
$$\therefore\ \sqrt{ab} \times \frac{a-b}{a+b} = 12;$$
$$\therefore\ \frac{a-b}{a+b} = \frac{12}{\sqrt{ab}}.$$

But by (I.) we have $a - b = 40$;

$$\therefore\ \frac{40}{a+b} = \frac{12}{\sqrt{ab}};$$
$$\therefore\ 40\sqrt{ab} = 12(a+b);$$
$$\therefore\ 10\sqrt{ab} = 3(a+b).$$

Substituting $b + 40$ for $a$, we have—

$$\begin{aligned} 10\sqrt{b(b+40)} &= 3(b + 40 + b) \\ &= 6b + 120; \end{aligned}$$

$$\therefore\ 5\sqrt{b^2 + 40b} = 3b + 60.$$

Squaring both sides, we have—

$$25(b^2 + 40b) = 9b^2 + 360b + 3600;$$

$$\text{i.e. } 25b^2 + 1000b = 9b^2 + 360b + 3600;$$

$$\therefore\ 16b^2 + 640b = 3600;$$

$$\therefore\ b^2 + 40b = 225;$$

$$\therefore\ b^2 + 40b + 400 = 625;$$

$$\therefore\ b + 20 = \pm 25;$$

$$\begin{aligned} \therefore\ b &= \pm 25 - 20 \\ &= +5 \text{ or } -45. \end{aligned}$$

$$\begin{aligned} \text{But } a &= b + 40 \\ &= (+5 + 40, \text{ i.e.}) \ 45; \\ &\text{or } (-45 + 40, \text{ i.e.}) -5. \end{aligned}$$

$\therefore$ *Answer* = **5 and 45, or −5 and −45.**

13. *Assuming the earth a sphere whose radius is 4000 miles, what decimal fraction of the surface is occupied by England and Wales, the area of which is 37324915 acres?*

$$\text{The surface of a sphere} = (\text{diameter})^2 \times \tfrac{22}{7};$$

$$\begin{aligned} \therefore\ \text{The surface of the earth} &= 64000000 \text{ sq. miles} \times \tfrac{22}{7} \\ &= 64000000 \times 640 \text{ acres} \times \tfrac{22}{7} \end{aligned}$$

$$\begin{aligned} \therefore\ \frac{\text{The surface of England and Wales}}{\text{The surface of the earth}} &= \frac{37324915}{64000000 \times 640 \times \frac{22}{7}} \\ &= \frac{37324915 \times 7}{64000000 \times 640 \times 22} \\ &= \cdot 00029 \text{ nearly, } \textit{Answer}. \end{aligned}$$

# APPENDIX.

# EXAMINATION PAPERS.

## I. Papers of the London University.

### No. 1. *Marticulation Examination.* *January*, 1870.

1. Find the value of $\frac{3}{8} - \frac{5}{14} + \frac{1}{20}$, and divide $\frac{19}{40}$ by the result.

Divide ·0075 by 25·6, and state the principle upon which you fix the position of the decimal point in the quotient.

2. Reduce nine inches and nine tenths to the decimal of a mile; and find the value of ·0625 of 1 ton 2 cwt. 3 qr. 12 lb.

3. A sells goods to B for £115, 19s. 2d., and gains 10 per cent. on the price he originally paid for them. B sells the same goods again, and loses 10 per cent. on the price at which he bought them. At what price did A buy the goods, and at what price did B sell them?

4. What annual income will be produced by investing £13000 in a $3\frac{1}{2}$ per cent. stock at 91? and by investing the same sum in a 4 per cent. stock at 96?

5. Extract the square root of 10074538384; and find the value of $\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$ to four places of decimals.

Prove that no square number can end with one of the digits 2, 3, 7, 8.

### No. 2. *Marticulation Examination.* *June*, 1870.

1. If a tax of 12 per cent. on the income of a country brings in £5,200,000, how much will an Income-tax of 5 pence in the pound bring in?

2. Express 3·1875 and the circulating decimal ·062636363 (where the figures 63 recur) by vulgar fractions reduced to their lowest terms.

3. Extract the square root of ·0001841449; and find to four places of decimals the square root of $269\frac{14}{25}$.

4. Explain the difference between Interest and Discount. Find how much £211·5 will produce at 4 per cent. Simple Interest in 13 years 6 months.

5. Three graziers, A, B, C, agree to hold a pasture in common. A puts in 8 oxen for 10 months, B 7 oxen for 12 months, C 20 oxen for 5 months, at a rent of £66 per annum. How much should they each respectively contribute towards the rent?

6. The national debt of a country was increased by one-third during a war. For a period of ten years after peace supervened £2,000,000 of the debt was annually paid off. At the end of the tenth year the rate of interest upon the debt was reduced from 6 to 5 per cent.; and it was found that the interest became precisely the same as it was before the war broke out. Find how much was added to the debt during the war.

## No. 3. *First B.A. Pass Examination. July,* 1870.

1. Prove the rule for finding the Greatest Common Measure of two numbers.

Reduce the fraction $\frac{17427}{24975}$ to its lowest terms.

2. (*a.*) If 640 acres go to a square mile, what is the length of the sides of a square plot of ground which contains 100 acres?

(*b.*) Find the square root of the circulating decimal ·111..........

In applying the ordinary method of extracting the square root to this example, state any law that you may notice in the form of the digits which express the successive remainders.

3. (*a.*) An express train, which ought to travel at uniform speed, after being an hour in motion, was delayed half-an-hour by an accident; after which it proceeded at three-fourths of its original rate of speed, and in consequence arrived at the end of its journey 1 hour 50 minutes behind time. Had the accident occurred (and the same delay and subsequent retardation taken place) after the train had travelled a distance of 60 miles, it would have been 1 hour 40 minutes behind time. Find the length of the line.

(*b.*) Supposing the above question were varied in the latter part of it by your being informed that "had the accident occurred when the train had gone *half-way*, it would have arrived 1 hour 20 minutes behind time," would that information have been incorrect? Would it have enabled you to determine the length of the line?

## No. 4. *Marticulation Examination.* *January,* 1871.

1. Simplify $\dfrac{\frac{2\frac{1}{2}}{54}+\frac{1}{8}-\frac{1}{216}+\frac{1}{12}}{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}$; and divide ·000133 by 8·75.

2. State and prove the rule for finding the Least Common Multiple of two given numbers.

Define a Prime Number. Express 364, 2520, and 5455 as products of powers of prime numbers.

3. Find the value of ·01625 of £204, 3s. 4d.; and reduce 8 lb. 5 oz. 14 drs. to the decimal of a quarter.

4. What fraction, when multiplied by itself, produces $\dfrac{4880\frac{1}{3}}{5043}$?

What is the length of each side of a square court which contains 43785·5625 square feet?

5. Supposing a gallon to contain 277¼ cubic inches, find approximately the number of gallons of water which would cover a square mile of ground to the depth of an inch.

## No. 5. *Marticulation Examination.* *June,* 1871.

1. If the circumference of a coach wheel measures 17 ft. 7⅛ in., how often will it turn round in traversing a distance of 8 miles 260 feet?

2. (*a.*) At what rate of Simple Interest will £325 amount to £379, 3s. 4d. in 5 years.

(*b.*) What rate of interest for money can be obtained from 3 per cent. stock when £100 of stock can be bought for £85 in cash?

3. Express $\dfrac{1887}{6993}$ and $\left(\dfrac{19}{30}-\dfrac{4}{21}+\dfrac{18}{35}-\dfrac{32}{105}\right)$ in their lowest terms.

4. Find the value in decimals of $\dfrac{1}{3+\dfrac{1}{7+\frac{1}{16}}}$; and the quotient of the recurring decimal ·2323... divided by the recurring decimal ·28752875...

5. Extract the square root of 32400005625; and find the value of $\dfrac{\sqrt{5}+3}{\sqrt{5}-3}-\dfrac{\sqrt{5}-3}{\sqrt{5}+3}$; in both cases to four places of decimals.

6. If a man can do a piece of work in 77 hours which a boy wants 121 hours for, in how many hours, minutes, and seconds can they do it conjointly?

7. Add together $(\sqrt{5}+\sqrt{3}+\sqrt{2}+1)^2$, $(\sqrt{5}+\sqrt{3}-\sqrt{2}-1)^2$, $(\sqrt{5}-\sqrt{3}+\sqrt{2}-1)^2$, $(\sqrt{5}-\sqrt{3}-\sqrt{2}+1)^2$.

8. A labouring man was hired upon the agreement that he should receive 2s. 8d. for every week day when he came to his work, and should forfeit 6d. for every such day when he stayed at home, except for illness, in which case he was neither to receive nor pay. At the end of eight weeks, during which time he was kept at home only one day on account of illness, his employer had to pay him exactly £5. How many days was he absent from work?

### No. 6. *First B.A. Pass Examination. July,* 1871.

1. Find the Greatest Common Divisor of 1287000 and 504504; and prove that every common divisor of two given numbers divides their greatest common divisor.

2. (*a.*) How long will an up-train and a down-train be in passing one another, if each of them be 44 yards long, and if each of them travels at the rate of 30 miles an hour?

(*b.*) A metre being 39·3708 inches, state accurately, as far as three places of decimals, what decimal fraction a foot is of a metre.

3. The population of a country is at present 32,000,000, and increases at the rate of 5 per cent. every year; what will it be at the end of 5 years?

## II. Questions proposed in the Examination Papers for Science and Art Schools and Classes, South Kensington.

### 1867.

1. If an arc of a circle measures 50 feet, while the radius is 18 feet, what is the area of a sector whose base is the given arc?

2. Find the L.C.M. of 16, 24, and 30, and explain the method.

### 1868.

1. Calculate to four places of decimals the value of this expression—

$$\frac{\frac{5}{8}\text{ of }\cdot 31416}{\sqrt[2]{\cdot 93}}.$$

2. If the circumference of a circle is 100 feet, what will be the area of the inscribed square?

3. If out of a triangle, all of whose sides are 10 feet, the inscribed circle be taken away, what is the area of the remaining portion?

4. Extract the square root of $37-20\sqrt{3}$.

## 1869.

1. Explain the method of turning a vulgar fraction into a decimal. Find the decimal fraction equivalent to $\frac{16}{113}$ to four places.

2. Extract the square root of 141·7 to two places of decimals.

3. Reduce $\dfrac{\sqrt[2]{\cdot071}}{4 \times (3\cdot7)^2}$ to a decimal, to three places, correctly.

## 1870.

1. Convert $\frac{1}{128}$ into a decimal fraction, and find the vulgar fraction corresponding to the recurring decimal $\cdot2\dot{2}9\dot{7}$.

2. Show, by proper attention to the value of the figures in multiplying one number by another, that the order in which the figures of the multiplier are taken is of no importance. Multiply 61·143 by 47·982 correctly to three places of decimals, beginning with the left-hand figure of the multiplier, and use as few figures as possible.

3. Reduce $\dfrac{\sqrt[3]{5\cdot12} + \sqrt[3]{\cdot03375}}{\sqrt[3]{80} - \sqrt[3]{\cdot01}}$ to an equivalent simple decimal.

## 1871.

1. A foreman has 116 half-crowns, and wants to pay 50 men, some at 6s. 6d. and some at 5s. 6d. each. He finds he can do this without change by paying them in groups. How many are there in each group?

2. Reduce $\dfrac{\sqrt{1\cdot75} - \sqrt{\cdot63}}{\sqrt{3\cdot5} - \sqrt{2\cdot1}}$ to its simplest form.

## 1872.

1. Show, by an easy example, that division of one whole number by another is equivalent to a series of subtractions.

Divide 1·02 by $\frac{17}{24}$ of ·144.

2. If a parcel of 12 lb. weight be carried 80 miles by rail for 2s. 4d., and the rate for the distance over 50 miles be two-thirds of the rate for the first fifty miles, how far can a parcel of 8 lb. be carried for 4d.?

3. If the 3 per cents. are at 91⅛, what interest does this give on £100? (Omit brokerage and fractions of a penny.)

4. How many lb. in ·321875 of a ton weight? Convert it into kilograms (omitting fractions), assuming that a cubic decimetre of distilled water weighs 15432·35 grains.

5. A train carrying three classes of passengers at 6d., 4d., and 3d., has eight times as many of the third class as of the second class, and seven times as many of the second as of the first class. The whole sum received was £19, 7s. 2d. How many first class tickets were issued?

## 1873.

1. Reduce $\frac{429}{40}$ and $\frac{1}{3125}$ to decimal fractions; then multiply them together, and divide the product by ·0000075. Reduce the quotient to a vulgar fraction.

2. A vessel steams 16 knots (or nautical miles) per hour. How many metres per second is this equivalent to, taking 1 metre as 3·2809 feet, and 1 knot, or nautical mile, as 6086·5 feet?

3. A tile of copper, 1 foot long, 9 inches wide, and $\frac{5}{8}$ of an inch thick, is rolled into a plate 6 feet long and 4 feet wide. How thick will the plate be? Give your answer both as a vulgar fraction and in decimals of an inch exactly.

4. I purchase $2\frac{1}{2}$ cwt. of tea at £13, 1s. 8d. per cwt., and $1\frac{1}{2}$ cwt. at £18, 12s. per cwt. I mix them, and sell the mixture at 3s. 6d. per lb. What is the profit per lb., the profit per cent. on the outlay, and the profit per cent. on the selling price? (Two decimal places sufficient.)

5. A cargo steamer leaves port at the rate of $8\frac{1}{2}$ miles per hour at 3 A.M. on Monday. At midnight a despatch boat starts in pursuit at the rate of 15 miles an hour. When will they be in sight of one another, supposing them to be visible 6 miles off?

6. Find by how much the square root of $9 + \cfrac{1}{1 + \cfrac{1}{7 + \frac{1}{8}}}$ differs from $\frac{355}{113}$. Which comes nearest to $3 + \frac{1}{10}\sqrt{2}$?

## III. Examinations in Government Training Colleges.

### No. 1. *Examination of Candidates for Admission.* 1859.

1. Explain, as you would to children, how to subtract 1607 from 10340.

2. How many francs, each $9\frac{1}{2}$d., are there in £2850?

3. A train, consisting of 3 first, 4 second, and 5 third class carriages, travels from London to York, a distance of 191 miles. Each first class carriage contains 18 persons, each second class 32, and each third class 48. The rates per mile are—1st class $2\frac{1}{4}$d., 2nd class $1\frac{3}{4}$d., 3rd class 1d. Find the sum of the fares paid by the passengers.

4. Find, by Practice, the cost of 1087½ pounds of butter at 11½d. per lb.; and of 15 acres 24 poles of land at £7, 13s. 9d. per acre.

5. Find the cost of cleaning 7260 square yards of ground at 1s. 9½d. per perch.

6. Find the Income-tax on £498, 15s. at 7d. in the pound.

7. If 18 horses eat 37 qrs. 7 bush. 3 pks. of corn in 45 days, in what time will 50 horses eat 25 qrs.?

8. A tunnel, $\frac{7}{16}$ mile long, is excavated at the rate of $\frac{11}{12}$ yard per day; in how many years will it be completed?

9. An estate, worth £10,000, is left to A, B, and C; $\frac{3}{8}$ to A, $\frac{2}{5}$ to B, and the remainder to C. Find C's portion, and its value.

10. State and prove the rules for pointing in Division of Decimals.
Find the value of £·000546875 ÷ ·175.

11. What is the interest on £154, 16s. 8d. for 4 years 219 days at 3½ per cent. per annum?

12. What sum of money, put out to interest, will amount to £194, 16s. 1½d. in 2¾ years at 4 per cent. per annum?

13. If I invest £1200 in the 3 per cents. at 72, what is my income, and how much per cent. do I get for my money?

14. Find the acreage of a triangular field whose sides are 45, 40, and 13 chains respectively.

15. Find the cost of covering with gravel, at 7½d. per square yard, a path 3 feet wide, round the outside of a bed whose diameter is 9 feet.

## No. 2. *Examination of First Year Students (Males).* 1859.

### *Section I.*

1. Add $\frac{5}{8}$ of $\frac{2}{7}$ to $\frac{8}{11}$ of $\frac{40}{63}$; multiply the sum by the difference between $\frac{2}{3}$ and $\frac{138}{208}$; and divide the product by 15 times the difference between $\frac{1}{216}$ and $\frac{1}{324}$.

2. Reduce $\frac{238}{475}$ of £233, 9s. 8d. to the decimal of £99, 3s. 4d.

3. What is the side of a square field containing 48 acres 10 poles 22½ square yards and 4 square feet?

### *Section II.*

1. If 17 cwt. 3 qrs. 26 lb. cost £41, 18s. 1½d., what is that per cwt.?

2. The following bill was paid by a certain number of persons contributing £1, 16s. 7d. each; how many were there?—23 articles at £9, 2s. 11d. per score; 54 at 11s. 8d. per score; 37 at 12s. 6d. per dozen; 15 at 4½d. each; and 11 at 7¼d. each.

3. A person taking a 1st class and a 2nd class ticket from A to B, pays 13s., what did he pay for each, if the 1st and 2nd class fares from A to C are 2s. 7d. and 1s. 9d. respectively, and the fares are proportional to the distance.

### *Section III.*

1. The interest on a certain sum lent for 85 days at £4, 6s. 8d. per cent. per annum is £3, 13s. 8d., what was the principal?

2. If the 3 per cents. are at 92⅛, at what price should the 4 per cents. be to pay the same interest on a given investment? And if I invest £1000 in each, what will be the aggregate amount (stock) which I shall possess?

3. A borrows a sum of money from B at 3½ per cent., B borrows the money from C at 3⅓ per cent. for 7 months, in order to lend it to A; A repays B the sum with interest before the 7 months are expired; and when B repays C, he finds he has neither gained nor lost anything. For how long did A require the money?

### *Section IV.*

1. Two casks, containing each 8 gallons of two fluids, A and B, are mixed in this manner: a gallon is taken out of each and put into the other. This is repeated 4 times; how much of the fluid (A) is there now in the first cask?

2. A square whose side = 500 feet, has a circular garden within it 400 feet in diameter; what will it cost to pave the part outside the garden at 2s. per square yard?

3. From a cylinder, height 12 feet, diameter 3 feet, a portion is cut off by a plane passing through a tangent line of its upper circular end, and making an elliptic section whose longer axis is 5 feet; find the portion of the solid content which is left, and its weight, if its specific gravity = 7·145.

### *Section V.*

1. Show how to multiply 3·14159 by 7·84567, when the result is required to be accurate to 5 places of decimals only, with the least number of figures.

2. Give a clear explanation, from first principles, of the fact that a fraction is not altered by multiplying both its numerator and denominator by the same number.

3. Explain, as for a pupil teacher, the rules for pointing in extracting square root and cube root, including the case of decimal fractions.

## No. 3. *Examination of First and Second Year Students (Females).* 1871.

(N.B.—All examples omitted here are worked in other parts of this book.)

### *Section I.*

1. From thirty-two thousand and eight pounds and sevenpence halfpenny take six thousand nine hundred and forty-seven pounds and sevenpence three-farthings, and write your answer in words.

4. If 1 metre (= 39·3708 English inches) be the ten millionth part of a quadrant of a meridian, how many miles are there in a meridian?

### *Section II.*

1. Find, by Practice, the value of 289 articles at £3, 18s. 4d. each.

2. Make out the following bill, and deduct one penny in the shilling as discount for cash:—25 score of potatoes at 15 pence per score; 11 lb. of honey at 10½d. per lb.; 17 lb. of bacon at 1s. 1d. per lb.; 13 lb. of cheese at 9¼d. per lb.; 15 lb. of sugar at 4¾d. per lb.

3. Find, by Practice, the value of 157 miles 3 fur. 24 poles of telegraph wire at £11 per mile; and explain the working of the example as you would to a class.

4. Make out the following bill, and deduct 10 per cent. discount for cash:—4½ lb. of green tea at 3s. 6d. per lb.; 5¼ lb. of black tea at 2s. 3d. per lb.; 7½ lb. of coffee at 1s. 7½d. per lb.; 13½ lb. of lump sugar at 6½d. per lb.; 19¼ lb. of moist sugar at 5d. per lb.; 2¾ lb. of pepper at 1s. 4d. per lb.

### *Section III.*

1. An express train to London, which travels at the rate of 45 miles an hour, leaves New Milford, 285 miles from London, at 8 h. 35 m. A.M.; when ought it to reach London?

3. A clock marked true time on the 24th June at 11.30 A.M.; on the 4th of July, at 9 h. 15 m. P.M. (true time), it had gained 9¼ minutes; at what date (true time) would it be a quarter of an hour too fast?

### *Section IV.*

2. A grocer bought a chest of tea containing 90 lb. at 2s. per lb.; another of 40 lb. at 2s. 6d. per lb.; and a third of 20 lb. at 4s. per lb.; mixed them together, and sold the mixture at 3s. per lb. What was his gain per cent. on the cost price?

### *Section V.*

1. What must be the nature of the denominator of a vulgar fraction in order that the vulgar fraction may be capable of being represented exactly by a decimal fraction? Give examples in illustration of your statement.

2. Reduce $3\frac{6}{7}$, $23\frac{13}{30}$, $5\frac{8}{11}$, $14\frac{9}{16}$, and $8\frac{17}{25}$ to decimals, and add the results together.

3. Reduce £116, 17s. 6d. to the form of a decimal of £1, and multiply the result by $1\cdot\dot{4}\dot{5}$.

4. Find the sum of $7\cdot168 \times 36\cdot95$ and $23892\cdot128 \div 3\cdot25$.

### *Section VI.*

1. What will be the interest of 729 francs 75 centimes for 5 years and 4 months at 3 centimes interest per annum for each franc lent? Explain carefully your working as you would to a class.

2. In 1870 the number of persons employed in factories was 410190 males and 563077 females; determine to four places the per-centage of males and females relative to the whole number employed.

3. What sum of money, put out for 3 years at 3½ per cent., simple interest, will amount to £384988, 18s. 1½d.?

4. A person sold £20700 in the 3 per cents. at 92½, and invested the proceeds in Canada 5 per cents. at 103½; what was his increase of income

## No. 4. *Examination of First Year Students (Males).* 1865.

### *Section I.*

1. Express in figures two hundred and four millions thirty thousand nine hundred and one (one amount), and in words 701000609.

2. Purchased 49 lb. at 8½d. a lb.; 17 ounces at 1s. 1½d. per oz.; 15 yards at 4¼d. per yard; 30 articles at 2s. 5½d. per dozen. How much change ought I to have left out of a £10 note?

3. If a tradesman buys an article at 5s. the lb., and retails it at 6d. an ounce, what gain will he make on an outlay of £6, and what is the gain per cent.?

4. A wall is 6 yards 2 feet long, and 5 feet 3 inches high; what will it cost to paper it with paper $\frac{3}{4}$ of a yard wide, which costs 3 farthings a yard?

5. How many square inches are there in 3 acres 2 roods 13 poles?

6. Make a bill of these articles:—13 loaves at $9\frac{1}{2}$d. a loaf; 5 yards 3 nails at 1s. 2d. per yard.; 17 gallons 3 pints at 11d. a gallon; 16 lb. 5 oz. at 10d. per lb.; and 6 cwt. 2 qrs. 21 lb. at 2s. 6d. per cwt.

*Section II.*

2. If the equatorial circumference of the earth be taken as 25000 miles, and the earth revolve in 24 hours exactly, how long will a point on the equator be in describing 1260 miles?

*Section III.*

1. Simplify $\dfrac{14\frac{1}{4} - 6\frac{1}{2}}{3\frac{1}{3} + 7\frac{1}{5}} \div \dfrac{4\frac{1}{12} + 6\frac{1}{8}}{9\frac{1}{5} - 3\frac{1}{4}} + [30\frac{7}{18} - 22\frac{3}{8}]$.

2. Suppose the number of first class passengers in a railway train to be 56, paying 3d. a mile; the number of second class 94, paying 2d. a mile; and the number of third class passengers 120, paying 1d. a mile; the first class passengers travel on an average 20 miles each, the second class 16 miles each, and the third class 10 miles each. What is the money received for this train?

3. Calculate the value of $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \frac{1}{4\cdot5\cdot6} + \ldots$ to four places of decimals.

*Section IV.*

1. Represent 10 tons 12 cwt. 2 qrs. 20 lb. in tons and decimals of a ton.

2. The rents of a parish amount to £2340, 17s. 6d.; a rate is levied of £137, 10s. 8d.; how much in the pound is the rate, and what will be paid on an estate rented at £143, 9s. 10d.?

3. If 48 feet of Cremona are equal to 56 English feet, and the French metre be 39·371 English inches; find the value of the foot of Cremona in metres, and the value of the metre in feet of Cremona.

*Section V.*

1. Find the value of 457 articles at £14, 17s. $9\frac{1}{2}$d. each, by Practice, and then turn the price into pounds and decimals of a pound, and do the same question by Multiplication.

3. If when I sell cloth at 10s. per yard I lose 5 per cent., how much shall I gain or lose per cent. by selling it at 12s. 6d. per yard?

*Section VI.*

1. A square court has to be paved with stone, at 1s. 2d. per square foot. The total cost is £48. What are its dimensions?

2. Find the value of 25 acres 1 rood 16 poles 10 square yards at £125, 10s. 6d. per acre.

3. If a man pays the same amount of Income-tax when it is raised from 6d. to 9d. per £1, how much per cent. has his income diminished? and if he pays twice as much when it is reduced to 4d. per £1, how much per cent. has it increased?

*Section VII.*

1. Find the simple interest for 2 years 9 months on £1784, 16s. 8d. at 3¼ per cent. per annum.

2. Find the discount on a bill of £546, 10s. 8d. due 4 months hence at 4½ per cent.

3. I invest £1000 in the 3 per cents. at 88½ (interest paid half-yearly); six months after I sell out at 92, and invest principal and interest in a 3 per cent. stock at 68·15, what will my income now be?

## No. 5. *Examination of Second Year Students (Males).* 1865.

1. How much money must a person invest in the 3 per cents., when the price is 90⅜ per cent., in order that his half-yearly income may be £50?

3. The length of $\frac{1}{360}$ of the earth's circumference is about $69\frac{1}{22}$ miles; what is the earth's diameter, assuming that $\frac{7}{22}$ expresses nearly the ratio of a circle to its circumference?

4. If the length of the year be taken at 365¼ days, the true length being 365·242264 days, how long will it be before the error amounts to a whole day?

5. Bought a quantity of goods for £47, 9s. 8d., payable 12 months hence; sold them again for £57, 8s. 2d., payable ¾ of a year hence; what was the gain in ready money, allowing discount at 4½ per cent.?

6. If 2 tons 3 cwt. 3 qrs. be sold for £112, 10s., what was the rate of purchase per cwt., the profit having been 45 per cent.?

7. A garrison was victualled for 30 days; after 10 days it was reinforced by 3000 men; it was then found that the provisions would only last 5 days; what was the original number of the garrison, and how many men must be sent back to enable the rest to hold out 10 days after being reinforced?

8. If I have to pay a bill of £370 at 6 months date from this time, and I pay £150 of it at once, what extension of time ought to be allowed for the payment of the remainder?

9. A lump of iron, containing 4 cubic feet, is rolled into a bar 16 yards long; what will be the diameter of the bar?

10. A man saves 3d. one week, 6d. the next, 9d. the next, and so on, always increasing his savings by 3d.; how long will it be before he has saved £50?

12. A grocer had 150 lb. of tea, of which he sold 30 lb. at 4s. 6d. per lb., and found he was gaining only $7\frac{1}{2}$ per cent.; at what price must he sell the remaining 120 lb. in order to gain 10 per cent. on his whole outlay?

## No. 6. *Examination of Second Year Students (Males).* 1866.

1. Prove the rule for dividing a fraction by an integer; and simplify the following expression:—$\dfrac{4\frac{11}{13}\text{ of }\frac{8}{95}\text{ of }7\frac{3}{7}}{12\frac{1}{35}-2\frac{3}{7}}+\dfrac{2\frac{1}{3}+1\frac{35}{48}}{9\frac{2}{7}-3\frac{39}{112}}$.

2. If a model of the earth were made with a diameter of 20 yards, express as the fraction of an inch the height of the highest mountain, taking the earth's diameter as 7900 miles, and the actual height of the highest mountain as 25000 feet.

3. Find the cost of surrounding a bowling-green, 80 ft. by 46 ft. 2 in., with a paved walk a yard and a-half wide, at 2s. 8d. per square foot.

4. Owing to leakage, the quantity of wine in a cask is less at the end of each year by $\frac{1}{10}$ part of what it was at the beginning of the year. How much will be lost from a cask, which now contains 100 gallons, in the course of 5 years?

5. Find the difference between the simple and compound interest on £4296, 10s. for 3 years at 4 per cent. per annum.

6. A and B enter into partnership; A contributes £5500, and B £3500. The agreement is that £120 is to be put by as an insurance fund annually, and the remaining profits to be divided in proportion to the capital subscribed. At the end of the year A gets £435, 8s. 4d. Find the percentage of the whole profit.

7. Assuming that the distance of the horizon from any station in miles is approximately equal to the square root of one and a-half times the number of feet in the height of the station above the sea level, find in miles the distance of the horizon from a balloon 25000 feet high to three places of decimals.

11. At a school inspection $\frac{4}{5}$ of the numbers in average attendance were eligible for examination, but 25 per cent. were absent. Of those present and examined 8 per cent. failed in reading, and 14 per cent. in each of

the two other subjects. The grant on examination was £52, 16s.—at the rate of 2s. 8d. for each pass. Find the number examined and the average for the year.

12. A farmer purchased a number of oxen for £112, and observed that if he had had one more for the same money, each of them would have cost him £2 less; required the number he purchased, and the cost of each?

## No. 7. *Examination of Second Year Students (Males).* 1871.

2. A person buys £1000 stock in the 4 per cents. which are at 95, and invests £656 in the 3 per cents. at 96. What income does he get? and if he sells out of both at £100, what is his gain?

3. Simplify $3\frac{7}{18}$ of $\frac{426}{427} \div 71 + 2\frac{3}{70}$ of $\frac{3}{35}$ of $\frac{5}{44} - \frac{7}{150}$ of $1\frac{1}{34}$.

4. Find the value of $\cdot 3\dot{8}\dot{7}$ of £8, 16s. 3d. + $6\frac{1}{2}$ of $\frac{18}{35}$ of 7s. $8\frac{1}{2}$d. − $(\cdot 42 \div 70)$ of 12s. 6d. − $\frac{5}{11}$ of 1d.

5. Reduce $\frac{\cdot 1\dot{8}}{289}$ of 7 tons 13 cwt. to the decimal of $\frac{2}{3}$ of a cwt.

6. If a franc piece weighs 5 grams, of which $\frac{9}{10}$ are pure silver, find the value of a £ sterling in francs, the gram being 15·438355 grains, and the pound weight of standard silver worth 62s.

7. A room is 24 ft. long, $13\frac{1}{2}$ wide, $9\frac{1}{2}$ high. The ceiling is whitewashed at 7d. the doz. sq. yds., the walls are papered at $\frac{3}{4}$d. the sq. ft. (deducting 16 per cent. for windows, doors, &c.), and the floor is carpeted at 3s. 7d. the sq. yd. Find the whole cost.

8. Find the compound interest on £3460 for 5 years at $3\frac{1}{2}$ per cent.

9. A dishonest publican uses measures which are false to the extent of 5 per cent.; but his brewer gave him in every barrel only 35 gallons. The publican buys at 21s. the barrel and sells at 2d. the pint; what does he gain in a year when he bought 950 barrels?

10. At a school, divided into 4 classes, $\frac{1}{7}$ of the children are in the 1st, $\frac{3}{14}$ in the 2nd, $\frac{5}{18}$ in the 3rd, and in the 4th 6 children more than half the number in the first three. Find the whole number.

11. A engages £15000, B 9000 in a business. The gross receipts for the first year are £5600, but of this 5 per cent. is paid for insurances, and $\frac{1}{7}$ for other expenses; of the rest, B receives a certain sum for managing, and the remainder is divided between them in proportion to the capital employed. If A receives £2500, find B's salary as manager.

12. B undertakes to run round a rectangular playground (sides 432 and 576 ft.) whilst A runs from one corner to the opposite and back. A has yet 54 ft. to run when B has completed his circuit at the rate of 8 miles per hour. Find A's speed.

13. Two partners, A and B, gained £140 by trade. A's money was 3 months in trade, and his gain was £60 less than his stock; and B's money, which was £50 more than A's, was in trade 5 months. What was A's stock?

## IV. Examinations at the Birkbeck Literary and Scientific Institution.

### No. 1. *Class Examination.* 1872.

1. Make out the following bill:—

168 lb. of rice, at 1½d.
224 lb. of sugar, at 4½d.
78 lb. of pepper, at 9½d.
92 lb. of coffee, at 1s. 5½d.
37 lb. of tea, at 2s. 10½d.
47 lb. of cocoa nibs, at 11½.

2. How many cubic feet to the acre does a rainfall of 1 inch represent?

3. Find the weight of 373 castings, each weighing 4 tons 17 cwt. 3 qrs. 16 lb.

4. What is the value of 1 cwt. of standard gold at £3, 17s. 10½d. the Troy ounce?

5. A vessel steams out of port at the rate of 12½ miles the hour, at 2 A.M., on May 1st. At 3·30 A.M. on May 3rd, another vessel, whose speed is 17¼ miles, starts in chase. At what time will she sight the first, supposing she can see her 10 miles off?

6. What sum must be put out at simple interest at 4½ per cent. to amount to £1000 in 10 years?

7. What will be the amount of £173, at 10 per cent. compound interest, in five years?

8. A bar of iron, 10 ft. long and 4 in. by 3 in., is rolled down to ¾ of an inch square. What will be its length?

9. Multiply 230258·5093 by ·004342944819 so as to retain eight places of decimals.

10. Reduce $\frac{4}{7}\frac{1}{3}$ of 11s. 6d. to a fraction of 6s. 8d.

11. Simplify the fraction—

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{11}}}}}}$$

12. I require to make an alloy containing by weight eight parts bismuth, five lead, and three tin. The only bismuth I can get is 1⅓ cwt. of an alloy containing eleven parts bismuth, three tin, and two lead. What weight of tin and lead must I add to this?

13. A pipe, whose section is 4⅓ square inches, delivers water into a reservoir with a velocity of 11 feet per second. In what time will it cover half-an-acre 19 inches deep?

14. A gallon measure contains fragments of a mixed metal, and these are found to weigh half-a-hundredweight. Two pints and a half of water are then poured in, and found to fill it exactly. What is the specific gravity of the metal? (Note that a gallon of water weighs 10 lb.)

## No. 2. *Examination for the Ravenscroft Prize.* 1872.

1. Find the value of $\dfrac{(\cdot075)^3 + (\cdot025)^3}{(\cdot075)^2 - (\cdot075)(\cdot025) + (\cdot025)^2}$.

2. In a division, the actual divisor is $\frac{1}{1000}$th of the true divisor too small, and the actual dividend is $\frac{1}{1000}$th of itself too large. Rectify the actual quotient, which is a million.

3. Multiply 3 ft. 2 in. by 2 ft. 3 in., showing the meaning of each term in the product by means of a small diagram.

4. The amount of £1 in 20 years @ 5 % is £2·6533. Find the amount of £1 per annum in the same time.

5. A builder signs a contract, estimating ⅔ of the whole cost for materials, and ⅓ for labour. When ⅓ of the time has elapsed, 30 % of the material rises 10 %; and on the expiration of half the time, 60 % of the labour obtains a rise from 8d. to 9d. an hour. Supposing a profit of 5 % to have been charged originally, what is the ultimate gain or loss %?

6. A has £3 more than ⅓ of the whole; B has £4 more than ¼ of the whole; and C has £5 less than ⅙. Find the sum divided.

# ANSWERS.

## Exercise 1.

1. 70; 100; 700; 900; 16; 19; 11; 13; 12; 52; 91; 43; 38; 164; 222; 789; 355; 819.

2. Thirty; two hundred; five hundred; four hundred; eight hundred; fourteen; seventeen; eighteen; sixteen; fifteen; thirty-nine; sixty-two; seventy-four; ninety-three; twenty-four; seven hundred and twenty-nine; four hundred and eighty-five; six hundred and nineteen; three hundred and twenty-four; seven hundred and thirteen; eight hundred and forty-two.

3. 42,981; 734,613; 4,625,337; 839,142,317.

4. Three thousand six hundred and forty-five; seven thousand three hundred and eighty-one; nine thousand one hundred and sixty-five; eighty-two thousand seven hundred and forty-three; six hundred and ninety-four thousand one hundred and seventy-two; thirty-one thousand four hundred and twenty-nine; seven hundred and forty-eight thousand three hundred and sixty-two; eight thousand one hundred and forty-two; seven million three hundred and eighty-one thousand four hundred and sixteen; two million eight hundred and thirty-nine thousand seven hundred and forty-three; twenty-eight million six hundred and fourteen thousand nine hundred and twelve; eight hundred and forty-three million nine hundred and sixteen thousand four hundred and eighty-five; seven hundred and twenty-one million nine hundred and sixteen thousand eight hundred and eleven; * three billion one hundred and forty-one million

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* According to the old notation, the following numbers would be read as—Three thousand one hundred and forty-one million six hundred and twenty-seven thousand one hundred and eighty-five; two hundred and thirty-nine thousand one hundred and eleven million two hundred and twenty-five thousand two hundred and eighteen; forty-three billion, two hundred and sixty-five thousand one hundred and forty-one million two hundred and seventy-five thousand nine hundred and eighty-eight.

**six hundred and twenty-seven thousand one hundred and eighty-five; two hundred and thirty-nine billion one hundred and eleven million two hundred and twenty-five thousand two hundred and eighteen; forty-three trillion two hundred and sixty-five billion one hundred and forty-one million two hundred and seventy-five thousand nine hundred and eighty-eight.**

5. **203; 601; 5400; 16019004; 400030012; 18340000; 200001001; 2000000016; 30100040050.**

6. **Four hundred and four; eight hundred and fifty; six hundred and three; seven thousand nine hundred and forty; six thousand and eighty-nine; seventeen thousand four hundred and eight; two hundred and nine thousand one hundred and six; eighteen million and three; four hundred million five hundred thousand and sixty; eight hundred and ten million ten thousand and ninety-nine; eighty-two billion four hundred and sixteen thousand eight hundred and twenty-nine; three trillion one billion twenty million one hundred and sixteen thousand four hundred and ten.**

7. **36; 694; 863; 4332; 74061; 74061; 238419; 238419.**

## Exercise 2.

1. **Two hundred, 200; ninety, 90; ninety-nine, 99; seventy-five, 75; two hundred and forty-three, 243; five thousand and four, 5004; one thousand thousand, *i.e.* one million, 1000000; one hundred and forty-two thousand and fifty-one, 142051.**

2. **XXX.; XVII.; MCXLII.; $\overline{\text{VI}}$XVIII.; $\overline{\text{XXIV}}$XXIX.; $\overline{\text{LXXXI}}$.**

## Exercise 3.

1. **16; 32; 52; 59.**
2. **13; 15; 15; 99; 16; 750.**
3. **96; 56; 30; 1024; 680.**
4. **6; 2; 16; 9; 37; 34.**
5. **72; 106; 479; 18; 980.**
6. **24; 136; 26; 11; 80.**
7. **26; 24; 132.**
8. **3; 84.**
9. **336; 3; 80.**

## Exercise 4.

1. 2250.
2. 24867.
3. 1881333.
4. 2163869.
5. 9208.
6. 137796.
7. 3072966.
8. 876710403.
9. 7338.
265920.
2628339.
10. 803983141597.
11. 30900.
12. 1088801.
13. 985269909.
14. 780992988.

## Exercise 5.

1. 44.
2. 17.
3. 314.
4. 622.
5. 6131.
6. 11223.
7. 718231.
8. 4811.
9. 56023.
10. 501532.
11. 395474.
12. 519972.
13. 69796696.
14. 10535391.
15. 72076326.
16. 893428.
17. 9299950993.
18. 98599383.
19. 983982.
20. 9985268.
21. 16474.
22. 305923965.
23. 3891741003.
24. 743115.
25. Nine million eighty-three thousand one hundred and ninety.
26. Fourteen million.
27. Fifty thousand six hundred and seven.
28. One million two hundred and nine thousand four hundred and thirty-six.

## Exercise 6.

1. 21729.
2. 45820.
3. 576891.
4. 398478.
5. 3257488.
6. 2273400.
7. 21201003.
8. 1008073297.
9. 74883936.
10. 557362.
11. 140976384.
12. 1290389313.
13. 7771410940.
14. 5755944.
15. 2289081510.
16. 495531999.
18. 20198651824018.
19. 2604307744898024.
20. 23552430606.
21. 1935464932.
22. 561216301449132.
23. 1967828032.
24. 62596113037.
25. 58152876407616.
26. 200987249035.
27. 83762.
28. 26058187744.
29. 80220588.
30. 507236523289227706.
31. 367579500245689572 to be subtracted.

## Exercise 7.

1. 342.
2. 3912.
3. 45342016.
4. 3596009.
5. 230574.
6. 2756390 + 2.
7. 232120.
8. 18325606.
9. 1287694.
10. 610021.
11. 5201834.
12. 3137689 + 1.
13. 10200312.
14. 409552416.
15. 89103864 + 3.
16. 2050071.
17. 26820849.
18. 957103090 + 3.
19. 4060708.
20. 2860265.
21. 28817348 + 9.
22. 67864104.
23. 59986890 + 3.
24. 34735584 + 11.
25. 2062351603$\frac{1}{3}$ times.
26. 765865316798.
27. 62909117.
28. 314639 ; 6382104 ; 8004103 ; 7103049 ; 6291483.
29. 401495$\frac{4}{15}$ ; 104567231116$\frac{28}{67}$ ; 280676$\frac{17}{25}$ ; 1816629395003 + 1.
30. 62917.
31. 2543223$\frac{18}{79}$.
32. 906108.
33. 830852$\frac{31948}{101101}$.
34. 3619728.
35. 1400445$\frac{47}{448}$.
36. 4168227$\frac{218}{638}$.
37. 42723532127$\frac{8}{47}$.
38. 836498.
39. 5852$\frac{2358}{6179}$.
40. 82070380.
41. 1038900$\frac{3754}{8147}$.
42. 21409.
43. 38959[illegible].
44. 802710[illegible].
45. 172579[illegible].
46. 297029$\frac{71}{101}$.
47. 190[illegible].
48. 906337 and 77834736 will remain.
49. 8109 times + 429064.
50. 100408056426$\frac{88}{191}$.

## Exercise 8.

1. 52496.
3. One thousand two hundred and seventy-five.
5. 6807.
6. 63575031.
7. 17129.
8. 12027.
9. 1129.
10. 168.
11. 41097703.
13. 200.
14. 2560000 ; 64000.
15. 4609.
16. 408003.
17. 6829407200000.
18. 85692.
19. 17293798.
20. 276732649923613370094 24.
21. 72.
22. 1000.
23. 6318490.
24. 701900.

## Exercise 9.

1. 29; 313; 1; 67; 413; 8016; 1; 2615; 94; 201.
2. 68; 30; 324; 910; 3111; 104; 1456; 2184; 1116864; 5754840.
3. 240; 248472; 74844.
4. $(314725 \times 9 \div 5) = 566505$.
5. $(148995 \div 9 \times 5) = 82775$.
6. 3 the greatest number of times, and 7905 the least.
7. 371280.
8. 4.
9. 1391.

## Exercise 10.

1. 88142 f.; 33720 f.; 147273 sixpences; 386376 h. d.; 6492 h. d.; 14445 fourpences; 5753 threepences; 56 fourpences; 1563 threepences; 1517586 h. c.; 1971 crowns.
2. £299, 6s. 2d.; £9, 17s. $1\frac{3}{4}$d.; 23764; 3786; 13604448.
3. 2392; 447.
4. £8072, 18s. 4d.; £16, 13s. 4d.; £32004, 17s. $8\frac{1}{4}$d.
5. $19\frac{6760}{525960}$ yrs.
6. 31556928 seconds.
7. 13360 poles; 40534565 inches; 12681792 inches; 157 m. 6 fur. 24 p. 5 yds. 2 ft. 4 in.; 1080303 fur. 1 p. 1 yd. 6 in.; 261882734 sq. in.; 4516300900 sq. in.; 186 ac. 3 roods 10 yds. 6 ft.; 428 yds. 18 ft. 128 in. (cubic).
8. 21411 drs. 617661 oz.; 23597760 drs.; 269 tons 1 qr. 26 lb. 3 oz.; 43 lb. 3 oz. 10 dwts. 22 grs.; 104928 grs.; 27901 tons 15 cwt. 2 qrs. 6 lb.
9. 180 pints; 160 pks.; 1129 pints; 9259 hhds. 14 galls.; 59886 qrs. 7 bush. 2 pks.; 2560230 galls. 1 qt.; 1148 h. pints; $1142\frac{3}{4}$ inches; 66 yds. 2 in.
10. 288 days.
11. 18 spoons.
12. 2.
13. £150000000.
14. $4739\frac{67}{127}$ times.

## Exercise 11.

1. £2, 0s. 3d.
2. £2, 17s. 1d.
3. £54, 17s. 11d.
4. £91515, 16s. 6d.
5. £99, 12s. 1¾d.
6. £26785, 19s. 8¼d.
7. £8238359, 9s. 3½d.
8. £204855, 7s. 3d.
9. 7812 tons 18 cwt.
10. 41784 tons 5 cwt. 4 lb.
11. 52937 cwt. 15 lb. 11 oz.
12. 1001 tons 2 cwt. 2 qrs. 16 lb. 13 oz. 8 drs.
13. 562 oz. 12 dwts. 6 grs.
14. 2892 oz. 19 dwts. 21 grs.
15. 12767 lb. 7 oz. 1 dwt. 12 grs.
16. 158 lb. 1 oz. 4 dwts. 21 grs.
17. 82270 yds. 1 ft. 9 in.
18. 231 miles 2 fur. 33 poles 2½ yds. 1 ft. 7 in.
19. 10634 lea. 3 fur. 85 yds. 4 in.
20. 153 ells.
21. 29793 sq. yds. 2 ft. 120 in.
22. 2892 ac. 11 p. 19 yds. 6 ft. 111 in.
23. 1245 c. yds. 26 ft. 199 in.
24. 1835 hhds. 5 galls. 1 qt.
25. 1218 yrs. 4 m. 1 wk. 2 days 12 hrs. 45 min. 26 sec.
26. 180 sq. yds.
27. 156 tons 8 cwt. 2 lb.

## Exercise 12.

1. £53, 3s. 1d.
2. £738, 9s. 9d.
3. £957, 15s. 5d.
4. £2200, 0s. 10d.
5. £3981, 5s. 5¼d.
6. £73561, 8s. 1¾d.
7. £400000, 7s. 5½d.
8. £1157, 19s. 11¾d.
9. 8099 tons 5 cwt. 13 lb.
10. 5292 tons 14 cwt. 91 lb. 10 oz.
11. 18 qrs. 13 lb. 2 oz. 2 drs.
12. 65 lb. 4 oz. 6 dwts. 3 grs.
13. 172 qrs. 6 bush. 2 pks. 1 gal.
14. 293 days 4 hrs. 53 min. 46 sec.
15. 4008 lea. 2 m. 7 fur. 30 p. 4½ yds. 2 ft. 2 in.
16. $285895\frac{23}{24}$ dwts.
17. 4 ells 1 qr. 3¾ nls.
18. 3 cub. yds. 1 ft. 1710 in.
19. 1598 lb. 3 oz. 10 grs.
20. 146 qrs. 3 bush. 2 pks.
21. 105 days 6 hrs. 46 min. 28 sec.
22. 327 galls. 1 qt. 1 pt.
23. 28586¼ sq. ft.
24. 88 ac. 3 r. 16 p. 4¼ sq. yds.
    190 ac. 2 r. 33 p. 6¼ sq. yds.
25. £199, 11s. 6d.
26. 2506 tons 19 cwt. 13 lb. 5 oz.
27. 531 yds. 3 qrs. 2 nls.

## Exercise 13.

1. £49, 5s. 9d.
£147, 17s. 3d.
£65, 14s. 4d.
£115, 0s. 1d.
£98, 11s. 6d.
£131, 8s. 8d.
£164, 5s. 10d.
£197, 3s.
£180, 14s. 5d.

2. £605, 13s. 2¼d.
£1816, 19s. 6¾d.
£1009, 8s. 7¾d.
£807, 10s. 11d.
£1413, 4s. 1¼d.
£1211, 6s. 4½d.
£1615, 1s. 10d.
£2018, 17s. 3½d.
£2220, 15s. 0¼d.
£2422, 12s. 9d.

3. £67565, 6s. 3d.
£57913, 2s. 6d.
£130304, 10s. 7½d.
£135130, 12s. 6d.
£101347, 19s. 4½d.
£56304, 8s. 6½d.
£43434, 16s. 10½d.

4. £223121, 0s. 2¾d.
£397737, 9s. 1¼d.
£252223, 15s. 0½d.
£329831, 1s. 2½d.
£1037997, 14s. 11¾d.

5. £456445971, 18s. 4½d.
£304265035, 5s. 6d.
£955258574, 6s. 1½d.
£16544532340, 11s. 10½d.

6. £160195, 19s. 10¾d.
£1979451, 10s.
£2506036, 7s. 1d.
£2664963, 9s. 4¾d.
£635708, 9s. 3d.
£3616622, 16s. 10¾d.

7. 115 tons 56 lb.
244 tons 8 cwt. 63 lb.
603 tons 17 cwt. 70 lb.
8698 tons 15 cwt. 35 lb.
273 tons 3 cwt. 77 lb.
2746 tons, 4 cwt. 49 lb.

8. 4582 tons 4 cwt. 1 qr. 21 lb. 8 oz. 12 drs.
10691 tons 17 cwt. 3 lb. 9 oz. 12 drs.
6182 tons 7 cwt. 1 qr. 1 lb. 8 oz. 4 drs.
46549 tons 11 cwt. 18 lb.

9. 383 lb. 4 oz. 6 dwts. 12 grs.
575 lb. 9 dwts. 18 grs.
1086 lb. 2 oz. 5 dwts. 2 grs.
5303 lb. 1 oz. 16 dwts. 14 grs.
25749 lb. 9 dwts. 22 grs.
45811 lb. 6 oz. 16 dwts. 18 grs.

10. 159 yds. 2 qrs.
119 yds. 2 qrs. 2 nls.
279 yds. 2 nls.
757 yds. 2 qrs. 2 nls.
2512 yds. 2 nls.
1914 yds.
518 yds. 1 qr. 2 nls.

11. 4655 ac. 1 r. 2 p. 28¼ sq. yds.
5801 ac. 1 r. 13 p. 24½ sq. yds.
79284 ac. 3 r. 28 p. 22¼ sq. yds.

12. 57 m. 3 f. 52 yds. 2 ft. 3 in.
106 m. 3 f. 99 yds. 1 ft. 11 in.

452 m. 6 f. 196 yds. 5 in.
1951 m. 6 f. 33 yds. 1 ft. 6 in.
848 m. 2 f. 119 yds. 1 ft. 7 in.

13. 302 galls.
320 galls. 3 qts. 1 pt.
358 galls. 2 qts. 1 pt.
7568 galls. 3 qts. 1 pt.
28312 galls. 2 qts.

14. 7882 qrs. 2 bush. 2 pks.
10900 qrs. 1 bush.
30883 qrs. 5 bush. 2 pks.
58134 qrs.

15. 1372 wks. 5 days 3 hrs. 15 min. 37 sec.
2905 wks. 14 hrs. 42 min. 49 sec.
5363 wks. 16 hrs. 5 mins. 12 sec.

16. 8895084768 oz.

## Exercise 14.

1. £48, 9s. 2d.
2. £509, 8s. 9d.
3. £47, 5s. 7d.
4. £2382, 4s. 5d.
5. £2252, 18s. 5d.
6. £1018, 13s. 3¼d.
7. £1240, 15s. 5d.
8. £117, 7s. 9¼d.
9. £307, 17s. 5¾d.
10. £414, 16s. 0¾d.
11. £4562, 16s. 8¼d.
12. £2752, 17s. 4½d.
13. £51, 16s. 1½d.
14. £35, 19s. 8¾d.
15. £801, 19s. 6¾d.
16. £28, 14s. 11¼d.
17. £2025, 3s. 10¼d. + 61.
18. £39, 17s. 10¾d.
19. £1936, 7s. 4½d.
20. £127, 6s. 9¼d.
21. £43928, 1s. 3¾d.
22. £73, 19s. 11¾d.
23. £63821, 12s. 5¾d. + 3038.
24. £81, 14s. 10¾d.
25. £42, 10s. 9¼d. + 6806.
26. 17s. 9¼d.
27. £3, 7s. 3½d.
28. £200004, 4s. 6½d.
29. £56, 0s. 1¼d. + 14.
30. £300003, 15s. 11¼d.
31. 14s. 3¼d.
32. 3s. 9¾d.
33. 17s. 3½d.
34. 7s. 11½d.
35. 45 tons 3 cwt. 1 qr.
36. 23 cwt. 2 qr. 27 lb.
37. 25 lb. 6 oz. 12 drs.
38. 86 lb. 7 oz. 16 dwts. 11 grs.
39. 67 yds. 1 qr. 2 nls.
40. 380 ac. 3 r. 32 po.
41. 57 galls. 3 qts.
42. 4 qrs. 6 bush. 2 pks.
43. 37 galls. 2 qts. 1 pt.
44. 175 fur. 17 po. 3 yds. 1 ft.
45. 78 ells 3 qrs. 2 nls. 1 in.
46. 1727 yds. 1 qr. 2 nls.
47. 349 hhds. 15 galls. 1 qt.
48. 345 ac. 1 r. 17 sq. po. 30 sq. yds.
49. 23 hrs. 45 min. 59 sec.
50. 875 times.
51. 47 times.
52. 32 cwt.
53. 936 times.
54. 47, and 8 drs. will remain.
55. 367 times.

## Exercise 15.

1. 1377 sq. ft.
2. 6975 sq. ft.
3. 7569 sq. ft.
4. 7421 sq. yds. 6 sq. ft.
5. 1008 yds.
6. 1094856 cub. in.
7. 18300 cub. ft.
8. 5169 cub. yds. 25 cub. ft. 1043 cub. in.
9. 213 children and 946 sq. in.; 198 children and 75240 cub. in.
10. 21 yds. 8 in.
11. 3 ft. $2\frac{1}{2}$ in. nearly.
12. 33 ft. 4 in.
13. 1 sq. yd. 3 sq. ft. 87 sq. in.
14. 15 ft. 2 in.
15. 246050.
16. 197 sq. yds. 6 sq. ft. 54 sq. in.; 284 yds. 2 ft. $1\frac{1}{5}$ in.

## Exercise 16.

1. £30, 18s. $9\frac{3}{4}$d. + $\frac{6}{7}$.
2. 159 sq. ft. 87 sq. in.
3. $183\frac{11}{35}$.
4. 7 cwt. 1 qr. 23 lb. 5 oz.
5. £5661, 18s.
6. 4s. $5\frac{1}{3}$d.
7. £2560.
8. 45 panes; 4s. $8\frac{1}{4}$d.
9. £20704, 5s. $6\frac{3}{4}$d.
10. 2s. 3d.
11. £15, 6s. 3d.
12. £1191, 10s. $1\frac{1}{5}$d.
13. $637\frac{1}{12}$.
14. £171, 9s. $10\frac{3}{4}$d.; £9, 0s. $6\frac{1}{4}$d.
15. £33, 2s. 6d.
16. £10601, 0s. $4\frac{1}{2}$d.
17. 238 ac. 20 p.
18. £1, 8s. 6d.
19. 270 tons 16 cwt. 2 qrs.
20. £838.
21. 425 qrs.
22. £5986, 0s. 8d.
23. £214.
24. $127\frac{6}{7}$.
25. 87672 hrs.
26. 36.
27. 7s. 10d.
28. 51 tons, 19 cwt. 2 qrs. 11 lb.
29. £4, 10s. 5d.
30. £3, 14s.
31. 4s. 2d.
32. 2s. 3d.
33. When it is a multiple of 792; when it is a multiple of 132.
34. 24.
35. 49 sq. ft. 62 sq. in.
36. 605 sq. ft. 72 sq. in.
37. 485 m. 37 po. 4 yds.
    3 cwt. 17 lb. $13\frac{18}{25}$ oz.
38. 10 wks. 6 days. 6 hrs. 59 min. 49 sec.
39. £1236, 9s. $4\frac{1}{2}$d.
40. 28160.
41. 40.
42. $\frac{5}{8}$ in.
43. 470.
44. £1, 4s.
45. £2, 7s. $7\frac{5}{13}$d.
46. £4185, 11s. $7\frac{23}{43}$d.
47. 402 oz.

48. $317\frac{1}{17}$ lb.
49. 4208.
50. 5017.
51. $69\frac{5}{12}$ sq. ft. ; 146328 cub. in.
52. £1, 2s. 6d.
53. 128 and 2304.
54. 1s. $10\frac{1}{2}$d.
55. 3054.
56. £177, 3s. 10d.
57. $11\frac{9}{13}$.
58. £29, 0s. $9\frac{1}{2}$d.
59. £2, 5s. $8\frac{1}{4}$d. +
60. 684263.
61. 1345193850.
62. 216 ; £233, 17s. 9d.
63. £45727, 15s. $2\frac{1}{4}$d.

## Exercise 17.

4. $\frac{107}{207}$.

5. $5$ ; $7$ ; $2\frac{3}{14}$ ; $511\frac{138}{161}$ ; $835$ ; $12\frac{2684}{8271}$ ; $3197\frac{241}{671}$.

6. $\frac{31}{7}$ ; $\frac{76}{9}$ ; $\frac{83}{5}$ ; $\frac{20447}{144}$ ; $\frac{1308109}{6000}$ ; $\frac{381053}{90725}$ ; $\frac{155320275}{94823}$.

7. $\frac{12,\ 9,\ 4}{36}$ ; $\frac{32,\ 18,\ 15}{48}$ ; $\frac{360,\ 96,\ 189,\ 280}{504}$ ; $\frac{780,\ 765,\ 891,\ 850,\ 325}{900}$ ;

$\frac{6930,\ 7546,\ 24745,\ 24206,\ 16016,\ 16874,\ 22295}{35035}$ ;

$\frac{1995,\ 600,\ 2278,\ 2023,\ 1810,\ 3383}{3570}$.

8. $\frac{4}{21}$ ; $\frac{4}{5}$ ; $\frac{14}{19}$ ; $\frac{7}{32}$ ; $\frac{40}{1463}$ ; $38\frac{1}{2}$ ; $1\frac{1}{6}$ ; $\frac{1}{2}$ ; $8\frac{203}{984}$ ; $251\frac{92}{133}$ ; $17\frac{11}{20}$ ; $134\frac{56}{221}$.

9. $\frac{14960}{289}$.

10. $1\frac{1}{3}$ ; $1\frac{5}{12}$ ; $5\frac{6}{7}$ ; $4\frac{121}{221}$ ; $16\frac{31}{74}$ ; $16\frac{7}{9}$ ; $79\frac{25}{134}$ ; $31\frac{800647}{1059637}$ ; $949\frac{8872959}{16222767}$.

11. $\frac{4}{13}$ ; $\frac{16}{147}$ ; $\frac{3}{101}$ ; $\frac{43}{203}$ ; $\frac{919}{71000}$ ; $\frac{618}{21233}$ ; $\frac{13}{21933}$ ; $\frac{161}{208719}$ ; $\frac{256}{53975}$.

12. $\frac{2}{3}$ ; $\frac{5}{7}$ ; $\frac{7}{13}$ ; $\frac{32494}{65127}$ ; $\frac{13}{101}$ ; $\frac{51}{230}$ ; $\frac{7048}{11781}$ ; $\frac{8}{11}$ ; $\frac{17}{19}$ ; $\frac{295}{911}$.

## Exercise 18.

1. $2$ ; $2\frac{2}{3}$ ; $3\frac{9}{13}$ ; $3$.

2. $1\frac{79}{216}$ ; $1\frac{64}{105}$ ; $2\frac{1951}{2100}$ ; $1\frac{412}{561}$ ; $3\frac{2225}{36036}$ ; $1\frac{2642287143}{3329623100}$.

3. $5\frac{16}{63}$ ; $16\frac{1}{6}$ ; $12\frac{7}{9}$ ; $31\frac{16}{17}$ ; $38\frac{401199}{3833830}$ ; $822\frac{2927}{3645}$.

4. $9\frac{7}{12}$; $16\frac{459}{847}$; $47\frac{1759}{5400}$; $17\frac{425}{462}$; $319\frac{3467}{6900}$.

5. $4\frac{17}{80}$; $12\frac{16}{35}$; $1040\frac{5077}{5454}$; $677\frac{195}{209}$; $14\frac{2}{5}$; $47\frac{1}{4}$; $1\frac{145}{544}$;
$\frac{1377}{1378}$; $5\frac{26}{315}$; $180\frac{9230}{20007}$; $2\frac{1891}{4950}$; $3\frac{11761}{217560}$.

6. $\frac{5294041}{10414656}$; $\frac{357098}{1064665}$.

## Exercise 19.

1. (*a.*) $\frac{4}{9}$; $\frac{2}{23}$; $\frac{4}{101}$; $\frac{1}{2}$; $\frac{1}{10}$; $\frac{1}{40}$.

(*b.*) $\frac{1}{8}$; $\frac{5}{11}$; $\frac{3}{20}$; $\frac{2}{7}$; $-\frac{91}{200}$; $\frac{179}{290}$; $\frac{56}{121}$; $\frac{5}{12}$; $\frac{2647}{15623}$; $\frac{1888}{4025}$;
$\frac{1}{38}$.

(*c.*) $\frac{19}{99}$; $\frac{5}{618}$; $\frac{31}{168}$; $\frac{1169}{1581}$; $\frac{11}{171}$; $\frac{98}{963}$; $\frac{395}{1384}$; $\frac{4429}{23229}$;
$\frac{87466}{459433}$; $\frac{90482041}{118159252}$; $\frac{697587}{1520000}$; $\frac{276043}{740838}$.

(*d.*) $\frac{13}{88}$; $\frac{2}{9}$; $\frac{128}{255}$; $\frac{689}{798}$; $\frac{1217}{2310}$; $\frac{2973}{7220}$; $\frac{691569}{1044500}$; $\frac{4}{95}$;
$\frac{120143}{6632274}$; $\frac{1199455760}{26509766549}$.

(*e.*) $\frac{3}{56}$; $\frac{73}{140}$; $\frac{13}{84}$; $\frac{6207}{31106}$; $\frac{29}{105}$; $\frac{5833}{14965}$; $\frac{131}{4060}$; $\frac{19}{36}$; $\frac{3}{380}$;
$\frac{1}{51}$; $\frac{15381}{18145}$; $\frac{960739}{48036006}$.

(*f.*) $\frac{1}{21}$; $-\frac{20}{117}$; $-\frac{133}{550}$; $-\frac{77}{120}$; $\frac{1455}{184221}$; $-\frac{13169}{28924}$; $\frac{1275}{5092}$;
$-\frac{1}{406}$; $-\frac{2}{3629}$; $\frac{183}{19034}$; $-\frac{1173}{10253}$; $\frac{19929}{155093}$.

(*g.*) $2\frac{1}{9}$; $16\frac{22}{133}$; $1\frac{1}{20}$; $13\frac{87}{133}$; $15\frac{2}{3}$; $9\frac{16}{17}$; $13\frac{1}{5}$; $5\frac{3}{4}$; $5\frac{4}{5}$;
$1\frac{1}{4}$; $\frac{4}{21}$; $\frac{7}{18}$; $\frac{9}{25}$; $11\frac{9}{44}$; $1\frac{53}{85}$; $2\frac{977}{3120}$; $1\frac{3867}{7001}$;
$13\frac{10798}{32439}$.

(*h.*) $-\frac{1}{2}$; $\frac{4}{15}$; $-\frac{1}{5}$; $\frac{1}{4}$; $11\frac{2}{7}$; $-\frac{7}{45}$; $-\frac{17}{117}$; $81\frac{22}{27}$; $28\frac{13}{35}$;
$-6\frac{1}{2}$; $13\frac{14}{17}$; $-20\frac{5}{19}$; $14\frac{1039}{1060}$; $-186\frac{149}{779}$;
$-13\frac{5231}{39360}$; $-22\frac{7}{40}$: $-161\frac{1}{171}$; $1\frac{1}{342}$.

2. (*a.*) $2\frac{17}{84}$; $\frac{41}{90}$; $1\frac{187}{1008}$; $-\frac{863}{880}$; $3\frac{2123}{13050}$.

(*b.*) $1\frac{3345}{15694}$; $8\frac{51597}{101920}$; $\frac{243999}{1082720}$; $-\frac{6669}{11036}$.

3. $\frac{71}{1008}$.

## Exercise 20.

1. (*a.*) $\frac{15}{56}$ ; $\frac{14}{45}$ ; $\frac{35}{54}$ ; $\frac{3}{14}$ ; $\frac{3}{10}$ ; $\frac{12}{77}$ ; $\frac{3}{25}$.
   (*b.*) $\frac{1890}{78139}$ ; $20\frac{124}{145}$.
   (*c.*) $\frac{24}{35}$ ; $1\frac{13}{27}$ ; $\frac{34}{135}$ ; 144 ; $148\frac{11}{138}$.
2. $10\frac{37}{40}$ ; $26\frac{5}{6}$ ; $20\frac{7}{40}$.
3. 1 ; 1.
4. 5 ; $\frac{239}{2202}$.
5. $\frac{1183}{1800}$ ; $6526\frac{127}{168}$ ; $18\frac{579}{1562}$.
6. 2.

## Exercise 21.

1. $1\frac{1}{21}$ ; $1\frac{7}{8}$ ; $1\frac{1}{55}$ ; $2\frac{2}{9}$ ; $2\frac{6}{7}$ ; $\frac{2}{3}$ ; $1\frac{5}{8}$ ; $\frac{3}{17}$ ; $2\frac{47}{71}$ ; 98 ; $2\frac{2}{33}$.
2. 8 ; $\frac{575}{972}$ ; $48\frac{524}{1725}$ ; $\frac{2}{3}$ ; $\frac{7}{180}$ ; $8\frac{752}{987}$ ; $\frac{2}{3}$ ; $2\frac{906322}{23086635}$ ; $\frac{1}{2}$ ; 1 ; 81 ; $5\frac{1}{16}$ ; $\frac{64}{121}$ ; $2\frac{6}{25}$ ; $\frac{694}{819}$.

## Exercise 22.

1. $1\frac{1}{3}$.
2. $\frac{307}{456}$.
3. $24\frac{7}{18}$.
4. $15\frac{3}{8}$.
5. $1\frac{63}{88}$.
6. $2\frac{213}{245}$.
7. $2\frac{36}{73}$ yrs.
8. A gets £3750 ; B gets £4000 ; C gets £2250.
9. $\frac{7}{18}$.
10. $27\frac{3}{5}$ pks.
11. 1.
12. $\frac{11}{15}$.
13. £39, 12s. $3\frac{1}{2}$d.
14. $\frac{422}{345}$.
15. $\frac{143360}{237861}$.
16. 103 cwt. 2 qrs. ; £11, 1s. $1\frac{1}{2}$a.
17. $\frac{15}{31}$.
18. 8932 : 9576 : 12673.
19. A Proper Fraction is less than Unity. No.
20. $\frac{1}{40}$.
21. $\frac{2022}{2222}$.
22. £10, 0s. 8d.
23. $\frac{37}{46}$ ; $10\frac{15}{49}$.
24. $\frac{107}{150}$ ; $\frac{108}{151}$ ; 16157 ; 16200.
25. $7\frac{311}{952}$ ; $14\frac{81}{160}$ ; $1\frac{907}{2013}$.
26. $\frac{337}{1600}$.
27. $\frac{23}{11250}$.
28. 2800 : 2793 : 3040.

29. £1500.
30. $\frac{23}{45}$ greatest, $\frac{3}{8}$ least.
31. £577, 10s.
32. The former, by £3, 8s. 7½d.
33. $\frac{3883}{2399904}$.
34. $\frac{15}{23}$.
35. $\frac{1055}{4234}$.
36. $\frac{1232}{5525}$.
37. $\frac{113389}{354478}$.
38. 3 cwt. 1 qr. 6 lb.; £566, 2s. 11d.
39. $\frac{1757}{3960}$.
40. £329; £51562, 10s.; 31296 tons, 6 cwt. 2 qrs.; £304722.
41. $20\frac{10603}{33486}$.
42. 7 inches.
43. £100.
44. 300 gallons.
45. £562, 3s. 2d.
46. 1; ½.
47. £16000.
48. £15000.
49. $-22\frac{4}{5}$ (about).

## Exercise 23.

1. ·18; ·0383; ·416; ·6343.

2. 1·086744.

3. (*a.*) ·4; ·17; ·13; ·2001; 1·801; ·007; ·000028; ·001041; ·00000006.
   (*b.*) ·04; ·054; ·042; ·003078; ·0002415.
   (*c.*) 181·05; 170·2304.

4. (*a.*) $\frac{3}{100}$; $\frac{451}{10000}$; $\frac{1}{4}$; $\frac{1}{200}$; $\frac{171}{1000000}$; $\frac{80807}{100000000}$; $\frac{71003}{1000000}$; $\frac{1}{16}$; $\frac{1}{10000}$.
   (*b.*) $28\frac{1}{100}$; $430\frac{1}{500}$; $71\frac{173}{800}$; $30824\frac{1}{2}$; $1\frac{1}{16}$; $1801\frac{329}{2000}$.
   (*c.*) $\frac{1107}{2000}$; $16\frac{1629}{2000}$; $73\frac{49}{200}$; $6\frac{229}{1250}$; $19\frac{3}{2500}$; $70\frac{65211}{125000}$.
   (*d.*) $21\frac{837}{1000}$; $620\frac{8341}{10000}$; $7\frac{121}{500}$; $\frac{3}{50}$; $2\frac{1}{10}$.
   (*e.*) $624\frac{1}{10}$; $638\frac{1}{10}$; $72816\frac{1}{2}$; $2139\frac{373}{500}$; 60.
   (*f.*) 8391082; 18100; $6381\frac{3}{5}$; $32\frac{113}{200}$; 18136491.

5. 1·1109317; 33156·20525; 185·986988; 652·2131; 76278·446; 26552086·965; 73337639·0727303.

6. (*a.*) 13·542.
   (*b.*) 43019·295107.
   (*c.*) 81076·936412.
   (*d.*) 230143·0353741.
   (*e.*) 113·8551.
   (*f.*) 2106·0948; 4074·0529; 189·585.
   (*g.*) 1572·0001; 25·992907; 24·98294.
   (*h.*) 99·276676.
   (*i.*) 5555·6049095.

## Exercise 24.

1. 1150·53503 ; 417962·601 ; 257·049 ; 408·04.
2. ·00043 ; ·036 ; ·03375 ; 6·21387 ; ·0279.
3. ·091791 ; ·51282 ; ·0314 ; 2304·0192 ; ·273.
4. ·1 ; 4 ; 4 ; ·105 ; 80.
5. ·064 ; ·01024 ; 2·56 ; ·014 ; ·04096.
6. 6057·25175 ; 1·0544661 ; 2·9322522.
7. ·006088056676 ; 95·9676096.
8. ·1792784 ; ·066594 ; 31·552101 ; ·0076473.
9. 1969·8 ; ·274228 ; 646477·78925 ; 9926·38522.
10. ·0122045998536 ; ·85120171788834.
11. 50·805792 ; 100325·8672772940465024.
12. ·4822533830268 ; ·000001638213228.

## Exercise 25.

1. 9·618 ; 23·81 ; 108·017 ; 21·00309 ; $\cdot 008105\frac{11}{18}$.
2. ·35 ; 6·3 ; 8·15 ; ·20008.
3. 68000 ; 12362241·53, &c. ; ·0000003.
4. ·000218 ; 8·2436 ; ·1382304, &c.
5. 34069·44 ; 1319·84 ; 1136·208 ; 129600.
6. 35·44 ; 11·024 ; 491416 ; 15·2.
7. ·9173.
8. $229\cdot 02232\frac{4}{7}$.
9. 18998·3995071.

## Exercise 26.

1. $\frac{3}{1000}$ ; $\frac{43}{1000}$ ; $\frac{159}{500}$ ; $\frac{1553}{50000}$ ; $\frac{91}{100000}$ ; $\frac{17859}{25000}$ ; $\frac{21}{200000}$.
2. ·4 ; ·18 ; ·5 ; ·15 ; ·010345 ; ·0000601 ; ·00005404 ; ·00108.
3. $2\frac{3}{10}$ ; $40\frac{9}{100}$ ; $2000\frac{8}{125}$ ; $50\frac{81}{100}$ ; $26\frac{149}{20000}$ ; $10080\frac{801}{1000000}$.
4. 4·2 ; 12·75 ; 17·05 ; 106·00025 ; 20 0031496, &c. ; 10007·00002493, &c.
5. $7\frac{16}{13}$ ; $106\frac{1}{10}$ ; $10\frac{2}{3}$ ; 8 ; $\frac{25}{99}$ ; $30\frac{28}{45}$ ; $201\frac{1316}{2475}$ ; $\frac{503}{49500}$ ; $\frac{24681}{49950}$ ; $20\frac{73}{99}$ ; $8\frac{3}{22}$ ; $2000\frac{1363}{15000}$ ; $7030\frac{82}{99}$ ; $106\frac{89719}{99900}$.
6. $\cdot\dot{4}2857\dot{1}$ ; $\cdot\dot{3}\dot{6}$ ; $\cdot 5\dot{6}$ ; $\cdot 00\dot{8}$ ; $\cdot\dot{1}\dot{7}$ ; $8\cdot\dot{3}0769\dot{2}$ ; $2\cdot 00\dot{1}4285\dot{7}$ ; $10\cdot 000\dot{1}4285\dot{7}$ ; $1\cdot 00\dot{2}10526315789473 68\dot{4}$ ; $\cdot\dot{4}2857\dot{1}$ ; $\cdot\dot{1}9753086\dot{4}$ ; $1\cdot\dot{4}70588235294117\dot{6}$ ; $\cdot 1\dot{8}82352941176470\dot{5}$ ; $\cdot 7\dot{8}$ ; $\cdot 002535885167464114 8\dot{3}$ ; $20\cdot\dot{1}4285\dot{7}$ ; ·4 ; ·075 ; ·2125 ; ·099167, &c. ; 1·587890625 ; $1\cdot 00\dot{3}9682\dot{5}$ ; $3\cdot 736\dot{1}$ ; $14\cdot\dot{7}7922\dot{0}$ ; $\cdot 1\dot{5}30612244897959183673469387755102040816\dot{3}2\dot{6}$.

## Exercise 27.

1. $416{\cdot}722\dot{9}6387\dot{2}$.
2. $42{\cdot}89581\dot{5}2170\dot{3}$.
3. $29{\cdot}9\dot{2}\dot{3}$.
4. $19{\cdot}085\dot{0}67\dot{7}$.
5. $83{\cdot}98\dot{9}\dot{6}$.
6. ${\cdot}09\dot{0}\dot{6}$.
7. $93{\cdot}282\dot{1}31914993 3 0\dot{1}$.
8. $1951{\cdot}13\dot{2}9775\dot{2}$.
9. $135{\cdot}42\dot{6}$; $17874{\cdot}\dot{7}\dot{8}$; $97{\cdot}6\dot{9}\dot{3}$; $14864{\cdot}08\dot{6}$; $2807{\cdot}9924\dot{1}0\dot{8}$; 634·166937, &c.
10. $5{\cdot}6\dot{7}0\dot{3}$; 21·1224489, &c.; 10·6840579, &c.; 9·1981132, &c.; ${\cdot}\dot{3}6630\dot{0}$.
11. $2{\cdot}7\dot{7}1428\dot{5}$; $5\frac{12}{37}$.
12. $3\frac{1}{2}$; $3\frac{49}{100}$; $3\frac{49}{99}$.
13. 3·5606885835, &c.; ·3628054869, &c.; 2·3703896611, &c.; 96·9913630929, &c.; ·0115059912, &c. [$0158931\dot{9}$;
14. $7490{\cdot}\dot{7}711379529561347743165924984106802288620470438652256834075$ ${\cdot}4\dot{5}2380\dot{9}$; ${\cdot}02\dot{6}0942\dot{7}$; ${\cdot}\dot{0}578512396694214876033\dot{3}$; ${\cdot}\dot{1}7460\dot{3}$.

## Exercise 28.

1. ·625; ·525; ·5125; ${\cdot}88\dot{3}$; ·775; ·875; $1{\cdot}5\dot{3}$; ·98125; ${\cdot}919791\dot{6}$; $14{\cdot}48\dot{3}$; 21·0125; $17{\cdot}986458\dot{3}$.
2. ·1375; ·7125; ·51875; 13·303125; 1·65; 17·9015625; ·8669642, &c.
3. ·525; ·56627; ${\cdot}0\dot{5}7142\dot{8}$; $1{\cdot}1\dot{6}$; ·008332 nearly; ·94956, &c.; ·3220088, &c.; 137·8461, &c.; ·06637, &c.; ·152758, &c.
4. ·6245, &c.
5. 4·693, &c.
6. 16s.; £2, 8s.; £17, 7s. 6d.; 12s. 6d.; £10, 1s. 3d.; £2, 16s. 8d; £101, 7s. 6d.; £166, 12s.; £1704; £270732, 9s. 7½d.; £4733, 8s. 9d.; £373, 0s. 3d.; £1386, 2s. 6d.; £24, 16s. 9·6d.; 3 cwt. 1 qr. 8·559824 lb.; £426, 6s. 6·965d.; £194, 10s. 9·6d.
7. £4, 19s. 4¾d.; 32 days 12 hrs.; 2261 ac. 3 r. 14 po.; £1, 11s. 6d.; £1, 17s.
8. ·04153, &c.
9. ${\cdot}\dot{6}$.
10. ${\cdot}053\dot{5}7142\dot{8}$.
11. 5·61, &c.
12. 2·27875.
13. 6s. 10·701d.
14. £45, 9s. 6¾d.
15. 1 m. 6 f. 21 po. $2{\cdot}8\dot{1}$ yds.
16. ·2625.

## Exercise 29.

1. 21399·9 met.
2. 392034·481 met.
3. 65479 centim.
4. 4710·59 ar.
5. 2210800 centig.
6. 1898666·39 gr.
7. 75·414 gr.
8. 1113576·31 st.
9. 244264 st.
10. 3 times.
11. 1209 lit.
12. $63{\cdot}09436\frac{4}{7}$ lit.
13. 8008 bottles.

## Exercise 30.

1. 179·34.
2. $39{\cdot}45\dot{6}$ yd.
   $\cdot 1793\dot{4}\dot{8}$ f.
   $\cdot 02241856\dot{6}\dot{0}$ m.
3. 343·993 persons.
4. £3, 17s. 10½d.
   ·9734375 ton.
5. 200.
6. 900000.
7. ·049.
8. ·046875.
9. £9570, 1s. 6¼·3d.
10. $24855{\cdot}\dot{3}\dot{0}$ m.
11. $55{\cdot}82850\dot{2}886\dot{0}$.
12. £170.
13. 7616·2816.
14. 10353·57100632.
15. 1600.
16. ·0605109.
17. ·2742498.
18. ·03493.
19. ·000643291.
20. $39{\cdot}70617\dot{4}\dot{2}$.
21. 2 cwt. 2 qrs. 16 lb.
22. 19·6714449.
23. 1·479816.
    6·3099.
24. 3103·1442.
    10826·9869.
    ·0307.
    14375.
    74·84.
    22600.
    6·24312.
    1000.
    6.
    £39, 5s.
    £192, 6s. 6d.
    ·06240312.
    100000.
    6000.
25. ·7875.
    $51{\cdot}93\dot{1}4285\dot{7}$.
    623·5.
    2·6667.
    666.
    $83{\cdot}30000\dot{0}3\dot{7}$.
    117389·88261.
26. £1·8672 or £1, 17s. 4·128d.
27. 43·039783 ft.
    6·1575164 yds.
    89·03894378 m.
28. 475·135024.
29. 1·240346.

30. ·890625.
2·0195s.
31. ·84375.
$\frac{616}{3125}$.
32. ·61837.
33. 4·00000896.
34. £·002256428571 or ½d. ·16617 +
35. 2·401.
36. 16s. 2¼d. 222f.
3 lb. 5 oz. 10 dwts. 3·86 grs.
37. £18, 6 fl. 2 c. $7{\cdot}08\dot{3}$ mils.
£12, 3 fl. 6 c. $9{\cdot}791\dot{6}$ mils.
£32, 8 fl. 2 c. 1·875 mils.
£6, 7 fl. 1 c. 0·416 mils.
38. $\frac{8416}{4868889}$.
39. £7, 10s. 6¼d. ·336.
40. £115, 9s. 1·67184d.
41. 65·510217.
42. 277·2738.
43. 5·0420168.
44. 1·40498.
45. £44.
46. £186, 4s.
47. 574·425.
48. ·0338235.
71·0038.
·49867.
49. ·10137475916.
50. $1141876{\cdot}8282\dot{1}\dot{0}$.
51. 95·240906819.

## Exercise 31.

1. (1.) £14.
(2.) £9.
(3.) £20, 4s.
(4.) £23.
(5.) £13, 10s.
(6.) £74.
(7.) £110.
(8.) £425.
(9.) £8211.
(10.) £2410.
(11.) £5265.
(12.) £4510.
(13.) £2496.
(14.) £9570.
(15.) £9801.
(16.) £129267, 10s.
(17.) £7350, 10s.
(18.) £14664.
(19.) £134442.
(20.) £4863, 10s.
(21.) £33.
(22.) £41205.
(23.) £6946.
(24.) £2604.
(25.) £866730.
(26.) £75361.
(27.) £1634561.
(28.) £450146.
(29.) £32216261.
(30.) £511677320.

2. (1.) £115, 10s.
(2.) £1808.
(3.) £3726.
(4.) £3239.
(5.) £1761, 6s.
(6.) £13668, 8s. 4d.
(7.) £666, 3s. 9d.
(8.) £2480, 4s. 6d.
(9.) £1625, 8s. 4d.
(10.) £4412, 12s. 8d.
(11.) £9861, 4s.
(12.) £8464, 10s.
(13.) £140.
(14.) £186, 13s. 4d.
(15.) £409, 10s.
(16.) £24, 18s. 8d.
(17.) £522, 6s. 4d.
(18.) £610, 17s. 6d.
(19.) £43, 7s. 11½d.
(20.) £620, 7s. 6¼d.
(21.) £151, 18s. 1¾d.
(22.) £639, 17s. 8¼d.
(23.) £321, 11s. 10¼d.
(24.) £43, 3s. 5¾d.
(25.) £2206, 4s. 4½d
(26.) £25629, 0s. 8d.

(27.) £89164, 18s. $5\frac{1}{4}$d.
(28.) £51478, 16s. $11\frac{1}{2}$d.
(29.) £2945, 13s. $0\frac{3}{4}$d.
(30.) £16485, 4s. 6d.
(31.) £127246, 0s. $10\frac{1}{2}$d.
(32.) £36152, 11s.
(33.) £110150, 1s. 9d.
(34.) £10322, 19s. $0\frac{3}{4}$d.
(35.) £148295, 19s. $3\frac{1}{4}$d.
(36.) £560824, 13s. 9d.
(37.) £20528, 15s.
(38.) £36671, 0s. $1\frac{1}{4}$d.
(39.) £27784, 14s. $9\frac{1}{2}$d.
(40.) £49747, 15s. $2\frac{1}{2}$d.
(41.) £9, 11s. $7\frac{1}{2}$d.
(42.) £159, 5s. $10\frac{5}{8}$d.
(43.) £2892, 15s.
(44.) £20532, 4s.
(45.) £941, 5s. 10d.
(46.) £458, 7s. $8\frac{10}{11}$d.
(47.) £5866, 3s. 4d.
(48.) £12491, 4s. 8d.
(49.) £45428, 4s. $9\frac{3}{7}$d.
(50.) £61, 3s. $3\frac{9}{13}$d.

3. (1.) £185, 5s.
(2.) £216, 15s. 7d.
(3.) £17, 14s. 2d.
(4.) £136, 1s. 8d.
(5.) 10s. $0\frac{3}{4}$d.
(6.) 14s. $2\frac{1}{2}$d.
(7.) £151, 0s. $7\frac{1}{2}$d.
(8.) £36, 15s. $0\frac{1}{8}$d.
(9.) £1401, 8s. 2d.
(10.) £53, 7s. $5\frac{13}{16}$d.
(11.) £12, 14s. 2d.
(12.) £431, 4s.
(13.) £1075, 16s.
(14.) £20, 15s. 9·6d.
(15.) £7512, 9s. $7\frac{1}{8}$d.
(16.) £799.
(17.) £175946, 8s. $6\frac{5}{7}$d.
(18.) £34, 7s. $11\frac{1}{2}$d.
(19.) £69, 9s. $5\frac{1}{8}$d.
(20.) £13, 10s. $7\frac{1}{2}$d.

4. (1.) £64, 13s. 9d.
(2.) £106, 13s.
(3.) £40, 17s. $1\frac{5}{8}$d.
(4.) £7, 18s. $10\frac{1}{2}$d.
(5.) £11, 5s. $6\frac{3}{4}$d.
(6.) £23, 5s. $7\frac{1}{8}$d.
(7.) £21, 1s. $10\frac{1}{2}$d.
(8.) £184, 2s. $3\frac{1}{2}$d.
(9.) £135, 7s. $5\frac{1}{10}$d.
(10.) £70, 8s. $5\frac{5}{8}$d.
(11.) £44, 9s. $4\frac{9}{10}$d.
(12.) £19, 19s. $0\frac{3}{4}$d.
(13.) £1668, 9s. 6·175d.
(14.) £111976, 2s. 6d.
(15.) £33439, 17s. 3·9d.
(16.) £618, 7s. 6d.
(17.) £14013, 11s. $10\frac{1}{2}$d.
(18.) £595, 10s. $7\frac{3}{4}$d.
(19.) £2289, 5s. $10\frac{7}{8}$d.
(20.) £34, 17s. $10\frac{53}{76}$d.
(21.) £15216, 13s. 4d.
(22.) £36174, 12s. 7·2576d.
(23.) £659, 14s. 2·754d.
(24.) £2, 9s. $0\frac{85}{132}$d.
(25.) £5, 5s. $10\frac{131}{1760}$d.
(26.) £111, 12s. $10\frac{11}{15}$d.
(27.) £87 17s. $9\frac{33}{46}$d.
(28.) £9, 3s. $10\frac{11}{14}$d.
(29.) £219, 9s. $3\frac{15}{16}$d.
(30.) £19312, 19s. $10\frac{3}{7}$d.

5. 118 ac. 20 poles.

6. 69 tons 60 lb.

7. 5827680 cub. ft.

8. £8, 18s. $4\frac{11}{16}$d.

9. $33\frac{33}{56}$ grs.

## Exercise 32.

1. £20, 16s. $8\frac{1}{4}$d. +
2. £47, 7s. $9\frac{1}{4}$d.
3. £80, 11s. 3d.
4. £1153, 19s. ; £444, 16s. 8d. ; £719, 11s. 8d. ; £1327, 4s. ; £311, 3s. 4d. ; £691, 5s.
5. 168.
6. £123, 10s.
7. $20\frac{35}{72}$ yds. ; £5, 12s. $8\frac{1}{2}$d.
8. £659, 0s. $0\frac{3}{4}$d.
9. £2, 17s. $6\frac{3}{8}$d.
10. £1119, 18s. 9d.
11. £1654, 13s. 5d.
12. £1, 16s. 9d.
13. £4114, 11s. 5d.
14. 186 m. 4 f. $76\frac{2}{3}$ yds.
15. £3142, 6s. $10\frac{1}{2}$d.
16. £213, 19s.
17. £1, 18s.
18. £163, 13s. $3\frac{1}{2}$d.
19. £75843, 1s. 6d.
20. £10236, 4s. 10d.
21. £33, 3s. $11\frac{3}{4}$d.
22. £40, 18s. $8\frac{1}{2}$d.

## Exercise 33.

2. 49 ; 315 ; 300 ; 30 ; 26 ; 165 ; 238 ; $\frac{3}{5}$ ; 243 ; 400 ; 84 ; 1000.
3. 8s. $1\frac{1}{2}$d.
4. 17 lb.
5. 13.
6. £1, 0s. $7\frac{5}{13}$d.
7. 45.
8. $34\frac{18}{25}$.
9. 10.
10. 17 lb.
11. £43, 0s. $10\frac{10}{11}$d.
12. $63\frac{1674}{2907}$ yds.
13. £382, 18s. 4d.
14. 1050.
15. 120 lb.
16. 5 hrs. $32\frac{4}{13}$ mins.
17. £9, 15s. $11\frac{1}{4}$d.
18. £670, 5s.
19. 16.
20. £9, 15s.
21. 302 miles.
22. 15s. 6d.
23. 2 yds. 2 qrs. $3\frac{4}{11}$ nls.
24. £552, 10s. 8d.
26. £2, 7s. $2\frac{10}{73}$d.
27. $9\frac{18}{151}$d. per week.
28. £3105, 3s.
29. £1810, 0s. $3\frac{3}{4}$d.
30. £33, 5s. 10d.
31. $2\frac{1}{2}$d.
32. $947\frac{1}{3}$ galls.
33. 40 lb.
34. £6, 11s. $7\frac{1}{6}$d.
35. £1, 19s. $3\frac{1}{4}$d. ·76.
36. £58, 2s. 6d.
37. $154\frac{49}{60}$ yds.
38. 2s. $3\frac{1}{2}$d. ; £6, 11s. 3d.
39. 216000 vibrations.
40. £7, 6s. 8d.
41. 750 men.
42. 14 lb.
43. $933\frac{1}{3}$ lb.
44. £17, 6s. $10\frac{1}{2}$d.
45. 28 cwt. 3 qrs. 8 lb.
46. £67, 10s.
47. $\frac{3}{4}$ second.
48. 8s.
49. £203, 9s. $4\frac{1}{2}$d.
50. £34, 6s.
51. 815·36 lb.
52. $2\frac{1}{4}$d.
53. 332800 lb.
54. £2, 3s. $6\frac{1}{2}$d.
55. 4 cwt. 1 qr. 5 lb. 8 oz.
56. $662\frac{22306}{138637}$ galls.
57. £2, 3s. 9d.
58. 73 lb.
59. £186, 4s.
60. 567 yds.
61. $123\frac{171}{838}$ sovs.
62. £300.

## Exercise 34.

1. 8 days.
2. 5 hrs.
3. £2, 14s. 1$\frac{4}{5}$d.
4. 10 cwt.
5. 480.
6. £640.
7. £1, 18s.
8. 300 minutes.
9. £413, 6s. 8d.
10. £732, 10s.
11. 3 ft. 2$\frac{558}{625}$ in.
12. £50, 8s. 4$\frac{2110}{2343}$d.
13. 160.
14. £20, 2s. 6d.
15. £18, 4s. 5$\frac{1}{3}$d.
16. 44 oz.
17. 13s. 1$\frac{1}{2}$d.
18. 280000.
19. 34$\frac{2}{27}$ lb.
20. 37$\frac{1}{2}$ days.
21. 13$\frac{1}{2}$ acres.
22. 54 tons 3 cwt. 72 lb.
23. 11 hrs.
24. 16$\frac{1}{8}$ ft.

## Exercise 35.

1. £13, 0s. 7$\frac{3}{4}$d.
2. 16.
3. 1$\frac{3}{5}$ hrs.
4. £70.
5. 215.
6. The first; 16 yds.
7. £97, 19s. 5·6d.
8. 9$\frac{1}{3}$ yds.
9. 138 days 10 hrs. 40 min.
10. 96 days.
11. 5 yrs.
12. 57$\frac{3}{5}$ hrs.
13. £3255.
14. £50, 10s.
15. 52.
16. 31 mins.
17. 4.
18. 1411141·2 nearly.
19. 16000.
20. 6$\frac{1}{4}$ galls.

## Exercise 36.

1. £18; £32; £24; £6; £86.
2. £30; £258; £3664; £260, 4s.; £414, 14s. 4·8d.
3. £28; £199, 10s.; £371, 17s. 6d.; £775, 15s.; £3636, 14s. 4$\frac{1}{2}$d.; £97, 6s.; £865, 10s. 9d.; £173, 13s. 0·8d.
4. £780; £1932; £643, 6s. 8d.; £2281, 5s. 6$\frac{1}{4}$d.; £2143, 3s. 3·28d.; £1253, 17s. 3·9075d.; £998, 8s. 10·3875d.; £4501, 11s. 4·44d.; £3917, 1s. 9·3d.; £760, 14s. nearly; £38277, 15s. nearly.
5. £81, 0s. 0$\frac{3}{4}$d.
6. 116·75 fr.
7. £24, 18s. 6·76d.
8. £85, 12s. 6d. nearly.
9. £88, 16s. 11d. +
10. £29, 17s. 8d.
11. £4, 0s. 5$\frac{1}{4}$d.
12. £833, 14s. 8d. nearly.
13. £31, 0s. 11·136d.
14. £502, 4s. 10$\frac{26}{73}$d.
15. £5, 15s. 3d. +
16. £6243, 1s. 10$\frac{1}{4}$d. nearly.
17. £40655, 6s. 4·2d.
    £115828, 0s. 4·8d.
    £58244, 5s. 3·12d.
18. £82, 10s.
19. £651, 1s. 6$\frac{2}{13}$d.
20. £811, 3s. 11·68d.
21. 8 p. c.
22. 5 p. c.
23. £649, 1s. 8$\frac{30}{31}$d.
24. £787, 10s.
25. £378, 19s. 1$\frac{511}{551}$d.
26. 2$\frac{1}{2}$ p. c.
27. £175.
28. £6307, 15s.
29. 4$\frac{1}{2}$ yrs.
30. £236333, 6s. 8d.
31. £3750.
32. £348406, 5s.
33. 8$\frac{5369}{20894}$ p. c.
34. 2$\frac{1}{2}$ yrs.
35. 3$\frac{1}{5}$ yrs.
36. 700 yrs.
37. 50000 yrs.
38. 4642 yrs.
39. 350 p. c.
40. £3012800.

## Exercise 37.

1. £82.
2. £2156, 4s. 4·8d.
3. £2224, 19s. 8$\frac{1}{4}$d. nearly.
4. £119704, 16s. 9$\frac{1}{4}$d. nearly.
   £88584, 1s. 9d.
   £58549, 14s. 2·112d.
   £490, 13s. 11$\frac{1}{2}$d. +
   £153, 4s. 5$\frac{3}{4}$d. nearly.
5. £6, 11s. 2$\frac{1}{2}$d. +
6. £2, 14s. 11·87744d.
7. 15s. 11·729864d.
8. £500.
9. 940 per cent.
10. £7900, 15s. 9$\frac{3}{4}$d.
11. £1025, 6s. 2$\frac{1}{2}$d., about.
12. £835, 17s. 9·31d., &c.

## Exercise 38.

1. £1, 1s. 3$\frac{3}{20}$d.
2. 12s. 8$\frac{19}{30}$d.
3. £38, 8s. 9$\frac{1}{8}$d.
4. £177, 2s. 4$\frac{1}{5}$d.
5. £1945, 1s. 1$\frac{191}{423}$d.
6. £2704, 13s. 10$\frac{18}{1087}$d.
7. £3, 7s. 3$\frac{3}{4}$d. nearly.
8. £1577, 17s. 4d. nearly.
9. £668, 9s. 2·91d.
10. £45, 8s.
11. £230, 0s. 3d.
12. £34, 2s. 3d.

## Exercise 39.

1. £16, 13s. 8$\frac{1}{2}$d.
   £16, 13s. 7$\frac{1}{2}$d.
   £3, 2s. 1d.
   £2, 10s. 3d.
   £2, 14s. 2$\frac{1}{2}$d.
   £7, 1s. 5d.
   £21, 13s. 2$\frac{1}{2}$d.
   £210, 19s. 4$\frac{1}{4}$d.
   £3, 2s. 6$\frac{2}{13}$d.
2. £208, 5s.
3. £289.
4. £80, 6s. 4d.
5. £604, 3s. 4d.
6. £176, 13s. 4d.
7. £3396, 5s.
8. £9, 0s. 3d.
9. £504, 3s. 4d.
10. £9, 13s. 4d.
11. £221, 5s.
12. £3, 12s. 8$\frac{8}{11}$d.
13. £19, 10s.
14. £107, 0s. 1$\frac{3}{4}$d. +
15. £31, 4s. 3$\frac{1}{2}$d.
16. £81, 16s. 4$\frac{1}{2}$d.
17. £18, 12s. 3$\frac{41}{77}$d.
18. £20, 10s. 1$\frac{191}{223}$d.; £382, 10s.
19. 4 p. c.
20. £37, 10s. 4$\frac{1}{2}$d.
21. £2, 4s. 11$\frac{1}{16}$d.
22. 20$\frac{1}{3}$ months, about.

## Exercise 40.

1. £85, 15s. 7$\frac{1}{2}$d.
2. £52, 14s. 2d.
3. £15, 12s. 9d.
4. £24, 8s. 4$\frac{580}{807}$d.
5. £85.
6. 3 p. c. stock.
7. £250, 13s. 4d.
8. £945 stock.
9. £639.
10. £133$\frac{1}{3}$.
11. £288.
12. £335, 8s.
13. £243.
14. £118$\frac{2}{11}$.
15. £55$\frac{5}{8}$.
16. £450, 18s.; gains 8s. 9·21d. per an.
17. Loses £8, 15s. 2$\frac{1}{10}$d. per annum.
18. 97$\frac{247}{1016}$.
19. £91, 6s. 1$\frac{1}{23}$d.; 3$\frac{6}{23}$ p. c.
20. 98$\frac{1}{4}$.
21. Railway shares.
22. £35, 12s. 5$\frac{7}{37}$d.
23. £58, 16s. 3d.
24. In the 3 p. c.; £3, 0s. 5$\frac{3}{4}$d. +
25. Loses £15; gains £45 per annum.
26. 2 p. c.
27. £100 less.
28. Loses £25; gains £47 per annum.
29. £6947$\frac{11}{12}$.
30. Increased by £24.
31. £10, 16s.
32. Loss of £61, 15s. 3d., about.
33. £13125; $\frac{25}{36}$ p. c.

## Exercise 41.

1. £2, 0s. $1\frac{1}{3}$d.
2. £1, 8s.
3. £4, 19s.
4. 192 lb.
5. $7\frac{7461}{[illegible]}$d.
6. At an advance of $31\frac{1}{4}$ p. c. upon the cost price.
7. 20 p. c.
8. $33\frac{1}{3}$ p. c.
9. £7462.
10. £3472, 13s. 4d. ; £65, 2s. 3d.
11. 25 p. c. ; 20 p. c.
12. $\frac{1}{4}$ p. c.
13. 4 p. c.
14. 2d.
15. 2s. 6d. per lb.
16. £84.
17. $7\frac{1}{3}$d.
18. Loss of $5\frac{1}{2}$ p. c.
19. £2, 10s. $9\frac{[illegible]}{[illegible]}$d.
20. 5d. and 2s. 6d. ; $4\frac{7}{8}$d. and 3s. 6d.
21. 25 for 18d., or, $\frac{18}{25}$d. each.
22. £1, 19s. 4d. ; $21\frac{52}{277}$ p. c.
23. $150\frac{6850}{10259}$ cwt.
24. The bookseller pays $\frac{3672}{6125}$ of published price, and ∴ makes $\frac{2453}{3672}$ of 100 p. c., *i.e.* $66\frac{737}{918}$ p.c.
25. $6\frac{2}{13}$ p. c.
26. Gains 4 p. c.
27. £3200.
28. £1672, 4s. $5\frac{1}{3}$d.
29. $16\frac{2}{3}$ p. c.
30. $23\frac{1}{13}$ p. c.
31. $47\frac{1}{2}$ p. c.
32. $1\frac{[illegible]}{[illegible]}$.

## Exercise 42.

1. £2, 15s. $6\frac{2}{3}$d.
2. $6\frac{1}{4}$.
3. £1 ; £3 ; £9 ; £13.
4. 5487 ; 33 p. c.
5. £3333, 16s. $11\frac{1}{3}$d.
   £666, 15s. $4\frac{2}{3}$d.
6. 3 : 4.
7. 52 days.
8. 6s. $1\frac{1}{13}$d. ; 4s. $7\frac{5}{13}$d. ; 9s. $2\frac{9}{13}$d.
9. £457, 10s. $10\frac{1}{4}$d.
   £374, 7s. $0\frac{3}{4}$d.
   £291, 3s. $3\frac{1}{4}$d.
10. 5s. $0\frac{71}{253}$d. ; 5s. $5\frac{162}{253}$d. ; 2s. $1\frac{17}{253}$d.
11. A gives away £14, 19s. 3d., of which £7, 19s. goes to B, and £7, 0s. 3d. to C.
12. £127, 10s. ; £354, 12s. ; £217, 18s.
13. £175, 3s. $11\frac{1}{2}$d. ; £17, 17s. 6d. ; 16s. $10\frac{3}{11}$d.
14. £300.
15. £450 to A ; £825 to B ; £1575 to C.
16. Gains 16 p. c.
17. £59, 1s. $11\frac{1}{4}$d., or, 286·62 dollars.
18. 3570.
19. $11\frac{2}{3}$ months.
20. 2 : 3.
21. £2747, 8s. $1\frac{747}{1189}$d.
    £2290, 3s. $2\frac{418}{1189}$d.
    £1962, 8s. $8\frac{72}{3567}$d.
22. 9 lb. of each price.
23. 40 of rye, 40 of oats, 20 of barley.
24. 18 Canary, $31\frac{1}{2}$ Malaga, $13\frac{1}{2}$ sherry, and 27 white wine.
25. 29 lb. at 5s. and at 9s., $14\frac{1}{2}$ lb. at 6s. and at 8s.
26. The circuitous is the more advantageous by 124·8325 milrees.

# EXAMINATION PAPERS.

## I. Papers of the London University.

**1.**

1. 7; ·00029296875.
2. ·00015625; 1 cwt. 1 qr. 20 lb.
3. £105, 8s. 4d.; £104, 7s. 3d.
4. £500; £541, 13s. 4d.
5. 100372; ·2265.

**2.**

1. £902777, 15s. $6\frac{2}{3}$d.
2. $3\frac{3}{16}$; $\frac{689}{11000}$.
3. ·01357; $16\cdot\dot{4}2857\dot{1}$.
4. Int. = £114, 4s. 2·4d.
5. A, £20; B, £21; C, £25.

**3.**

1. $\frac{157}{225}$.
2. 695·7 yds. + ; $\cdot\dot{3}$.

The remainder consists in each case of as many 2's as there are figures in the quotient.

3. (*a*.) 200 miles.
   (*b*.) Yes; no.

**4.**

1. $\frac{3}{8}$; ·0000152.
2. 7 × 13 × 2 squared; 5 × 7 × 3 squared × 2 cubed; 5 × 1091.
3. £3, 6s. $4\frac{1}{4}$d.; ·298828125.
4. $\frac{121}{123}$ or $\frac{40\frac{1}{3}}{41}$; 209·25ft.
5. 14479674 approximately.

**5.**

1. $2414\frac{17}{22}$ times.
2. $3\frac{1}{3}$ p. c.; $3\frac{9}{17}$ p. c.
3. $\frac{17}{63}$; $\frac{137}{210}$.
4. ·3183098, &c.; ·008.
5. 180000·0156.
6. 47 hrs. 3′ 20″.
7. 44.
8. 8 days.

**6.**

1. 10296.
2. $1\frac{1}{2}$ sec.; ·304.
3. 41870010.

## II. Papers of the Science and Art Department.

**1867.** 1. 450 sq. ft. 2. 240.

**1868.** 1. ·2036. 2. $506\frac{24}{121}$ sq. ft. 3. $60\frac{43}{105}$ sq. ft. 4. $5-2\sqrt{3}$.

**1869.** 1. ·1415, &c. 2. 11·90, &c. 3. ·004, &c.

**1870.** 1. ·0078125; $\frac{33}{148}$. 2. 2933·763, &c. 3. ·5.

**1871.** 1. 35 at 5s. 6d. and 15 at 6s. 6d. 2. 1·2548, &c.

**1872.** 1. 10. 2. 15 miles. 3. £3, 5s. 10d.; 4. 721 lb.; 327 kilog. + 5. 23.

**1873.** 1. $\frac{7984}{15} = 532\frac{4}{15}$. 2. 8·245 metres. 3. $\frac{5}{256}$; ·01953125 in. 4. $9\frac{17}{32}$d.; $29\frac{369}{1036}$ p. c. 5. $11\frac{1}{2}$ hrs. after the dispatch-boat started. 6. By $\frac{1}{781}$; the first.

## III. Examinations in Government Training Colleges.

### No. 1.

2. 7200.
3. £2, 8s. 9$\frac{1}{2}$d.
4. £52, 2s. 2$\frac{1}{4}$d.
   £116, 9s. 3$\frac{3}{4}$d.
5. £21, 10s.
6. £14, 10s. 11$\frac{1}{4}$d.
7. 10$\frac{2}{3}$ days.
8. In 840 days, *i.e.* 2$\frac{11}{73}$ yrs.
9. $\frac{9}{40}$; £2250.
10. ·003125.
11. £24, 18s. 6·76d.
12. £175, 10s.
13. £50; 4$\frac{1}{8}$ p. c.
14. 25$\frac{1}{3}$ acres.
15. 3s. 5$\frac{1}{4}$d.

### No. 2.

#### *Section I.*

1. $\frac{2561}{4361}$.
2. 1·17 nearly.
3. 1447 ft.

#### *Section II.*

1. £2, 6s. 7$\frac{307}{1007}$d.
2. 8.
3. 7s. 9d.; 5s. 3d.

#### *Section III.*

1. £365.
2. 122$\frac{5}{6}$; £1899$\frac{437}{777}$.
3. 6$\frac{2}{3}$ months.

#### *Section IV.*

1. 5$\frac{13}{128}$ galls.
2. £1380$\frac{20}{21}$.
3. $\frac{5}{8}$; 14 tons 1 cwt. 3 qrs. 22 lb. 5$\frac{4}{7}$ oz.

#### *Section V.*

1. 24·64787, &c.

### No. 3.

#### *Section I.*

1. Twenty-five thousand and sixty pounds, nineteen shillings and elevenpence three farthings.
4. $248553{\cdot}\dot{0}\dot{3}$ miles.

#### *Section II.*

1. £1131, 18s. 4d.
2. £3, 8s. 11$\frac{3}{4}$d.
3. £1731, 19s.
4. £2, 12s. 10$\frac{1}{2}$d.

#### *Section III.*

1. 2 hrs. 55′ P.M.
3. July 11th, at 8.30 A.M.

#### *Section IV.*

2. 25 p. c.

#### *Section V.*

2. $3{\cdot}\dot{7}1428\dot{5}$; $23{\cdot}1\dot{4}$; $5{\cdot}\dot{7}\dot{2}$; $14{\cdot}562\dot{5}$; 8·68; their sum $=55{\cdot}8285\dot{0}2886\dot{0}$.
3. £116·875; £170.
4. 7616·2816.

#### *Section VI.*

1. 116·76 francs.
2. 42·1456; 57·8543.
3. £348406, 5s.
4. £304.

### No. 4.

#### *Section I.*

1. 204030901; seven hundred and one million six hundred and nine.
2. £6, 14s. 8$\frac{1}{2}$d.
3. £3, 12s.; 60 p. c.
4. 11$\frac{2}{3}$d.
5. 22463892.
6. £3, 2s. 7d.

### *Section II.*

2. 1 hr. 12 min. 34·56 sec.

### *Section III.*

1. $8\frac{87457}{99540}$.
2. £31, 10s. 8d.
3. ·2492, &c.

### *Section IV.*

1. $10\cdot633\dot{9}28571\dot{4}$.
2. 1s. $2\frac{1886}{187227}$d.; £8, 8s. $7\frac{93937}{280905}$d.
3. ·355 met.; 2·812 ft. C.

### *Section V.*

1. £6804, 10s. $9\frac{1}{2}$d.
3. $18\frac{3}{4}$ p. c.

### *Section VI.*

1. Area $= 822\frac{6}{7}$ sq. ft.; one side $= \frac{288\sqrt{10}}{\sqrt{7}}$ in. $= 344\cdot3$, &c., inches.
2. £3182, 6s. $4\frac{208}{605}$d.
3. $33\frac{1}{3}$ p. c.; 200 p. c.

### *Section VII.*

1. £159, 10s. 4·675d.
2. £8, 1s. $6\frac{90}{203}$d.
3. £46, 10s. 2d. nearly.

## No. 5.

1. £3012, 10s.
3. $7908\frac{102}{121}$ miles.
4. 129·2657704239, &c., yrs.
5. £10, 1s. $10\frac{3}{4}$d. +
6. £1, 15s. $5\frac{125}{203}$d.
7. 1000; 2000.
8. $4\frac{1}{11}$ months.
9. (If $\pi = \frac{22}{7}$) $\frac{12\sqrt{7}}{\sqrt{66}} = 3\cdot9$ in. +
10. 88·4 wks., really 89 wks.
12. 4s. $7\frac{49}{85}$d.

## No. 6.

1. 1.
2. $\frac{375}{869}$ in.
3. £162, 8s.
4. 4·90099501 galls.
5. £20, 17s. 11·56224d.
6. $9\frac{1}{4}$ p. c.
7. $50\sqrt{15}$ miles $= 193\cdot649$ miles.
11. 150; 250.
12. 7; £16.

## No. 7.

2. £60, 10s.; £77, 6s. 8d.
3. $\frac{1}{60}$.
4. £4, 0s. $8\frac{82}{110}$d.
5. ·15.
6. 26·601, &c., francs.
7. £8, 8s. $1\frac{7}{8}$d.
8. £649, 7s. $10\frac{3}{4}$d. nearly.
9. £1335, 16s. 8d.
10. 126.
11. £520.
12. 3·74 miles + per hour.
13. £100.

## IV. Examinations at the Birkbeck Literary and Scientific Institution.

### No. 1.

1. £22, 12s. 4d.
2. 3630.
3. 1825 tons 14 cwt. 4 lb.
4. £4306, 4s. 4$\frac{34}{35}$d.
5. May 8th, at 11 hrs. 39$\frac{2}{13}$ min. A.M.
6. £689, 13s. 1$\frac{7}{25}$d. (at simp. int.)
7. £278, 12s. 4·3752d.
8. 71 yds. 4 in.
9. 999·99999999, &c.
10. $\frac{2829}{3920}$.
11. $\frac{11676}{15831}$.
12. Lead 51 lb. 3 oz.; tin 11 lb. 13 oz.
13. 27 hrs. 52′.
14. 17·92.

### No. 2.

1. 1.
2. 998001.
3. 7$\frac{1}{3}$ sq. ft.
4. £33, 1s. 3·84d.
5. 2$\frac{5}{13}$ p. c.
6. £9$\frac{3}{13}$.

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